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Review

The Centenary of the Omori Formula for a Decay Law of Aftershock Activity

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The Omori formula $n(t)=K(t+c)^{-1}$ and its modified form $n(t)=K(t+c)^{-p}$ have been successfully applied to many aftershock sequences since the former was proposed just 100 years ago. This paper summarizes studies using these formulae. The problems of fitting these formulae and related point process models to observational data are discussed mainly. Studies published during the last 1/3 century confirmed that the modified Omori formula generally provides an appropriate representation of the temporal variation of aftershock activity. Although no systematic dependence of the index p has been found on the magnitude of the main shock and on the lowest limit of magnitude above which aftershocks are counted, this index (usually $p=0.9$–1.5) differs from sequence to sequence. This variability may be related to the tectonic condition of the region such as structural heterogeneity, stress, and temperature, but it is not clear which factor is most significant in controlling the p value. The constant c is a controversial quantity. It is strongly influenced by incomplete detection of small aftershocks in the early stage of sequence. Careful analyses indicate that c is positive at least for some sequences. Point process models for the temporal pattern of shallow seismicity must include the existence of aftershocks, most suitably expressed by the modified Omori law. Among such models, the ETAS model seems to best represent the main features of seismicity with only five parameters. An anomalous decrease in aftershock activity below the level predicted by the modified Omori formula sometimes precedes a large aftershock. An anomalous decrease in seismic activity of a region below the level predicted by the ETAS model is sometimes followed by a large earthquake in the same or in a neighboring region.

1. Introduction

One hundred years have passed since Omori (1894a,b) proposed a formula to represent the decay of aftershock activity with time. This formula and its modified form have been widely used as one of a few established empirical laws in seismology. Any theory for the origin of aftershocks must explain this law, which is unique for its power law dependence on time. The power law implies the long-lived nature of activity in contrast to the exponential function appearing in most decay laws in physics. To celebrate the centenary of the original Omori formula, this paper reviews studies related to the

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formula, the modified one, and point process models using these formulae, mainly from the viewpoint of model fitting. The physical explanation of the formulae, related observations such as the afterslip on a fault ruptured during the main shock, the acoustic emission after the main fracture of a rock sample, etc., and all sorts of simulation studies are outside the scope of this paper.

Usually aftershocks having a magnitude above a certain fixed level are adopted as data, except in early papers in which all aftershocks felt by a person or recorded instrumentally at a certain station or a network of stations were counted. In this paper, it is assumed that the lowest limit of magnitude, M_z, is fixed and all aftershocks with magnitude equal to or larger than M_z are counted.

We sometimes find difficulty in identifying a shock as an aftershock of a specific main shock. This difficulty is apparently removed if we define an aftershock identification scheme for each main shock (for example, define the size of a space-time window as a function of the main shock magnitude). However a satisfactory definition of such a scheme is again difficult. It may be said, however, that the ambiguity in identifying aftershocks does not affect the conclusion significantly in many studies. It is desirable to use a model in which identification of aftershocks is unnecessary in the estimation of parameters. If a set of data on an aftershock sequence contain background seismicity and/or shocks of other aftershock sequences, we can take such circumstances into the model and estimate the parameters for the aftershock sequence together with the parameters for the background seismicity and/or the other sequences without classifying the individual shocks.

2. The Original Omori Formula

Omori (1894a,b) studied the decrease of half-day and monthly frequencies of felt aftershocks with time following the 1891 Nobi (Mino-Owari), central Japan, earthquake and two other earthquakes in Japan. He showed that the frequency of aftershocks per unit time interval $n(t)$ at time t is well represented by

$$n(t)=K(t+c)^{-1}, \qquad (1)$$

where K and c are constants. Omori used the letter h in place of c, but we use c since it is commonly used in recent years. It is also common for t to be measured from the origin time of the main shock, but Omori used a different time origin in his papers, where $t=0$, $t=1$, $\cdots$corresponds to the first, second,$\cdots$unit time intervals, respectively.

He first tried to fit the exponential function but it was unsatisfactory, and reached the conclusion that Eq. (1) provides a good fit to the data. He considered that Eq. (1) is the simplest case of the equation

$$n(t)=K(t+c)^{-1}+K'(t+c)^{-2}+\cdots \qquad (2)$$

or

$$n(t)=K\{(t+c)+k'(t+c)^2+\cdots\}^{-1}, \qquad (3)$$

and claimed that Eq. (3) adopting the first and second terms is more satisfactory in the case of the 1889 Kumamoto earthquake.

Omori (1900a, 1902) reported that Eq. (1) fitted well to the Nobi aftershocks for a time interval as long as ten years after the main shock. Figure 1 (top) shows the decay of the occurrence rate of felt shocks at Gifu (a city within the aftershock zone) using Omori's data from October 28, 1891, 18 h 00 m JST (0.474 days after the main shock) through the year 1899. Moreover, the rates of felt shocks recorded at JMA's Gifu Observatory during 1900–1991 have been plotted. Figure 1 (bottom) is a graph of the cumulative number of felt shocks using the same sets of data. The curves in the top and the bottom figures represent the Omori formula and its integrated form fitted to these 100 years' data, respectively. Not all the felt shocks at Gifu were the Nobi aftershocks, but the effect of the inclusion of outsiders seems insignificant.

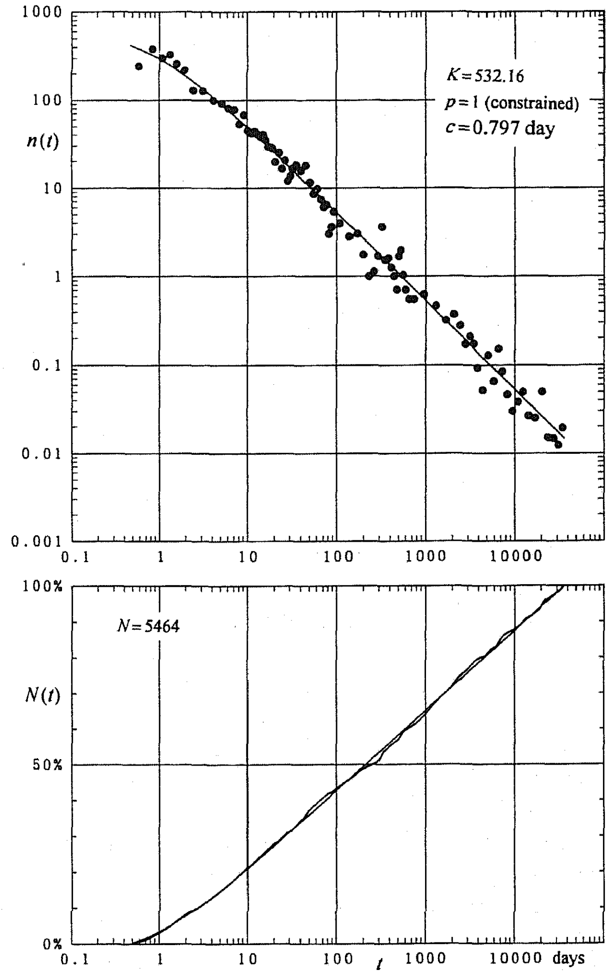


Fig. 1. Occurrence rate (top) and cumulative number (bottom) of felt earthquakes at Gifu after the Nobi earthquake of 1891. Smooth curves represent the Omori formula fitted to the data.

The Omori formula was successfully applied to many other aftershock sequences including those of the 1854 Nankai earthquake (Omori, 1900c), the 1894 Nemuro-Oki earthquake (Omori, 1900b,c), the 1904 Formosa earthquake (Omori, 1906), the 1847 Zenkoji and the 1830 Kyoto earthquakes (Omori, 1908a), the 1909 Anegawa earthquake (Omori, 1909; Imamura, 1910), the 1915 Avezzano earthquake (Friedlander, 1918), the 1922 Shimabara earthquake (Nakamura, 1923), the 1923 Kanto earthquake (Nakamura, 1925; Shiratori, 1925), the 1927 Tango earthquake (Sagisaka, 1927; Nasu, 1929; Jeffreys, 1938), and the 1933 Sanriku earthquake (Matuzawa, 1936).

These papers simply described that aftershock activity of respective earthquakes decreased more or less regularly according to the Omori formula often referred to as the hyperbolic law. It was remarkable that a large aftershock (April 1, 06 h 08 m JST) of the Tango earthquake of March 7, 1927 was accompanied by many aftershocks of its own (called second-order aftershocks or secondary aftershocks), which also fitted the Omori formula (Jeffreys, 1938; also see Utsu, 1970, pp. 215–216). Sagisaka (1927) noted that this large aftershock might be an independent event because other large aftershocks of comparable size were not followed by any appreciable secondary activities.

For more about the original Omori formula and other aftershock studies by Omori, see a review paper by Ikegami (1982).

3. The Modified Omori Formula

3.1 Development of the formula

Hirano (1924) adopted an equation

$$y(x)=b(x+a)^{-c} \tag{4}$$

to represent the daily frequencies of nearby earthquakes recorded at Kumagaya (a city north of Tokyo) from the day of the great Kanto earthquake of September 1, 1923 through December 31, 1923. In this equation, $y(x)$ denotes the frequency of shocks on the xth day, taking September 1 as the first day. He obtained $a=0$, $b=603$, $c=1.6$ for the first 19 days, and $a=4.0$, $b=60$, $c=0.8$ for the later period. His data seems to contain many earthquakes which do not belong to the aftershock sequence of the Kanto earthquake.

