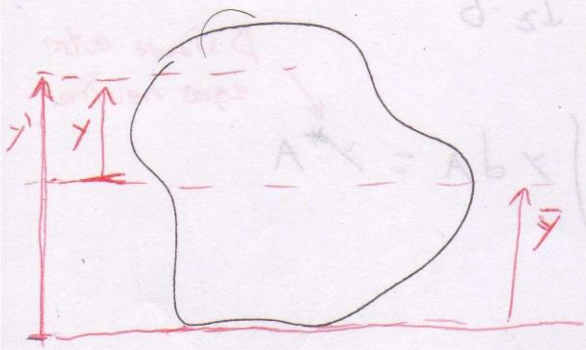
* Cálculo de propiedades de Área (I_z, Q)

↳ 2º momento de área ($\bar{I}_z$)



$$\bullet\ \bar{y}\ (\text{Eje neutro}) = \frac{\int y'\, dA}{A}$$

$$\bullet\ I_z = \int y^2\, dA = \int (y' - \bar{y})\, dA$$

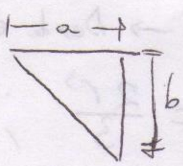
• Si hay varias fig. geométricas

$$\bar{Y}_T = \frac{\sum \bar{Y}_i \cdot A_i}{\sum A_i}$$

• Traslado de eje de rotación (Teorema de los ejes paralelos)

$$\bar{I}_z = I_z + \delta^2 A$$

* Encontrar eje neutro y I_z de:



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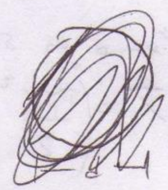

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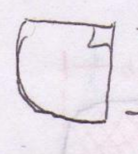
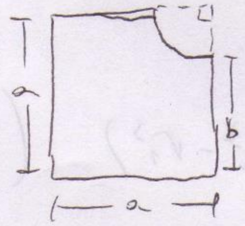
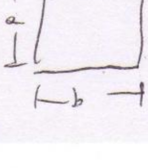


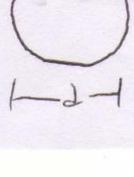


* Secciones conocidas:



$$I_z = \frac{b a^3}{12}$$

,



$$I_z = \frac{\pi d^4}{64}$$

• Cálculo de deflexión por integración:

$$\frac{d^4\hat{y}}{dx^4} = -\frac{w(x)}{EI}$$ ← Fza distribuida

* Modelación de fzas mediante función escalón ($r(x-x_0)$) para fzas distribuidas, y delta de Dirac ($\delta(x-x_0)$) para fzas puntuales

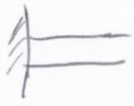
* Lo que ocurre en los extremos de la viga se reemplaza por condiciones de borde (Son necesarias 4 cond. de borde)

* Las cond. de borde son condiciones en la deflexión en los extremos, el ángulo de deflexión ($\frac{d\hat{y}}{dx}$), el momento ($\frac{d^2\hat{y}}{dx^2}$) ó de la fza. de corte ($\frac{d^3\hat{y}}{dx^3}$)

$$\hookrightarrow \boxed{M = EI\frac{d^2\hat{y}}{dx^2}} \wedge \boxed{V = -EI\frac{d^3\hat{y}}{dx^3}}$$

* Algunos bordes:

⟹ No hay deflexión → $\hat{y}(0) = 0$
No hay momento → $\frac{d^2\hat{y}}{dx^2} = 0$

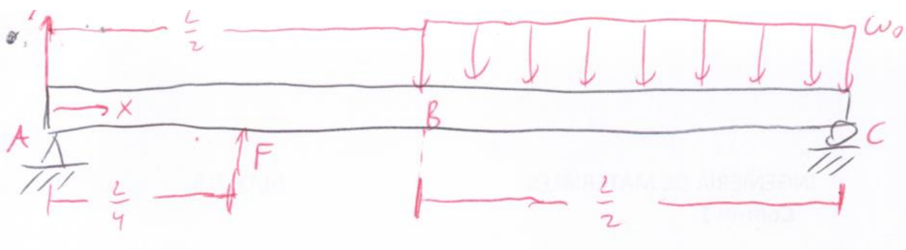


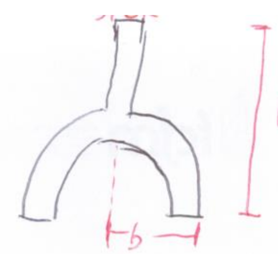
⟹ No hay deflexión → $\hat{y}(0) = 0$
No hay ángulo → $\frac{d\hat{y}}{dx}(0) = 0$

