A Klee-Minty Example

Klee-Minty example number n is

$$
\begin{array}{ll}
\text{maximize} & \displaystyle\sum_{j=1}^{n} 10^{n-j} x_j \\
\text{subject to} & \displaystyle 2\sum_{j=1}^{i-1} 10^{i-j} x_j + x_i \le 100^{i-1} \text{ for } 1 \le i \le n \\
& \text{all } x_j \ge 0
\end{array}
$$

Using the most-negative-entry pivoting rule, this requires $2^n - 1$ pivots to find an optimal solution.

In the case $n = 3$, the problem is:

$$
\begin{array}{lrrrcr}
\text{maximize} & 100x_1 & +10x_2 & +x_3 & & \\
\text{subject to} & x_1 & & & \le & 1 \\
& 20x_1 & +x_2 & & \le & 100 \\
& 200x_1 & +20x_2 & +x_3 & \le & 10000 \\
& \multicolumn{5}{l}{x_1, x_2, x_3 \ge 0}
\end{array}
$$

Initial tableau:

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	-100	-10	-1	0	0	0	0	$=$	z
0	1	0	0	1	0	0	1	$=$	s_1
0	20	1	0	0	1	0	100	$=$	s_2
0	200	20	1	0	0	1	10000	$=$	s_3

First pivot: x_1 enters, s_1 leaves the basis.

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	0	-10	-1	100	0	0	100	$=$	z
0	1	0	0	1	0	0	1	$=$	x_1
0	0	1	0	-20	1	0	80	$=$	s_2
0	0	20	1	-200	0	1	9800	$=$	s_3

Second pivot: x_2 enters, s_2 leaves.

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	0	0	-1	-100	10	0	900	=	z
0	1	0	0	1	0	0	1	=	x_1
0	0	1	0	-20	1	0	80	=	x_2
0	0	0	1	200	-20	1	8200	=	s_3

Third pivot: s_1 enters, x_1 leaves.

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	100	0	-1	0	10	0	1000	=	z
0	1	0	0	1	0	0	1	=	s_1
0	20	1	0	0	1	0	100	=	x_2
0	-200	0	1	0	-20	1	8000	=	s_3

Fourth pivot: x_3 enters, s_3 leaves.

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	-100	0	0	0	-10	1	9000	=	z
0	1	0	0	1	0	0	1	=	s_1
0	20	1	0	0	1	0	100	=	x_2
0	-200	0	1	0	-20	1	8000	=	x_3

Fifth pivot: x_1 enters, s_1 leaves.

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	0	0	0	100	-10	1	9100	=	z
0	1	0	0	1	0	0	1	=	x_1
0	0	1	0	-20	1	0	80	=	x_2
0	0	0	1	200	-20	1	8200	=	x_3

Sixth pivot: s_2 enters, x_2 leaves

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	0	10	0	-100	0	1	9900	=	z
0	1	0	0	1	0	0	1	=	x_1
0	0	1	0	-20	1	0	80	=	s_2
0	0	20	1	-200	0	1	9800	=	x_3

Seventh pivot: s_1 enters, x_1 leaves.

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	100	10	0	0	0	1	10000	=	z
0	1	0	0	1	0	0	1	=	s_1
0	20	1	0	0	1	0	100	=	s_2
0	200	20	1	0	0	1	10000	=	x_3

This is optimal.