Jeffreys (1938) tried to fit $n(t)\propto(t-\beta)^{-1+k}$ to the data of Nasu (1929) on the 1927 Tango aftershock sequence, but he finally set $k=0$ and estimated only the β values for the primary sequence and the secondary sequence starting on April 1.

Utsu (1957) emphasizes that the decay of aftershock activity of several earthquakes is somewhat faster than that expected from the original Omori formula, since the slope of the curve of cumulative number of aftershocks, $N(t)$, plotted against $\log t$ tends to decrease with time (see the bottom diagram in Fig. 2 for a new example). The original Omori formula predicts a constant slope K for large t, because $N(t)$ for Eq. (1) takes the form

$$N(t)=\int_0^t n(s)\,\mathrm{d}s=K\ln(t/c+1)\,. \tag{5}$$

Utsu (1957) showed that the occurrence rate of aftershocks fits the equation

$$n(t)=Kt^{-p} \tag{6}$$

with p around 1.4. Since this function diverges at $t=0$, he recommended the use of

$$n(t)=K(t+c)^{-p} \tag{7}$$

with additional small positive constant c.

The cumulative number $N(t)$ for this equation is expressed by

$$N(t)=K\{c^{1-p}-(t+c)^{1-p}\}/(p-1) \tag{8}$$

for $p\neq 1$. When $p>1$, $N(t)$ tends to a constant level $N_\infty=K/\{(p-1)c^{p-1}\}$ as $t\to\infty$. When $p\leqq 1$, $N(t)\to\infty$ as $t\to\infty$.

Utsu (1961) called Eq. (7) the modified Omori formula and estimated p and c values for 41 aftershock sequences mostly in Japan. Utsu (1969) added the estimates for 10 more sequences. The p values fall in the range from about 0.9 to 1.8, and the values 1.1 to 1.4 are most frequent. No correlation with the main shock magnitude is apparent. The c values scatter from less than 0.01 days to over 1 day, with a median of about 0.3 days (see discussions on the c value in Sec. 6).

Utsu (1961, 1969) considered that the p and c values were parameters characterizing individual main shock-aftershock sequences, and discussed the relation between these and other quantities such as the main shock magnitude, magnitude difference between the main shock and the largest aftershock, and Gutenberg-Richter's b value for aftershock magnitudes.

3.2 *p-value estimates from various sequences*

During the last 33 years, more than 200 p-value estimates have been published in at least 50 papers for aftershock sequences in various parts of the world. They are distributed from 0.6 to 2.5 with a median of about 1.1. Not all of them are quoted here, but several estimates are listed below. Several other papers (Mogi, 1962; Ogata, 1983; Kisslinger and Jones, 1991; etc.) will be referred to in later sections.

$p=0.9\pm0.1$ by Adams and Le Fort (1963) for the 1962 Westport earthquake in New Zealand. This is the first estimate of the p value by seismologists outside of Japan.

$p=1.13$–2.5 by Papazachos *et al.* (1967) for 19 aftershock sequences in Greece in the years 1926–1964. This is the first comprehensive statistical study of aftershocks outside of Japan. The validity of the method for estimating p values is somewhat questionable, because they used log cumulative number versus $\log t$ plots (see the first paragraph of Sec. 5). $p=0.83$–1.86 by Papazachos (1975a) for six aftershock sequences in Greece are more reliable, since these were obtained from ordinary $\log n(t)$ versus log t plots.

$p=1.4$ by Lukk (1968) for the 1965 Hindu Kush intermediate-depth earthquake ($h=219\,km$, $m_B=7.5$). This was the first report that such a deep earthquake was accompanied by many aftershocks obeying the modified Omori relation. According to the graph by Pavlis and Hamburger (1991), p values for this and two other Hindu Kush intermediate-depth earthquakes seem to be nearly equal to or slightly less than 1.

$p=1.14\pm0.06$ by Page (1968) for the great Alaska earthquake of 1964. This is an early example of thorough studies of aftershocks including microaftershocks of a single great earthquake.

$p=0.79$ by Ryall and Savage (1969) for the aftershocks accompanying the nuclear explosion Boxcar in Nevada in 1968. For aftershocks of a small blast in a mine, Iio (1984a) obtained $p=1.12$ for the period from 10 s to 1,000 s after the blast.

$p=0.8$–1.7 by Usami (1975) for aftershocks of 15 historical earthquakes in Japan including the 827 Kyoto, 1498 Meio (Tokai), 1854 Ansei (Nankai), 1855 Edo earthquakes based on the frequencies of felt aftershocks found in historical documents.

$p=0.84$ by Iio (1984b) for aftershocks following a rock fracture event in a mine, which corresponds to an earthquake of $M=-2$. McGarr and Green (1978) observed about 140 aftershocks ($M>-3.5$) within 10 h of each of two rock bursts in a gold mine ($M=1.5$ and 1.2). They noted that the frequency decayed according to the Omori formula ($p=1$).

3.3 Dependence of the p value on the lowest limit of magnitude

If the p value is significantly dependent on the lowest limit of magnitude M_z, the proportion of small to large aftershocks must vary with time. This means that the b value of the Gutenberg-Richter relation, or the mean magnitude of the aftershocks, varies with time.

Some seismologists including Omori (1894a,b) seem to believe that the size of aftershocks gradually drops off with the frequency. Richter (1958, p. 69) attached little importance to the Omori formula, because he thought that its meaning was doubtful due to 'the simple counting aftershocks without regard to their rapidly diminishing magnitudes.' Richter recommended Benioff's strain release curves (Benioff, 1951), but we rather say that its physical meaning is not clear, because the shape of the curve depends on the choice of the minimum magnitude M_z (Utsu and Hirota, 1968). The strain release curve tends to the cumulative frequency curve with decreasing M_z, because the increase in the number of shocks is faster than the decrease in so-called strain release if $b>0.75$ (half of the coefficient in the energy-magnitude relation $\log E=1.5M+\text{const}$). Otsuka (1985) showed two examples indicating that the frequency of felt aftershocks decays more rapidly than that of unfelt aftershocks. The dependence of the decay constant on magnitude level, if it is found in some cases, does not seem to be so systematic.

Utsu (1962) investigated the p values for aftershock sequences of the 1957 Aleutian earthquake ($M_W=9.1$), the 1958 Central Alaska earthquake ($M_S=7.3$), and the 1958 southeastern Alaska earthquake ($M_S=7.9$) by taking three different levels of M_z, and found no evidence of dependence of the p and c values on M_z. This result is consistent with the magnitude stability (constancy of the mean magnitude) during an earthquake sequence reported later by Lomnitz (1966a), Ranalli (1969), Arroyo and Udías (1972), Papazachos (1974), and others.

Independence of p on M_z was also reported by Yamakawa *et al.* (1969) and Hamaguchi and Hasegawa (1970b) for the aftershock sequences of the 1963 southern Kuril earthquake and the 1968 Tokachi-oki earthquake, respectively. Papazachos (1974) obtained almost equal p values ($p \fallingdotseq 0.70$) for the dam-induced Kremasta earthquake of 1966 for three different levels of M_z, 2.3, 2.9, and 3.5. Motoya and Kitagamae

Table 1. Maximum likelihood estimates of the parameters of the modified Omori formula for the aftershocks of the 1993 Hokkaido-Nansei-Oki earthquake for various cutoff magnitudes M_z.

M_z	N	K	p	c (day)
3.2	2,363	906.50±97.52	1.256±0.028	1.433±0.170
3.4	1,582	510.43±54.93	1.250±0.030	0.879±0.124
3.6	902	237.38±26.83	1.242±0.035	0.477±0.088
3.8	449	91.26±10.77	1.235±0.042	0.190±0.052
4.0	210	34.73± 4.68	1.246±0.054	0.075±0.034
4.2	117	16.30± 2.46	1.304±0.071	0.029±0.024
4.4	81	11.79± 2.22	1.310±0.087	0.037±0.033
4.6	54	6.60± 1.34	1.157±0.083	0.004±0.024
4.8	30	3.55± 0.96	1.107±0.106	0.000±0.031
5.0	14	1.59± 0.60	1.283±0.184	0.000±0.035

Data from 0.03 days to 262 days after the main shock have been used. N is the total number of aftershocks with $M \geqq M_z$ listed in this period.

(1971) reported that the p value for the 1970 Hidaka Mountain earthquake increased slightly with increasing M_z, i.e., $p=1.1$ for $M_z \geqq 1.8$ and $p=1.2$ for $M_z \geqq 2.8$. Eaton (1990) studied temporal distribution of the aftershocks of the Coalinga, California, earthquake of 1983 above three magnitude levels $M_z \geqq 1.7$, $M_z \geqq 2.5$, and $M_z \geqq 3.0$. His figure shows that the p value ($p \fallingdotseq 1.0$) is independent of M_z.

Table 1 lists the maximum likelihood estimates (see Sec. 5) with standard errors of K, p, and c for the aftershocks of the Hokkaido-Nansei-Oki, Japan, earthquake of July 12, 1993 ($M=7.8$) for various M_z from 3.2 to 5.0. Data between 0.03 days (43.2 min) and 262 days from the main shock taken from the JMA catalog have been used. Considering the standard errors, no systematic variation of p with M_z is evident, and a value $p \fallingdotseq 1.25$ seems to be appropriate for this sequence. The c value decreases with increasing M_z and becomes zero for $M_z \fallingdotseq 4.8$. This is because the small aftershocks in an early stage of the sequence are missing in the catalog. The c value for this particular sequence seems to be very close to 0, probably less than 0.03 days. Figure 2 shows the curves of $n(t)$ and $N(t)$ fitted to the data of $M_z \geqq 3.2$ and $M_z \geqq 4.0$. The slopes of two $n(t)$ curves are almost equal for large t.

