# Pauta P4 - Control 3

P1|

a) $F(x,y,z) = x^2\,\hat{\imath} - y^2\,\hat{\jmath} + z^2\,\hat{k}$

- $x^2 + y^2 = a^2$ (1)
- $x + y + z = a$ (2)
- $z \geq 0$ (3)

Parametrización

Por (1): $x = a\cos(t)$
$y = a\,\mathrm{sen}(t)$ /0,5

Reemplazando en (2):

$\Rightarrow z = a(1 - \cos(t) - \mathrm{sen}(t))$ /0,5

Por (3): $\frac{\pi}{2} \leq t \leq 2\pi$ /0,5

Luego,

$$\vec{r}(t) = \begin{pmatrix} a\cos(t) \\ a\,\mathrm{sen}(t) \\ a(1-\cos(t)-\mathrm{sen}(t)) \end{pmatrix}$$

$$\vec{r}\,'(t) = \begin{pmatrix} -a\,\mathrm{sen}(t) \\ a\cos(t) \\ a(\mathrm{sen}(t)-\cos(t)) \end{pmatrix} dt$$

$$\vec{F}(\vec{r}) = \begin{pmatrix} a^2\cos^2(t) \\ -a^2\,\mathrm{sen}^2(t) \\ a^2(1-\cos(t)-\mathrm{sen}(t))^2 \end{pmatrix}$$

Así

$$\int_C \vec{F}\cdot d\vec{r} = \int_{\pi/2}^{2\pi} \begin{pmatrix} a^2\cos^2(t) \\ -a^2\,\mathrm{sen}^2(t) \\ a^2(1-\cos(t)-\mathrm{sen}(t))^2 \end{pmatrix} \begin{pmatrix} -a\,\mathrm{sen}(t) \\ a\cos(t) \\ a(\mathrm{sen}(t)-\cos(t)) \end{pmatrix} dt$$

/1.0

$$= a^3 \int_{\pi/2}^{2\pi} -\cos^2(t)\,\mathrm{sen}(t) - \cos(t)\,\mathrm{sen}^2(t) + 2(\mathrm{sen}(t)-\cos(t))\begin{pmatrix} 1+\cos(t)\,\mathrm{sen}(t) \\ -\cos(t)-\mathrm{sen}(t) \end{pmatrix} dt$$

$$= a^3 \int_{\pi/2}^{2\pi} \cos(t)\,sen^2(t) - 3\cos^2(t)\,sen(t) + 2\,sen(t) - 2\cos(t) + 2\cos^2(t) - 2\,sen^2(t)\,dt$$

$\vdots$

$$= a^3 \left[ \frac{sen^3(t)}{3} + \cos^3(t) - 2\cos(t) - 2\,sen(t) \right]_{\pi/2}^{2\pi} + 2\int_{\pi/2}^{2\pi} \underbrace{\cos^2(t) - sen^2(t)\,dt}_{\to 0}$$

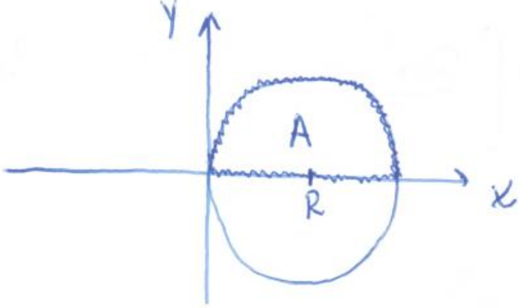
$$= \frac{2}{3} a^3 \quad //$$

/0,5

b) $\int_C (xy + ye^{xy})\,dx + (x^2+y^2+xe^{xy})\,dy$

$(x^2+y^2-2Rx=0)$

$(y \geq 0)$




- $x^2+y^2-2Rx+R^2-R^2=0$

  $\Leftrightarrow (x-R)^2+y^2=R^2$

- $M\,dx = (xy+ye^{xy})\,dx \; ; \; M(x,y) = xy+ye^{xy}$
- $N\,dy = (x^2+y^2+xe^{xy})\,dy \; , \; N(x,y) = x^2+y^2+xe^{xy}$

$\Rightarrow \frac{\partial M}{\partial y} = x + e^{xy} + xye^{xy}$

$\Rightarrow \frac{\partial N}{\partial x} = 2x + e^{xy} + xye^{xy}$

/1,5

Luego,

$$\int_C (xy+ye^{xy})\,dx + (x^2+y^2+xe^{xy})\,dy = \iint_A \left(2x + e^{xy} + xye^{xy} - x - e^{xy} - xye^{xy}\right)$$

$$= \iint_A x\,dx\,dy$$

$$= \int_0^{2R}\int_0^{\sqrt{R^2 - (x-R)^2}} x\, dy\, dx = \int_0^{2R} x\sqrt{R^2-(x-R)^2}\, dx$$ /1.0

~~[illegible]~~

Sustitución:

$R\,sen(t) = x - R$

$\Rightarrow x = R(sen(t)+1)$

$dx = R\cos(t)$

Luego,

$$= \int_{-\frac{\pi}{2}}^{\pi/2} R(sen(t)+1)\sqrt{R^2 - R^2 sen(t)}\cdot R\cos(t)\, dt$$

$$= \int_{-\pi/2}^{\pi/2} R^3(sen(t)+1)\cos^2(t)\,dt = R^3\int_{-\pi/2}^{\pi/2} \cos^2(t)sen(t) + \cos^2(t)\, dt$$

$$= R^3\left[ \cancel{-\frac{\cos^3(t)}{3}}_{0} \Big|_{-\pi/2}^{\pi/2} + \int_{-\pi/2}^{\pi/2} \cos^2(t)\,dt \right] = R^3\left[\frac{1}{2}(x + \cancel{\cos(x)sen(x)}_{0})\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{\pi}{2}R^3 \;//$$ /0,5