⟹ No hay momento → $\frac{d^2\hat{y}}{dx^2}(0) = 0$

Balance de fzas → $V(0) = -F = -k\hat{y}(0)$

$$\therefore -EI\frac{d^3\hat{y}}{dx^3}(0) = -k\hat{y}(0)$$





a)

$$\bar{Y}_T = \frac{\sum A_i Y_i}{\sum A_i} = \frac{(Y_1 \cdot A_1 - Y_2 \cdot A_2) + Y_3 \cdot A_3}{A_1 - A_2 + A_3}$$

(1): Semicírculo de radio b completo

(2): Semicírculo de radio (b-e)

Desde la base!

$$* \; Y_1 = \frac{4b}{3\pi} \qquad * \; Y_2 = \frac{4(b-e)}{3\pi} \qquad * \; Y_3 = \frac{(h-b)}{2} + b = \frac{b+h}{2}$$

$$A_1 = \frac{\pi b^2}{2} \qquad A_2 = \frac{\pi (b-e)^2}{2} \qquad A_3 = e(h-b)$$

$$\boxed{\therefore \; \bar{Y}_T = 0{,}07468 \; \cancel{m} = 7{,}468 \text{ cm}}$$

b) $\bar{I}_T = \bar{I}_1 - \bar{I}_2 + \bar{I}_3$ ← C/r a eje neutro de fig. completa $\bar{Y}_T$

$$\sim\!\!> I_1 = 0{,}1098\, b^4 \implies \bar{I}_1 = I_1 + (Y_1 - \bar{Y}_T)^2 \cdot A_1$$

$$= 0{,}1098\, b^2 + \left(\frac{4b}{3\pi} - \bar{Y}_T\right)^2 \cdot \frac{\pi b^2}{2} = 1643{,}56 \text{ cm}^4$$

$$\sim\!\!> I_2 = 0{,}1098\,(b-e)^4 \implies \bar{I}_2 = I_2 + (Y_2 - \bar{Y}_T)^2 A_2$$

$$= 0{,}1098\,(b-e)^4 + \left(\frac{4(b-e)}{3\pi} - \bar{Y}_T\right)^2 \cdot \frac{\pi (b-e)^2}{2}$$

$$= 2413{,}88 \text{ cm}^4$$

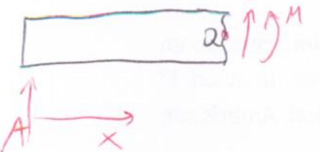
$$\sim\!\!> I_3 = \frac{e \cdot (h-b)^3}{12} \implies \bar{I}_3 = I_3 + (Y_3 - \bar{Y}_T)^2 A_3$$

$$= \frac{e(h-b)^3}{12} + \left(\frac{(h+b)}{2} - \bar{Y}_T\right)^2 \cdot e(h-b) = 283{,}28 \text{ cm}^4$$

$$\boxed{\therefore \; \bar{I}_T = 4340{,}72 \text{ cm}^4 = 4{,}341 \cdot 10^{-5} \text{ m}^4}$$

c) Momento interno $M(x)$:

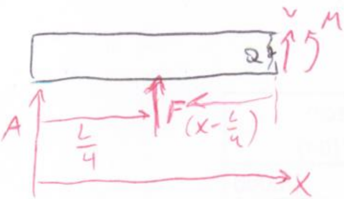
i) $0 < x < \frac{L}{4}$



$$\sum F_y = 0 \Rightarrow V(x) = A$$

$$\sum_Q M_z = 0 \Rightarrow -Ax + M = 0 \Rightarrow \boxed{M(x) = Ax}$$

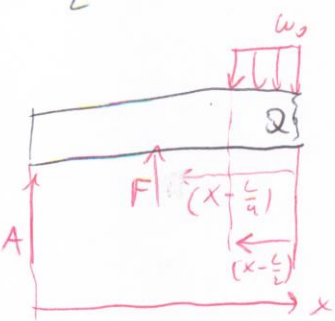
ii) $\frac{L}{4} < x < \frac{L}{2}$



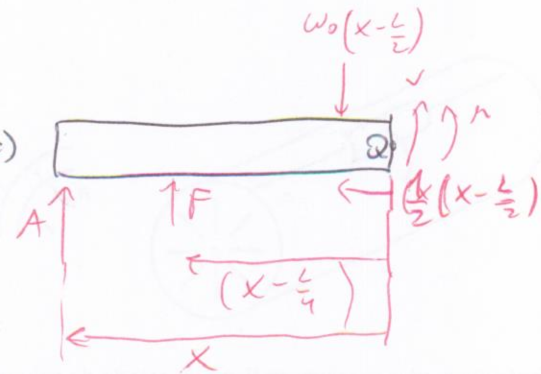
$$\sum_Q M_z = 0 \Rightarrow -Ax - F\left(x-\frac{L}{4}\right) + M = 0$$

$$\boxed{\therefore M(x) = Ax + F\left(x-\frac{L}{4}\right)}$$

iii) $\frac{L}{2} < x < L$



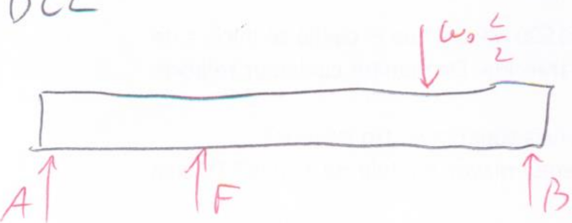
$\sum \Rightarrow$



$$\sum_Q M_z = 0 \Rightarrow -Ax - F\left(x-\frac{L}{4}\right) + \frac{w_0}{2}\left(x-\frac{L}{2}\right)^2 + M = 0$$

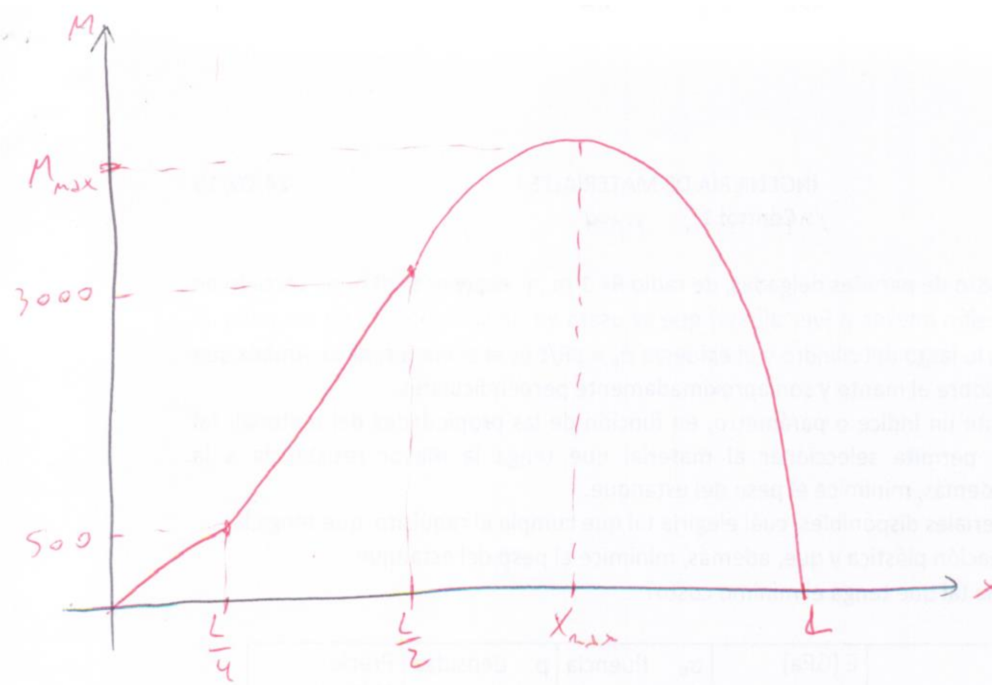
$$\boxed{\therefore M(x) = Ax + F\left(x-\frac{L}{4}\right) - \frac{w_0}{2}\left(x-\frac{L}{2}\right)^2}$$