Fluctuation of the b value during an aftershock sequence was occasionally observed (e.g., Gibowicz, 1973), but a clear tendency has been scarcely found for the frequency of larger aftershocks to decay more rapidly than that of smaller aftershocks. No trend was found in the b value during the aftershock sequences studied by Motoya (1970), Hamaguchi and Hasegawa (1970a), Drakopoulos (1971), McGarr (1976) and others.

4. Complex Cases

4.1 Occurrence of secondary aftershock sequences and some other anomalous activities

Some papers reported cases where the original or modified Omori formula did not fit the observed variation in aftershock frequency, or generally speaking, the aftershock activity did not decrease regularly with time.

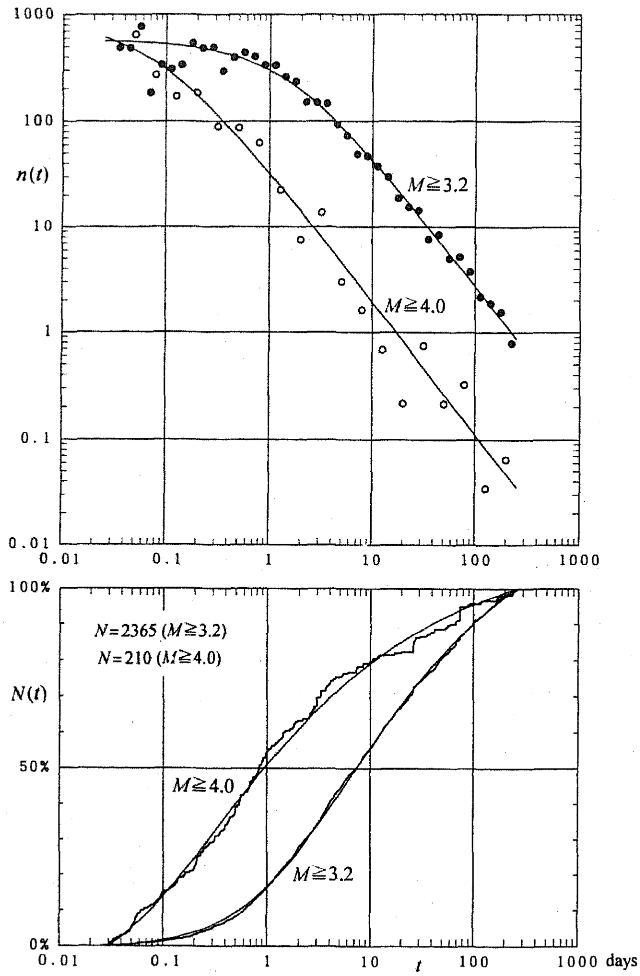


Fig. 2. Decay of the aftershock activity of $M \geqq 3.2$ and $M \geqq 4.0$ of the Hokkaido-Nansei-Oki earthquake of July 12, 1993 ($M=7.8$). Smooth curves represent the modified Omori formula whose parameters are shown in Table 1. It is evident that many aftershocks of $M<4.0$ within 1 day are missing. Almost all aftershocks of $M<4.0$ within 0.1 days have not been detected. Such incomplete detection results in a large c value for $M \geqq 3.2$. Note that the largest aftershock ($M=6.3$, off-fault event) occurred 26 days after the main shock.

For instance, Omori (1900d) reported that the aftershock activity of the Fukuoka double earthquakes of August 10 and 12, 1898 decreased rapidly, but in 1899 the activity became nearly constant, and in 1900 it rather showed a tendency to increase. Tams (1915) pointed out the anomalous increase of aftershock activity of the San Francisco earthquake of April 18, 1906 during July, 1906 and January, 1907. Hamilton and Healy

(1969) described a remarkable increase in aftershock frequency around 20 days after the nuclear explosion Benham.

It is not rare that one or more large aftershocks accompanied by many secondary aftershocks occur in an aftershock sequence. Such multiple sequences can not be represented by a simple formula, but may be expressed by a combination of the modified Omori formulae (Utsu, 1970; Ogata, 1983) such as

$$n(t)=K(t+c)^{-p}+H(t-T_2)K_2(t-T_2+c_2)^{-p_2}+H(t-T_3)K_3(t-T_3+c_3)^{-p_3}, \quad (9)$$

where $H(\cdot)$ denotes a unit step function, i.e., $H(s)=0$ for $s<0$ and $H(s)=1$ for $s\geqq 0$. The first term on the right-hand side represents the aftershock activity directly following the main shock (the primary sequence). The second and third terms represent the secondary aftershocks of large aftershocks occurring at time T_2 and T_3, respectively.

Kisslinger and Hasegawa (1991) studied aftershocks of the Aleutian earthquake of March 21, 1987 ($M_W=6.4$, depth = 105 km) and the Iwate earthquake of January 9, 1987 ($M_W=6.6$, depth = 75 km) recorded by regional high-sensitivity networks. The maximum likelihood estimates of p values are 0.923 and 0.955, respectively. A new sequence of earthquakes started at 64 days after the former earthquake, which fit the modified Omori formula with $p=1.036$ but no large triggering event was found. The latter sequence was also overlapped by a 63-day-long period of increased activity starting 120 days after the main shock. It is not clear what triggered these two secondary activities.

It is also possible that swarm-type activities occur in an aftershock sequence. Moreover, anomalous decreases in aftershock activity lasting for a certain period of time are observed occasionally, which will be discussed in Sec. 9.

4.2 A large main shock occurring within a swarm

Some earthquake swarms are caused by rapid increases of magma or water pressure (sometimes accompanied by the intrusion of such fluids into cracks within the crust). The temporal pattern of seismic activity in such cases is mainly controlled by the temporal behavior of the fluid. When the fluid activity stops, the seismic activity falls off rapidly. If a large earthquake occurs during such a swarm in the same area, the activity following this earthquake may be regarded as its aftershocks. Frequency of such aftershocks does not decrease regularly, because it is actually a mixture of the swarm events and the genuine aftershocks triggered by the large earthquake.

Such a mixed occurrence of a swarm and a main shock-aftershock sequence is occasionally found in the Izu area and other volcanic areas of Japan. The Kita-Izu earthquake of November 26, 1930 is a well known example for which the Omori formula failed to represent the frequency of shocks following the main shock. A large earthquake of $M=7.3$ occurred during an intense swarm and was followed by relatively weak aftershock activity which decayed very rapidly. The level of aftershock activity seemed to fall below the level of swarm activity within a day.

Detailed case studies are available for the Izuhanto-Toho-Oki earthquake sequence of 1980. The main shock of $M=6.7$ occurred 5 days after the commencement of a swarm. After careful relocation and examination of waveforms, Matsu'ura (1983) selected the aftershocks out of the swarm events and found $p=1.9$ and $c=0.07$ days for the aftershock sequence. An independent study by Ishida (1984) based on a different set of data indicated that $p=1.5\pm0.3$.

5. Estimation of Parameters

Formerly, the p and c values were estimated graphically using $\log n(t)$ versus $\log t$ plots. Otherwise only the p value was estimated using Eq. (6) for data of $t \gg c$. Some researchers used the number of shocks $N(t)$ that occurred until time t, or the number of shocks $N^*(t)$ that occurred at and later than t. The curve of $\log N(t)$ or $\log N^*(t)$ against $\log t$ is drawn and the slope s of the curve is measured. Such cumulative number methods often yield very inaccurate estimation, because the true cumulative number is difficult to know owing to incompleteness of data for very early aftershocks or very late aftershocks. Note that the relation between s and p, $s=1-p$, is valid only when $p<1$ for the $\log N(t)$ curve and $p>1$ for the $\log N^*(t)$ curve, respectively.

It is preferable to compute p and c values by the maximum likelihood method using the occurrence times of individual shocks. Maximum likelihood estimates of p values by Ranalli (1969) and Suhadolc (1982) were restricted to the particular case where $c=0$.

If a non-stationary Poisson process is assumed for N aftershocks occurring at time t_i $(i=1, 2, \cdots, N)$ between T_s and T_e with intensity $\lambda(t)=n(t)$, where $n(t)$ is given by Eq. (7) or (9), the likelihood function for the process is written as

$$L=\left\{\prod_{i=1}^{N} \lambda(t_i)\right\} \exp\left\{-\int_{T_s}^{T_e} \lambda(t)\mathrm{d}t\right\}. \quad (10)$$

Ogata (1983) obtained the maximum likelihood estimates of the parameters in Eq. (7) or (9) by maximizing $\ln L$ using Davidon-Fletcher-Powell optimization procedure. For the computer programs, see the last section.

It is sometimes preferable to add a constant term μ to Eq. (7) or (9) to represent background seismicity which may be mixed in the aftershock data. Ogata's method provides a stable solution even if secondary sequences and background seismicity are involved. For the 1927 Tango sequence, which included two secondary sequences starting at $T_2=0.25$ days and $T_3=24.49$ days, he obtained $p=p_2=p_3=1.009\pm0.022$, $c=0.036\pm0.400$, $c_2=0.087\pm0.439$, $c_3=0.011\pm0.441$ ($T_s=0$, $T_e=200$ days). The three p values are constrained to be equal, since AIC is smaller for this case than when p values are separately estimated. AIC is the Akaike information criterion (Akaike, 1974), a powerful tool for model selection.

