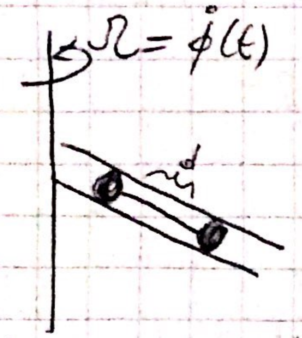
Pauta Ejercicio 2



Definimos las posiciones como:

$$\vec{r}_1 = \rho_1 \hat{\rho}$$
$$\vec{v}_1 = \dot{\rho}_1 \hat{\rho} + \rho_1 \dot{\phi} \hat{\phi}$$
$$\vec{a}_1 = (\ddot{\rho}_1 - \rho_1 \Omega^2)\hat{\rho} + 2\dot{\rho}\Omega\hat{\phi}$$

$\rho_1(0) = 0$; $\rho_2(0) = d$

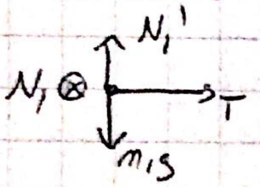
$\dot{\rho}_1(0) = \dot{\rho}_2(0) = 0.$

$\dot{\phi} = \Omega \Rightarrow \ddot{\phi} = 0$

$\rho_2 = \rho_1 + d$

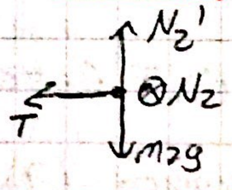
$$\vec{r}_2 = (\rho_1 + d)\hat{\rho}$$
0.5
$$\vec{v}_2 = \dot{\rho}_1 \hat{\rho} + (\rho_1 + d)\Omega \hat{\phi}$$
$$\vec{a}_2 = (\ddot{\rho}_1 - (\rho_1 + d)\Omega^2)\hat{\rho} + 2\dot{\rho}_1 \Omega \hat{\phi}$$

DCL m_1



0.5

DCL m_2



$\Rightarrow \vec{F}_1 = T\hat{\rho} + N_1 \hat{\phi} + (N_1' - m_1 g)\hat{k}$ $\qquad$ $\vec{F}_2 = -T\hat{\rho} + N_2\hat{\phi} + (N_2' - m_2 g)\hat{k}$

a) Por Newton, tenemos que:

$$\vec{F}_1 = m_1 \vec{a}_1 \Rightarrow T\hat{\rho} + N_1\hat{\phi} + (N_1' - m_1 g)\hat{k} = m_1\left[(\ddot{\rho}_1 - \rho_1\Omega^2)\hat{\rho} + 2\dot{\rho}\Omega\hat{\phi}\right].$$

$\Rightarrow$ $\hat{\rho}\rfloor$ $\quad T = m_1(\ddot{\rho}_1 - \rho_1\Omega^2)$ (1)

$\hat{\phi}\rfloor$ $\quad N_1 = m_1 2\dot{\rho}\Omega$ (2) $\qquad$ 0.5

$\hat{k}\rfloor$ $\quad N_1' = m_1 g$ (3)

$$\vec{F}_2 = m_2\vec{a}_2 \Rightarrow -T\hat{\rho} + N_2\hat{\phi} + (N_2' - m_2 g)\hat{k} = m_2\left[(\ddot{\rho}_1 - (\rho_1 + d)\Omega^2)\hat{\rho} + 2\dot{\rho}_1\Omega\hat{\phi}\right].$$

$\Rightarrow$ $\hat{\rho}\rfloor$ $\quad -T = m_2(\ddot{\rho}_1 - (\rho_1 + d)\Omega^2)$ (1)

$\hat{\phi}\rfloor$ $\quad N_2 = m_2 2\dot{\rho}_1\Omega$ (2) $\qquad$ 0.5

$\hat{k}\rfloor$ $\quad N_2' = m_2 g.$ (3)

Tenemos las 6 ecuaciones escalares.

b) Ahora teniendo en cuenta que por (1) $T = m_1(\overbrace{\ddot{e}_1 - e_1\Omega^2}^{\alpha})$

(2) $-T = m_2(\ddot{e}_1 - (e_1+d)\Omega^2)$

Así, (1)+(2) $\Rightarrow$ $0 = (m_1+m_2)\ddot{e}_1 - \Omega^2(m_1+m_2)e_1 - m_2 d\Omega^2$

$$\Rightarrow \underbrace{\ddot{e}_1 - \Omega^2 e_1}_{\alpha} = \frac{m_2 d\Omega^2}{(m_1+m_2)} \quad (7)$$

1.0

Por (1) $T = m_1\alpha \Rightarrow \boxed{T = \frac{m_1 m_2 d\Omega^2}{(m_1+m_2)}}$ Constante!

Por lo cual, sin importar cuánto tiempo transcurra, la cuerda NO se cortará.

c) Primero notamos que $e_2 = e_1 + d$ 0.5

Ahora, volviendo a la ecuación (7)

$\ddot{e}_1 - \Omega^2 e_1 = \frac{m_2 d\Omega^2}{(m_1+m_2)}$; Nos damos una solución del tipo.

$$e(t) = A\cosh(\Omega t) + B.$$
$$\Rightarrow \dot{e}(t) = A\Omega\sinh(\Omega t)$$
$$\ddot{e}(t) = A\Omega^2\cosh(\Omega t)$$

$$\Rightarrow A\Omega^2\cosh(\Omega t) - \Omega^2 A\cosh(\Omega t) - B\Omega = \frac{m_2 d\Omega^2}{(m_1+m_2)}$$

$$\Rightarrow -B\Omega^2 = \frac{m_2 d\Omega^2}{(m_1+m_2)} \rightarrow B = -\frac{m_2 d}{(m_1+m_2)}.$$

Imponiendo condiciones iniciales, se tiene que:

$$e_1(0) = R = A - \frac{m_2 d}{(m_1+m_2)} \Rightarrow A = \frac{R(m_1+m_2)+m_2 d}{(m_1+m_2)}.$$

$$\Rightarrow \boxed{\begin{aligned} e_1(t) &= \frac{R(m_1+m_2)+m_2 d}{(m_1+m_2)}\cosh(\Omega t) - \frac{m_2 d}{m_1+m_2}. \\ e_2(t) &= e_1(t) + d. \end{aligned}}$$

1.0