The maximum likelihood estimates of p and c values in Eq. (7) can also be obtained by numerically solving the simultaneous equations

$$\sum_{i=1}^{N} \ln(t_i+c) - \frac{N}{p-1} - N\frac{\ln(T_s+c)(T_s+c)^{-p+1} - \ln(T_e+c)(T_e+c)^{-p+1}}{(T_s+c)^{-p+1}-(T_e+c)^{-p+1}} = 0, \quad (11)$$

$$p\sum_{i=1}^{N} \frac{1}{t_i+c} - \frac{N(p-1)\{(T_s+c)^{-p}-(T_e+c)^{-p}\}}{(T_s+c)^{-p+1}-(T_e+c)^{-p+1}} = 0. \quad (12)$$

The K value can be determined from the equation

$$K=N(p-1)/\{(T_s+c)^{-p+1}-(T_e+c)^{-p+1}\}. \tag{13}$$

Equations (11)–(13) have been derived from $\partial \ln L/\partial K=0$, $\partial \ln L/\partial p=0$, and $\partial \ln L/\partial c=0$, where L is given by Eq. (10).

6. Remarks on the c Value

Smaller shocks in the early stage of an aftershock sequence are apt to be missing due to overlapping on the seismogram. For strong main shocks, the failure of observing systems at stations close to the epicenter causes lowering of detectability of small events. In such circumstances, the c value may be overestimated. True c values are difficult to estimate accurately unless very careful observation is started immediately after the main shock. When data are taken from ordinary earthquake catalogs, the estimated c value may partially reflect the effect of incomplete detection of small aftershocks shortly after the main shock.

Hamaguchi and Hasegawa (1970b) investigated the frequency of aftershocks of the Iwateken-Oki earthquake of June 12, 1968 ($M=7.2$) from the initial stage of the sequence. The estimated c value varies from 0.99 days to less than 0.01 days corresponding to a 10^3-fold increase in the minimum amplitude level at station KRM. This suggests that early aftershocks with small amplitudes are not completely detected. See also Eaton (1990) and Table 1.

There is an opinion that the c value is essentially 0 and all reported positive c values result from the above-mentioned incompleteness. If $c=0$, $n(t)$ in Eq. (7) diverges at $t=0$. Kagan and Knopoff (1981) explained this difficulty by considering that the main shock is a multiple occurrence of numerous subevents occurring in a very short time interval.

However, positive c values have been obtained for adequately observed and selected data. Yamakawa (1968) noted that the aftershock data of the 1964 Niigata earthquake recorded at TSK (Miyamura and Tsujiura, 1964), which were almost complete for shocks with an amplitude of 5 mm or more occurring more than 0.01 days after the main shock, yielded a positive c value around 0.4 days. Hirota (1969) showed that the frequency of aftershocks of the 1969 Shikotan-Oki earthquake recorded at KMU corrected for missing events is still too few shortly after the main shock (0.02–0.5 days) as compared with the expected frequency from $n(t)=Kt^{-p}$.

Yamakawa (1968) considered that the larger c value reflects the more complex feature of the rupture process of the main shock. If the expansion of aftershock area occurs in an early stage, a relatively large c value may be obtained. For simple aftershock sequences following relatively small main shocks, estimated c values are usually small ($c \leqq 0.01$ days). Examples are found in Motoya (1970, 1974). Lu (1983) reported that the decay of activity was in general slow in the early stages of the aftershock sequences following several large earthquakes in China.

Consider a case where the aftershock activity obeys the law $n(t)=Kt^{-p}$ but the observational data fit the relation $n(t)=K(t+c)^{-p}$ owing to the above-mentioned incompleteness. In this case, $100(1-2^{-p})$ percent of aftershocks is considered to be missing at $t=c$ (50% if $p=1$ and 60% if $p=1.3$). Such a large percentage of missing events seems unlikely for many of the aftershock sequences to which relatively large c values have been estimated.

7. Regional Variation of the p Value

Mogi (1962) estimated p values in Eq. (6) (he used the notation h instead of p) for 31 aftershock sequences in Japan, and discussed their geographical distribution. In general, p values are high for sequences on the Japan Sea side and low on the Pacific Ocean side. Since this distribution is, as a whole, similar to the distribution of surface heat-flow values, he thought that the aftershock activity decays faster ($p \geqq 1.3$) in regions of higher crustal temperature where the stress relaxes faster (Mogi, 1967a).

Kisslinger and Jones (1991) obtained maximum likelihood estimates of p, c, and b values for 39 aftershock sequences in southern California. The p values scatter from 0.7 to 1.8 with a mean of 1.11 ± 0.25. Several high p values are obtained in the Salton Trough where the heat flow is high, and one low p value is obtained in a region where the heat flow is low. Most of the p values around the mean are found in areas of normal heat flow. These observations are generally consistent with Mogi's hypothesis. Creamer and Kisslinger (1993) investigated the relation between the p value and the temperature T at the depth of the hypocenter for 23 aftershock sequences in the Tohoku district of Japan. The lowest T and p values are found in the subducting slab while the highest values are found west of the volcanic front. They obtained a regression $p=0.692+0.000994T$ (°C).

Wang and Wang (1983) studied p values for nine aftershock sequences following large earthquakes ($M \geqq 7.0$) in eastern China after 1966. They emphasize that p values are relatively low ($p=0.56$–0.99) for these earthquakes including the 1976 Tangshan earthquake, the only exception is the 1969 Bohai earthquake for which a high value $p=1.40$ was obtained. Wei *et al.* (1984) also obtained $p=1.5$ for this sequence.

Relatively small p values have also been found for aftershock sequences in intraplate China in a study by Zhao *et al.* (1992). They obtained maximum likelihood estimates of p and c values for 32 aftershock sequences. The p value ranges from 0.63 to 1.54 with a mean of 0.95. These p values are not related to the main shock magnitude.

Matsu'ura (1993) obtained maximum likelihood estimates of K, p, and c for aftershocks of earthquakes with $M \geqq 6.0$ in and near Japan during 1969–1991. The median p values for 30 interplate earthquakes and 24 intraplate ones are 1.16 and 1.12, respectively.

Three major thrust type earthquakes of similar size ($M=7.5$–7.8) occurred in 1940, 1983, and 1993 along the mobile belt running north-south off the Japan Sea coast of northern Japan. The maximum likelihood estimates of p value are, from north to south, $p=1.55$ for the 1940 Shakotan sequence (Matsu'ura, unpublished data, 1994), $p=1.25$ for the 1993 Hokkaido-Nansei-Oki sequence (Subsec. 3.3), and $p=1.04$ for the 1983 Nihonkai-Chubu sequence (using aftershocks of $M \geqq 3.5$ listed in the JMA catalog between 0.1 days and 3,200 days from the main shock). Heat flow data are not so abundant in this zone, but no appreciable differences are evident among the source regions of these earthquakes. The p values may correlate with the degree of heterogeneity of the fault zone of the main shock as suggested by Mikumo and Miyatake (1979) in their simulation study of seismic activity. Comparison of the seismic intensity distribution with surface wave magnitude suggests that the radiation of high-frequency seismic waves were relatively weak, medium, and strong for the 1940, 1993, and 1983 earthquakes, respectively. However, the data are too few to discuss the relation between the p value and the fault heterogeneity or complexities in faulting, stress drop of the main shock, regional level of crustal stress, etc.

Regional variation of the p value can also be studied by using superposed sequences (see next section) and the ETAS model (see Sec. 13).

8. p Values from Superposed Sequences

There are many earthquakes followed by only a small number of recorded aftershocks. It is impossible to estimate p and c values for each of these cases. If these aftershocks can be regarded as samples from a population obeying the modified Omori relation, a set of superposed occurrence times measured from each main shock must fit the modified Omori formula at least approximately. p and c values have been estimated for such superposed sequences by several researchers.

Utsu (1969) showed that the time interval t_1 between the main shock and the largest aftershock for about 250 aftershock sequences distributed according to the Omori formula ($p \fallingdotseq 1$) for t_1 from 0.01 to 20 days. The frequency of t_1 falls more rapidly after about 20 days. The time distribution of the largest aftershocks was studied by Papazachos (1975a) and Tsapanos *et al.* (1988) using the cumulative number method (see the first paragraph of Sec. 5). These papers showed that $s=-0.23$–0.3 (i.e., $p=1.23$–1.30).

Papazachos (1974) obtained $p=1.13$ for a superposed sequence of 2,544 aftershocks from 37 earthquakes in the region of Greece with magnitude $M \geqq 5.5$ (for 1911–1965) or $M \geqq 5.0$ (for 1966–1972). Each sequence includes at least 17 aftershocks.

Davis and Frohlich (1991) studied p values for superposed sequences. They selected aftershocks with the single-link cluster method. This method links earthquakes occurring within 40 ST-km (space-time kilometers, assuming that 1 day in time corresponds to 1 km in space). Any pair of earthquakes with a space-time distance of 40 ST-km or shorter should belong to the same cluster. A total of 47,489 earthquakes of $M_b \geqq 4.8$ taken from the ISC catalog were used. A maximum likelihood estimate $p=0.868 \pm 0.007$ (c was fixed at 0.03 days) was obtained from the data from $t=0.1$ to 20 days. For $t>20$ days the frequency decays nearly exponentially. For shallow subduction zone earthquakes $p=0.890 \pm 0.009$, for ridge-transform fault earthquakes $p=0.928 \pm 0.024$, and for deep earthquakes (depth >70 km) $p=0.539 \pm 0.022$. For shallow subduction zone groups, p values of 0.777, 0.832, and 0.831 were obtained for superposed sequences of $N=1$, $N \leqq 2$, and $N \leqq 5$, respectively, where N is the number of aftershocks in a sequence. Rapid decay for $t>20$ days may be mainly due to the scheme of aftershock selection. The above scheme seems to make it difficult to include late aftershocks.

White and Reasenberg (1991) obtained p values for microearthquakes in the Garm area of Tazihkistan by superposing the data for several periods and areas. They obtained a small value for the Peter-I fault zone ($p=0.77$), and relatively large values for regions north and south of the fault zone ($p \fallingdotseq 1.0$).

Utsu (1992) studied p and c values for superposed aftershock sequences in Japan based on the JMA catalog of shallow earthquakes between 1926 and 1991. Using the data of 0.01 days $\leqq t \leqq$ 100 days and $M \geqq 4.0$, he obtained $p=0.919$ and $c=0.485$ for $N \geqq 1$, $p=1.020$ and $c=0.107$ for $N \geqq 5$, and $p=1.070$ and $c=0.211$ for $N \geqq 20$. These values were obtained for the data to which some careful collation was done to remove background seismicity. The small p values especially for the case of small N suggest that the effect of background seismicity has not completely been removed.

Shaw (1993) obtained a p value slightly less than 1 for a superposed sequence of 228 main shocks of M_L 3 to 6 using the USGS central and northern California catalog ($M_L \geqq 1.5$, 1969–1990).

As described above, small p values have often been reported for superposed sequences. The superposed sequences consist of mostly small-sized sequences (one or a few aftershocks). A portion of these may not be real aftershocks; they may only represent background seismicity. From our experience, well developed aftershock sequences in Japan (say $N \geqq 50$) have p value near 1.0 or greater. If such sequences are superposed, the composite sequence also has a p value near 1.0 or greater.

It is possible to estimate the average p and c values for earthquakes in a region using the ETAS model (see Subsec. 12.3) without selecting aftershocks. p Values for shallow seismicity estimated on the basis of the ETAS model are not so small. They usually fall between 0.9 and 1.4, if the number of earthquakes used is several hundred or more.

9. Anomalies in Aftershock Rate and Their Significance in Earthquake Prediction

In Sec. 4, we have discussed cases in which seismic activity following a main shock can not be represented by a single formula due to mixing of different series of activities. In this section, we review cases in which aftershock activities, as a whole, decay according to the modified Omori formula but depart temporarily from the formula causing a relative quiescence or activation.

Gu *et al.* (1979) fitted data on 28 aftershock sequences to the relation $\log \tau = A \log t + B$, where τ is the time interval between successive aftershocks at time t from the main shock. They considered that if $\log \tau$ is related linearly to $\log t$, the sequence is normal and unusually large events are not expected. When the relation deviates from a straight line on the log–log plots, a large event will occur and its time can be predicted from the plots. Since this linearity corresponds to the modified Omori formula for $t \gg c$, the deviation from the straight line means the departure of the occurrence rate from the formula.

Fu (1981) reported that the aftershock activity of the 1975 Haicheng earthquake decayed very rapidly after an M6.0 earthquake on May 18, 1978, which occurred in a seismic gap within the aftershock zone. Similar changes in the decay rate were observed in the aftershock sequences of 1966 Xingtai and 1978 Tangshan earthquakes.

Wang (1979), Li *et al.* (1980), and Zhou *et al.* (1982) reported the relative increase in the rate of aftershocks of the 1966 Xingtai earthquake one to two years prior to some large earthquakes in north China such as the 1969 Bohai earthquake and the 1975–1976 Haicheng-Tangshan series. They suggested that the aftershock activity is a good measure to monitor the change in the tectonic stress of the region.

According to Xu (1984), Yu S.-J. reported in 1981 that when the decay of aftershock activity became slower, a large aftershock followed. This pattern was observed in seven large earthquakes of $M \geqq 7$ in China since 1966 including the Longling, Tonghai, Xingtai, Songpan, and Tangshan earthquakes. The Bohai and Haicheng earthquakes were exceptions. Wang and Wang (1983) reported cases in which the aftershock activity became anomalously low during a few to ten days prior to a large aftershock, and it returned to the normal level after the large aftershock. The hypocenters of aftershocks before a large aftershock concentrated near the hypocenter of the forthcoming large aftershock, or a gap was created around it.

Ohtake (1970) observed the aftershock sequences following a couple of earthquakes near Kamikochi, central Japan, on August 31 ($M=4.7$) and September 2 ($M=5.0$), 1969. A remarkable decrease in aftershock activity began 0.6 days after the former shock. The low activity continued until the occurrence of the latter shock. A similar feature was reported by Robinson (1994) in the case of Weber, New Zealand, earthquakes of $M_L=6.1$ and 6.2 on February 19 and May 13, 1990. Aftershock activity of the former decayed remarkably during a 35-day period prior to the latter.

Schenkova *et al.* (1982) reported three cases of abnormally decreased aftershock activity followed by a large event. The aftershock activity of the Friuli, Italy, earthquake ($M=6.4$) in May of 1976 became quiescent near the end of August, and it revived gradually in September. A series of destructive earthquakes occurred on September 11 ($M=5.9$) and 15 ($M=6.1$ and 6.0). A large foreshock ($M=5.9$) of the 1978 Thessaloniki earthquake in Greece was followed by its aftershocks and their activity became quiescent a few days before the main shock ($M=6.6$). A similar quiescence in aftershock activity following a large foreshock was observed for the 1979 Monte Negro earthquake of Yugoslavia.

Matsu'ura (1986) closely investigated the precursory decrease and recovery in aftershock activities before large aftershocks and proposed a method to detect such precursors statistically. She studied aftershock activities of large main shocks ($M \geqq 7.0$) which occurred in Japan and were accompanied by large aftershocks of magnitude $M-1.2$ or more, where M is the main shock magnitude. Most of these large aftershocks were accompanied by considerable secondary aftershock activities. Out of eighteen examples from nine such sequences, fourteen cases show a precursory decrease from (and recovery to) the level expected from the modified Omori formula. In the case of the largest aftershock ($M=6.3$) which occurred 22.5 h after the 1984 Western Nagano earthquake ($M=6.8$) and the large aftershock off Katsuura ($M=7.3$) which occurred 24 h after the 1923 Kanto earthquake ($M=7.9$), the quiescence was especially clear. Even for cases where the change is not so clear at a glance, the proper choice of the magnitude threshold and use of a model selection method for point-processes make it possible to detect quiescence. If the aftershock activity actually changed, *AIC* for the model using different parameter values for the modified Omori formula before and after the time of change (onset of the quiescence) becomes significantly smaller than *AIC* for the model which ignores the change. When the frequency-linearized time (which is equivalent to the theoretical cumulative number of aftershocks computed from the modified Omori formula with the estimated parameter values) is used in the time axis, the precursory change can be demonstrated more obviously.

Figure 3 shows a recent example of the precursory quiescence and recovery in an aftershock sequence. A pair of $M6.9$ earthquakes (2 min apart) occurred on July 18, 1992 off the coast of Iwate Prefecture, Japan. A large aftershock of $M6.3$ occurred on July 29 (10.83 days after the first main shock), which was accompanied by considerable secondary aftershock activity. A noticeable decrease in aftershock activity began 0.7 days

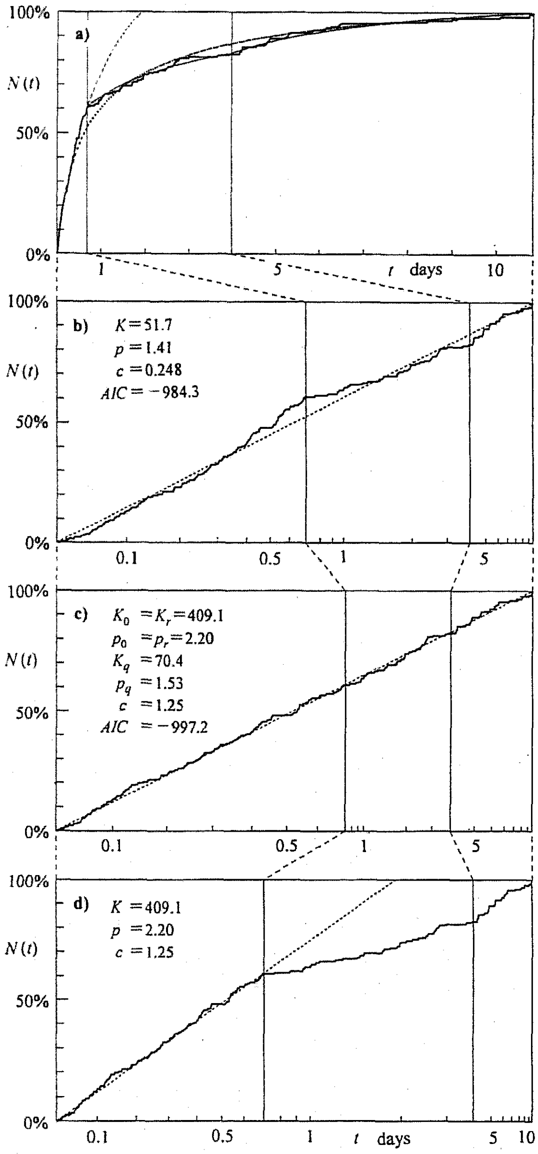


Fig. 3

after the main shock pair (see a kink in the cumulative number curve in Fig. 3(a)). When the single modified Omori formula is applied to the data (shocks of $M \geqq 3.3$ occurring between 0.01 days and 10.83 days from the first main shock), we obtain the maximum likelihood estimates of the parameters given in Fig. 3(b). $AIC = -984.3$ for this simple model. The best model for the same data can be achieved by using the modified Omori formula with different parameter values during the quiescent period (0.7 days $\leqq t \leqq$ 4 days). The K and p values for the first and last periods are constrained to be equal. The c values are kept equal for the three periods. The parameter estimates are given in Fig. 3(c). AIC ($= -997.2$) for this model is considerably smaller than AIC for the first model. The quiescence and recovery are clearly seen in Fig. 3(d) in which the frequency-linearized time axis based on the first part of the best model is used for the whole period.

Zhao *et al.* (1989) applied Matsu'ura's method to aftershocks of the Haicheng, Tangshan, Songpan, and Longling earthquakes in China and some of their foreshocks and obtained the similar results. Latoussakis *et al.* (1991) applied the same method to some Greek earthquakes.

Schreider (1990) who discussed the precursory quiescence in his own manner commented that aftershock activity also became quiescent before some large aftershocks. According to Levshina and Vorobieva (1992), I. Vorobieva and T. Levshina in 1991 made an algorithm which distinguished aftershock sequences containing large aftershocks from those not containing large ones on the basis of the seismicity before the main shock and in the early stage of the aftershock sequence. They applied this algorithm to 42 earthquakes of the world, and succeeded in 36 cases. However, we do not yet know the details of their work.

10. Duration of Aftershock Activity

It has been known since the beginning of this century that aftershock activities following large earthquakes continue ten years or more (e.g., Omori, 1902, see Fig. 1). It seems reasonable to define the duration of aftershock activity as the time required

Fig. 3. Quiescence and recovery in the aftershock sequence of the 1992 Iwateken-Oki earthquake pair (two M6.9 events 2 min apart). 173 aftershocks of $M \geqq 3.3$ from 0.01 to 10.83 days taken from the JMA catalog are used. A large aftershock of M6.3 occurred at 10.83 days (right end). Two vertical lines indicate the times of beginning and end of the quiescence (0.7 days and 4.0 days), respectively. (a) Cumulative number on a linear time axis. Dotted, thin, and broken lines represent theoretical curves calculated from the simple model, the best model, and the first part of the best model, respectively (see text). (b) Cumulative number on the transformed (frequency-linearized) time axis based on the simple model. Broken straight line is the theoretical curve. (c) Cumulative number on the transformed time axis based on the best model. (d) Cumulative number on the transformed time based on the first part of the best model. Theoretical curve (broken line) corresponds to the broken line in (a).

for aftershock activity to decrease to the level of background seismicity. The aftershocks may continue after this time but they become indistinguishable from the background.

There are several ways to estimate the background seismicity of the aftershock zone. Shiratori (1925) assumed the background seismicity was equal to the average seismicity observed before the main shock. Watanabe (1989) compared the aftershock activity with seismic activity of the surrounding region of the aftershock zone. He considered that the aftershock activities of the 1891 Nobi, 1927 Tango, 1943 Tottori, 1948 Fukui, 1962 Miyagi, 1983 Nihonkai-Chubu earthquakes still continue, because the microearthquake activity within the aftershock zones of these earthquakes are higher than those in the surrounding regions.

The following method provides the duration using the level of background seismicity μ. We fit a set of aftershock data to two models $n(t)=K(t+c)^{-p}$ and $n(t)=K(t+c)^{-p}+\mu$ and compute the maximum likelihood estimates of the parameters and the *AIC* values for each model. If $\mu>0$ for the second model and the *AIC* for the second model is smaller than the first, the duration t_d can be defined by $K(t_d+c)^{-p}=\mu$, where K, p, c, and μ are the estimates for the second model.

The duration based on the above definitions is dependent on the regional level of background seismicity. To avoid such dependence, some authors defined the duration as the time when the rate decreased to some fixed level, say 1 event/day, for a given threshold magnitude M_z (Zhou *et al.*, 1982), or the time of the last aftershock with a magnitude $M-D$, where M is the main shock magnitude and D is a positive constant, say 1.7 (Fu, 1982).

Ogata and Shimazaki (1984) estimated the time when the aftershock activity merged into the normal activity for the aftershock sequence accompanying the 1965 Aleutian earthquake ($M_W=8.7$). The largest aftershock ($M_W=7.6$) 54 days after the main shock was followed by a series of secondary aftershocks. They fitted the aftershock data of $m_b\geqq 4.7$ from 3 h to 1,000 days to the modified Omori formula with a secondary sequence and estimated the parameters. Then the cumulative frequency curve was drawn using the frequency-linearized time and the estimated parameters ($K=82.28$, $K_2=6.12$, $p=p_2=1.079$, $c=c_2=0.176$ days). The curve which is almost a straight line deviates upwards at about 2,200 days from the main shock. They concluded that a transition from aftershock activity to background activity occurred at this time.

11. Other Related Studies

11.1 Time interval between successive aftershocks

Tomoda (1954) reported that the time interval τ between successive shocks in earthquake swarms at Asama Volcano and two aftershock sequences (1927 Tango and 1948 Fukui) had a distribution of the form

$$f(\tau)=k\tau^{-q}, \tag{14}$$

where k and q are constants. For the above two sequences $q=1.23$ and 1.48, respectively.

Senshu (1959) showed that if the aftershocks are represented by a non-stationary Poisson process whose intensity is given by Eq. (6), the distribution (14) can be derived directly from Eq. (6) and the relation between p and q should be

$$q = 2 - p^{-1} . \tag{15}$$

q values for various sequences were reported by several researchers including Senshu (1959), Comminakis *et al.* (1968), Hirota (1969), and Utsu (1970). These q values are usually slightly larger than the q values calculated from Eq. (15) using p values estimated from observation.

11.2 Dependence among aftershocks

Jeffreys (1938), in his study of aftershocks of the 1927 Tango earthquake, concluded that the aftershocks were mutually independent events. Lomnitz (1966b) and Page (1968) found that there was clustering in small aftershocks, but larger aftershocks were independent events. We occasionally notice that aftershocks, especially small ones, occur in clusters. The small difference between observed and predicted q values mentioned above may reflect a weak tendency of clustering in aftershocks.

In the ETAS model (see Sec. 14), every aftershock may produce its own aftershocks. When the magnitude is assigned to each aftershock, the ETAS model usually provides a smaller *AIC* than the modified Omori formula for the same aftershock sequence, even when the sequence does not seem to include conspicuous secondary sequences. This indicates that significant dependence exists among aftershocks. However, the modified Omori formula remains to be a useful model for its simplicity.

11.3 Expected number of aftershocks and probability of large aftershocks

If the modified Omori formula is adopted, the expected number N of aftershocks occurring between time T_a and T_b is given by

$$N = \int_{T_a}^{T_b} n(t)\mathrm{d}t = K\{(T_a + c)^{1-p} - (T_b + c)^{1-p}\}/(p-1) . \tag{16}$$

It is natural to relate K to the main shock magnitude M_m and the lower limit of magnitude M_z by

$$\log K = A - b(M_m - M_z) , \tag{17}$$

where b is the coefficient in the Gutenberg-Richter relation A is a constant. Both A and b are considered to be common to all aftershock sequences (Utsu, 1970). The probability P that at least one aftershock of $M \geqq M_z$ occurs between T_a and T_b is given by

$$P = 1 - \exp(-N) . \tag{18}$$

If $N < 0.1$, $P \fallingdotseq N$.

This equation combined with Eqs. (16) and (17) is useful in evaluating the probability of coming large aftershocks. Numerical examples are found in Reasenberg and Jones (1989, 1994), Abe (1991, 1994), and Hosono and Yoshida (1992).

12. Comparison with Other Decay Formulae

Otsuka (1985, 1987) proposed a compound formula

$$n(t)=Ke^{-\alpha t}(t+c)^{-p}, \qquad (19)$$

which is the product of the modified Omori formula and the exponential function. For large t, the effect of the exponential function predominates. Exponential decay of activity in later periods of some aftershock sequences was suggested by Utsu (1957), Mogi (1962), Watanabe and Kuroiso (1970), and Otsuka (1985). In a model by Kagan and Knopoff (1987), activity decreases as $t^{-1.5}$ in the early period and it turns to an exponential decrease later (they did not show actual examples).

Utsu (unpublished data, 1992) applied Eq. (19) to several aftershock sequences and obtained maximum likelihood estimates of the parameters. The α values became zero, indicating that this complication was unnecessary. Equation (19) may be useful in some limited cases. In the model considered by Otsuka (1985) when introducing Eq. (19), the α value is strongly dependent on the magnitude level, i.e., the decrease of activity is very different between small and large aftershocks. This seems somewhat unrealistic.

Some functions used in statistics show a decrease proportional to t^{-1} in a wide range of t, and a more rapid decrease for larger t (curves on log-log graphs are shown in Utsu, 1992). For example, the function (Weibull distribution)

$$f(t)=\alpha\beta t^{\beta-1}\exp(-\alpha t^{\beta}) \qquad \alpha>0\,,\ \beta>0 \qquad (20)$$

has such properties if $\beta \ll 1$, and can be used to represent aftershock activity. The gamma distribution and the log-normal distribution also have similar characteristics in a limited range of parameter values.

Souriau *et al.* (1982) used the Weibull distribution to represent the time distribution of an aftershock sequence in the Pyrenees.

Kisslinger (1993) used the stretched exponential function for the expression of the number of aftershocks that have not occurred until time t

$$N^*(t)=N\exp\{-(t/t_0)^q\}\,, \qquad t_0>0\,,\ 1\geqq q>0 \qquad (21)$$

where $N(=N^*(0))$ is the total number of aftershocks, and $N^*(t)=N-N(t)$, where $N(t)$ is the cumulative number of aftershocks at time t. The density function, $\mathrm{d}N(t)/\mathrm{d}t(=-\mathrm{d}N^*(t)/\mathrm{d}t)$, is nothing but the Weibull distribution (20), because $f(t)=\mathrm{d}N(t)/\mathrm{d}t$ if $\alpha=t_0^{-q}$ and $\beta=q$. His estimation of the parameters for 29 aftershock sequences (mostly in California) was such that $q=0.3$–0.6 (median: 0.403), and $t_0=1$–300 days. A comparison of AIC between the modified Omori formula and Eq. (21) indicates that the former fits better in most cases if $T_s=0$. If T_s is set to be some positive value, Eq. (21) became a better model for about half the cases.

13. Application to Foreshock Sequences

It has been known from the early years of seismology that the temporal distribution of foreshocks is quite variable and no proper formula is found to represent it (e.g., Omori, 1908b, 1910; Suzuki, 1985). The foreshock sequences can be classified roughly into the following three types: Type 1, activity increases towards the main shock; Type 2, foreshock sequence itself has the form of a main shock-aftershock sequence or multiple occurrence of such sequences; and Type 3, irregular variation of activity like an earthquake swarm. Mogi (1967b) classified foreshock sequences into C and D types. The C type (the main shock occurs in a gradually increasing stage of foreshock activity) corresponds to type 1, and the D type (the main shock occurs in a decreasing or decreased stage of foreshock activity) corresponds to types 2 and most of type 3.

The proportion of the three types is not known exactly, but the examination of foreshock data (e.g., Suzuki, 1985) reveals that foreshock sequences of type 1 are rather rare.

For type-2 sequences, p values can be obtained in the same manner as aftershock sequences. Liu (1984) obtained unusually small p values ($p=0.3$–0.8) for 7 out of 14 type-2 sequences he examined. His method of determining p values seems inaccurate as Page (1986) commented.

For type-1 sequences, two types of increasing function have been tested by several researchers: the exponential function by Comminakis *et al.* (1968) and Ke *et al.* (1977), and the power law type function

$$n(t)=K(t_0-t)^{-p} \tag{22}$$

by Papazachos (1973). If the time is measured reversely from the main shock, Eq. (22) takes the same form as the modified Omori formula. Papazachos (1973) obtained high p values ranging from 1.53 to 2.60 for four foreshock sequences of dam-induced earthquakes. It is hard to understand why he included the number of aftershocks occurring within a half unit time (5/2 days) from the main shock among the foreshock data.

Varens (1989) proposed an equation for seismic moment release in a foreshock sequence

$$\mathrm{d}(\textstyle\sum\sqrt{M_o})/\mathrm{d}t=C(t_f-t)^{-n}, \tag{23}$$

where $\sum\sqrt{M_o}$ represents the sum of the square roots of seismic moment (or energy) released until time t, and C, t_f and n are constants to be estimated from the observational data. The main shock is expected to occur at $t=t_f$. Similar methods have been used in predicting volcanic eruptions for more than 30 years by several volcanologists.

p values for superposed foreshock sequences have been obtained by several researchers (Papazachos, 1974, 1975b; Jones and Molnar, 1979; von Seggern *et al.*, 1981; Davis and Frohlich, 1991; Utsu, 1992). It is puzzling that the superposed sequence fits the modified Omori formula with $p=0.7$–1.3 fairly closely in most cases, though individual well-developed foreshock sequences are mostly irregular and do not fit the reversed modified Omori formula at all.

14. Point Process Models Incorporating the Modified Omori Formula

14.1 The trigger model

Since the aftershocks account for a considerable portion of shallow seismicity, their effect must be included in the point process modeling of shallow seismicity. A Neyman-Scott type model proposed by Vere-Jones and Davies (1966) in a study of seismicity of New Zealand was the first of such models (see also Vere-Jones, 1970).

In this model the seismicity consists of two kinds of events, primary and secondary. The primary events (main shocks) distribute as a Poisson process with constant occurrence rate μ. Each primary event occurring at time t_i triggers a series of secondary events (aftershocks) whose intensity is expressed by

$$g(t)=Af(t-t_i)\,, \qquad \text{for } t\geqq t_i\,, \tag{24}$$

$$g(t)=0\,, \qquad \text{for } t<t_i\,. \tag{25}$$

Here, $f(t)$ is a normalized function, i.e., $\int_0^\infty f(t)\mathrm{d}t=1$. The total number A of secondary events triggered by a primary event is a random variable with mean a and variance v. To apply this model, the functional form of $f(t)$ must be specified, but it is not necessary to identify individual events as primary or secondary.

Vere-Jones and Davies (1966) tried the exponential and the modified Omori type functions for $f(t)$. They showed that the latter (p is fixed to 1.25) is more appropriate for New Zealand seismicity by comparing the theoretical and observed spectral curves.

A generalized Poisson model proposed by Shlien and Toksöz (1970) is a simplified trigger model where the delta function is used for $f(t)$, i.e., $f(t)=\delta(t)$. This model was partially successful to represent certain properties of seismicity, and was used by several researchers. The trigger model was tried by Utsu (1972), Sase (1974), and Hawkes and Adamopoulos (1973). The first two papers used the modified Omori type function, while the last paper used the sum of two exponential functions representing short- and long-term components.

In the trigger model, an aftershock never triggers its own aftershock activity. Magnitudes of individual events are not considered, so that the size of the aftershock sequence A is not related to the main shock magnitude. The maximum likelihood method can not be used in parameter estimation, because the ordinary likelihood function is too complicated to be computed. Instead, Hawkes and Adamopoulos (1973) calculated the approximate spectral likelihood for the model.

14.2 The ETAS model

Hawkes and Adamopoulos (1973) also compared the above trigger models with a self-exciting point process model in which every event can produce offspring events. The rate of occurrence at time t is expressed by

$$\lambda(t)=\mu+\sum_{t_i<t} g(t-t_i)\,, \tag{26}$$

where μ and $g(t-t_i)$ represent the constant rate seismicity and the rate of activity at time t triggered by an event at time t_i, respectively. They adopted the sum of two exponential functions for $g(t)$. This model did not surpass the trigger model with the sum of two exponential functions, when both models were compared using earthquake data in 14 regions of the world.

The ETAS (epidemic type aftershock sequence) model (Ogata, 1986, 1988, 1989, 1992, 1994) is a self-exciting point process model which covers all the weak points of the trigger model mentioned in the last paragraph of Subsec. 14.1.

This model deals with the earthquakes of magnitude M_z and larger occurring in a region during a period of time from T_s to T_e. The total number of such earthquakes is denoted by N. The seismic activity $\lambda(t)$ includes Poissonian background activity with occurrence rate μ, as given by Eq. (26). Every earthquake is followed by its aftershock activity, though only shocks of magnitude M_z and larger are included in the data set. The aftershock activity is represented by the modified Omori formula. The rate of aftershock occurrence at time t following the ith earthquake occurring at time t_i with magnitude M_i is given by

$$g(t-t_i)=K\exp\{\alpha(M_i-M_z)\}(t-t_i+c)^{-p}, \qquad \text{for} \quad t>t_i \tag{27}$$

where K, α, c, and p are constants common to all aftershock sequences.

The cumulative number of earthquakes at time t since T_s is given by

$$\Lambda(t)=\int_{T_s}^{t}\lambda(s)\mathrm{d}s=\mu(t-T_s)+K\sum_{t_i<t}\exp\{\alpha(M_i-M_z)\}\{c^{1-p}-(t-t_i+c)^{1-p}\}/(p-1)\,. \tag{28}$$

Five parameters μ, K, c, α, and p represent characteristics of seismic activity of the region. Maximum likelihood estimates of the five parameters can be obtained by maximizing the likelihood function L which is given in the same form as Eq. (10). The Davidon-Fletcher-Powell optimization procedure generally provides a stable solution. See the last section for the computer program.

Most p and c values obtained for various earthquake data sets fall in the range between 0.9 and 1.4, and between 0.003 and 0.3 days, respectively (Ogata, 1986, 1988, 1992). The parameter α, mostly ranging from 0.2 to 3.0, measures an efficiency of a shock with a certain magnitude in generating its aftershock activity. Swarm-type activity usually has a small α value. If α is large, it represents typical main shock-aftershock activity. Utsu (unpublished data, 1993) divided Japan into 16 regions and applied the ETAS model to shallow earthquakes (depth $<$ 100 km) in each region during the years 1926–1992. To keep temporal homogeneity of data, the lowest limit of magnitude M_z was chosen as 4.45, 4.95, or 5.45 depending on the region. The p and c values for the 16 regions range from 0.95 to 1.22 and from 0.003 to 0.28 days, respectively (an example is shown in Fig. 4).

The comparison between the trigger model and the ETAS model cannot be made by using *AIC*, because of the difficulty of the likelihood computation for the trigger model. However, computation of *AIC* for the restricted trigger model (Ogata, 1986, 1988) indicates the superiority of the ETAS model over the trigger model for all sets of data tested.

14.3 Residual analysis from the ETAS model

If the observed rate of occurrence is compared with the ETAS model fitted to the data, a period of anomalously decreased or increased seismic activity (relative quiescence or activation) can be recognized easily. To demonstrate such departure, we use transformed (frequency-linearized) time τ defined by $\tau=\Lambda(t)$ where $\Lambda(t)$ is given by Eq. (28). The length of the whole period is N (the total number of events) on the transformed time. If the observed frequency of events in an interval of length h on the

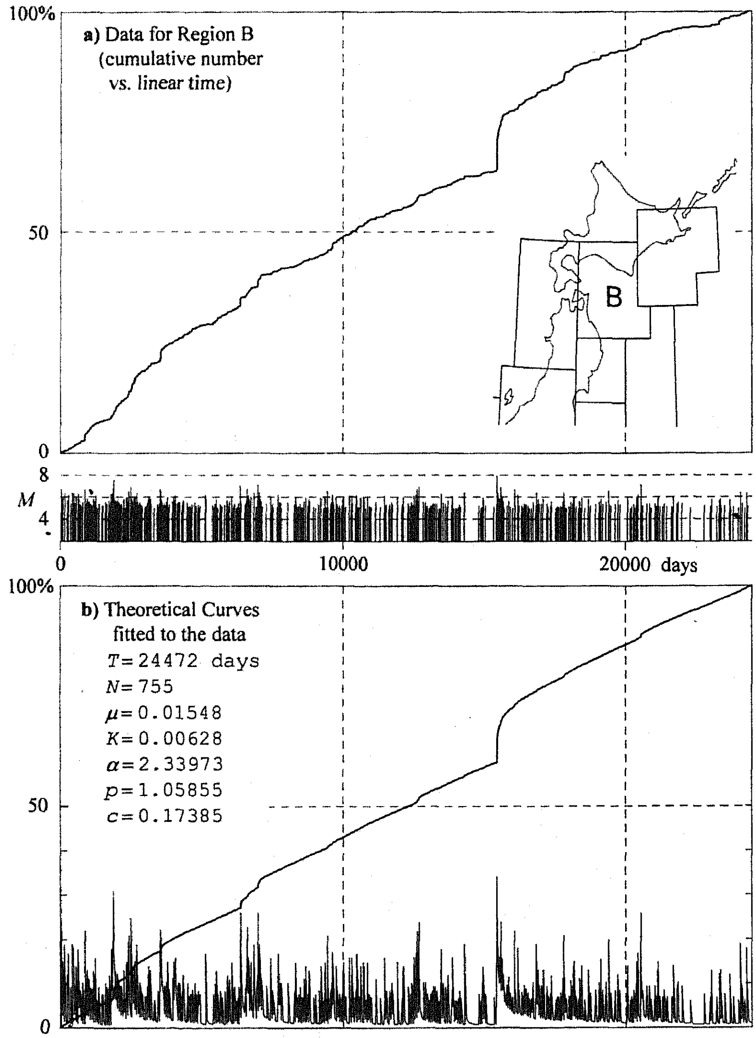
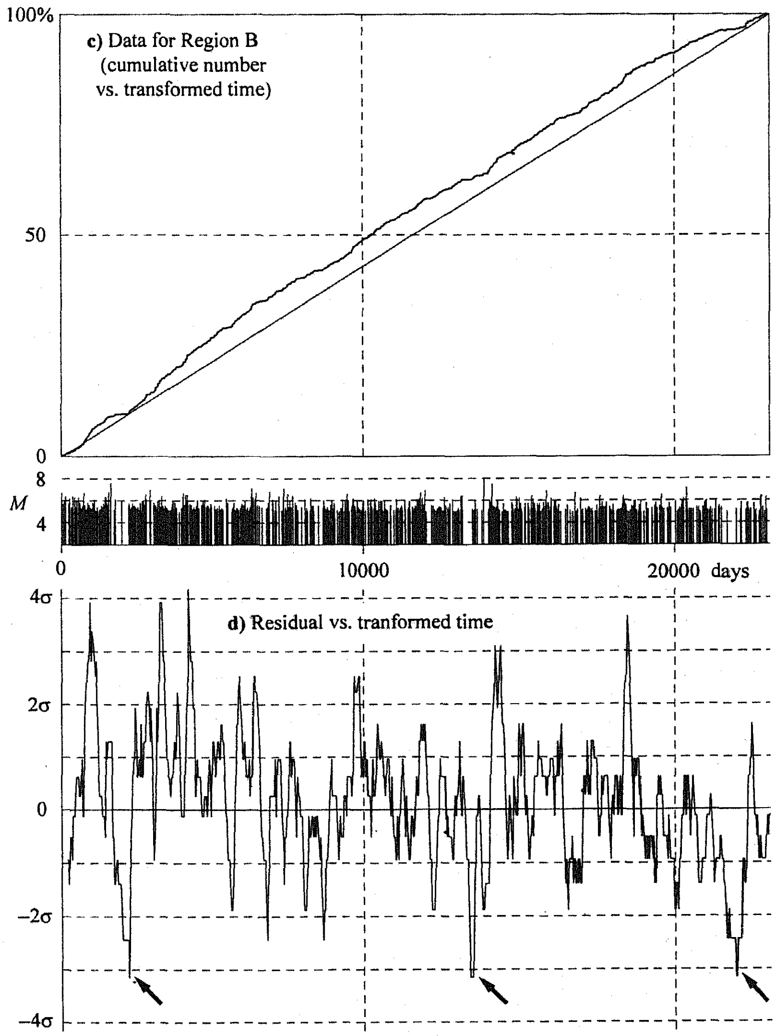


Fig. 4. ETAS model applied to shallow earthquakes (depth$\leqq$100 km) of $M \geqq 5.0$ vs. time diagram (100% corresponds to the total number $N=755$) and of cumulative number $\Lambda(t)$ and intensity $\lambda(t)$ using the maximum likelihood the time axis is scaled using the transformed (frequency-linearized) time. The scale. (d) Residual δn vs. the transformed time. The residual is defined by h in the transformed time. By the definition of the transformed time, h is the has been transformed so that the distribution of δn becomes approximately of remarkable quiescence are indicated by arrows.

transformed time is significantly smaller or larger than h (which is the expected frequency of events in that interval), we recognize respectively relative quiescence or activation in that interval (see Fig. 4(d)).



in region B (see inserted map) for the years 1926–1992. (a) Cumulative number magnitude vs. time diagram in linear time. (b) Computed theoretical curves estimates of the parameters shown in the figure. (c) Similar diagram as (a) but theoretical cumulative curve become a straight line (the diagonal) on this time $\delta n = n - h$, where n is the observed number of events in a time interval of length theoretical number of events in the interval of length h. The vertical scale normal with standard deviation of σ. See Ogata (1988) for details. Three cases

More than 50 papers on precursory seismic quiescence have been published since Inouye (1965) reported this phenomenon for the 1964 Niigata and 1952 Tokachi-Oki earthquakes. Ogata (1992) reported the relative quiescence using the ETAS model before some great earthquakes including the 1960 Chilean, 1957, 1965 and 1986 Aleutian, and

1952 and 1968 Tokachi-Oki earthquakes.

Figure 4 shows the result for one of the 16 regions of Japan mentioned previously to which the ETAS model was fitted. Three periods of relative quiescence are seen (indicated by arrows in Fig. 4(d)). The first one around 1931 is due to deficiency of aftershocks of the 1931 earthquake ($M=7.6$). This earthquake was followed by relatively few aftershocks for its large magnitude. This may be due to somewhat deep hypocenter and/or overestimated magnitude of the main shock. The second one before the 1968 Tokachi-Oki earthquake ($M=7.9$) may be a precursory quiescence. The third one around 1988 may be related to a large earthquake swarm in the fall of 1989 near the southeastern border of this region. The swarm included an $M=7.1$ earthquake. This event might be a greater one as Kawasaki *et al.* (1992) suggested from a few strainmeter records that a large aseismic deformation might have taken place there at that time.

15. Conclusion

The decay of aftershock activity with time is generally represented by the modified Omori formula, Eq. (7), with the exponent p usually between 0.9 and 1.5. This unique nature of aftershock activity provides a strong constraint in constructing a physical theory of aftershock generation. In point process modeling of shallow seismicity in the time domain, the effect of the aftershock activity, most suitably represented by the modified Omori formula, must be considered. At present, the ETAS model seems to be useful for representing the general seismicity of a region with relatively few parameters. Parameters of the modified Omori formula and the ETAS model may correlate with tectonophysical conditions (structural heterogeneity, stress state, temperature, etc.), therefore they may vary spatially and in some cases temporally. A decrease of the observed activity from the level predicted by the modified Omori formula or the ETAS model may be followed by a large earthquake.

Personal computer programs for maximum likelihood estimation of parameters of the modified Omori formula and the ETAS model will be released as a part of the forthcoming volume of IASPEI Software Library. The main parts of these programs were converted from FORTRAN programs written by Y. Ogata and K. Katsura. Some additional functions were provided by T. Utsu. An algorithm developed for high-speed computation of the ETAS model parameters (Ogata *et al.*, 1993) is adopted in one of these programs.

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