* DCL



$$\sum_B M_z = 0 \Rightarrow -A \cdot L - F \cdot \frac{3L}{4} + \frac{w_0 L^2}{8} = 0$$

$$\boxed{\therefore A = \frac{w_0 L}{8} - \frac{3}{4}F = 500\ N}$$



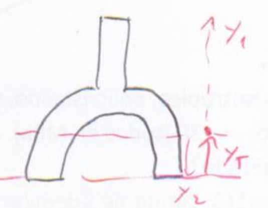
· Si el máximo se encuentra en el 3º tramo:

$$\left.\frac{dM(x)}{dx}\right|_{M_{max}} = 0 \Rightarrow A + F - \omega_0\left(x - \frac{L}{2}\right) = 0$$

$$X_{max} = \frac{A+F}{\omega_0} + \frac{L}{2} = 2,625 \text{ m}$$

$$\Rightarrow \boxed{M_{max} = 3781,25 \text{ N·m}}$$

· Esf. máximo $\Rightarrow \sigma = -\frac{M_{max}}{I_z} \cdot y_{max} \rightarrow$



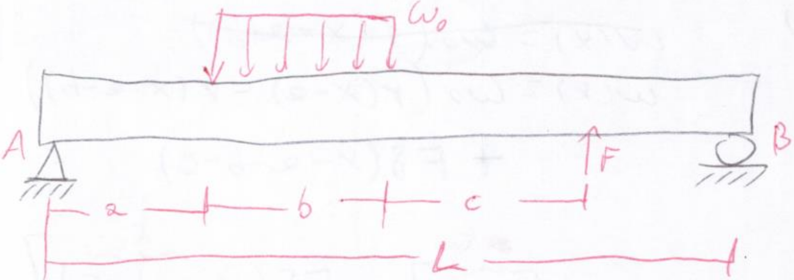
$y_1 = h - \overline{y_T}$

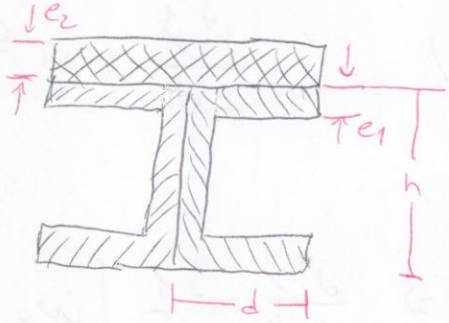
$y_2 = -\overline{y_T}$

$\Rightarrow \boxed{y_1 = 9,532 \text{ cm}}$; $y_2 = -7,468$ cm

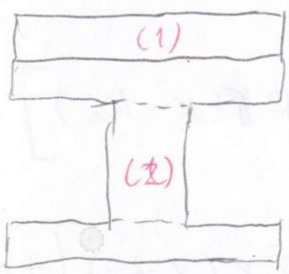
$$\therefore \sigma = -\frac{3781,25 \text{ N·m}}{4,341 \cdot 10^{-5} \text{ m}^4} \cdot 9,532 \cdot 10^{-2} \text{ m} \Rightarrow \boxed{\therefore \sigma_{max} = -8,303 \text{ MPa}}$$

(Compresión)





a)



$$\overline{Y_T} = \frac{\sum \overline{Y_i} \cdot A_i}{\sum A_i} \quad \Big/ \quad \overline{Y_1} = h + \frac{e_2}{2} \; ; A_1 = 2de_2$$

$$\overline{Y_2} = \frac{h}{2} \; , A_2 = 4de_1 + 2e_1(h-2e_1)$$

$$\Rightarrow \overline{Y_T} = \frac{(h+\frac{e_2}{2}) \cdot 2de_2 + \frac{h}{2}\left(4de_1 + 2e_1(h-2e_1)\right)}{2de_2 + 4de_1 + 2e_1(h-2e_1)}$$

$$\boxed{\overline{Y_T} = 0{,}177 \, m}$$

$$\rightsquigarrow \overline{I_z} = \overline{I_1} + \overline{I_2}$$

$$\hookrightarrow \overline{I_1} = I_1 + (\overline{Y_1} - \overline{Y_T})^2 \cdot A_1 = \frac{2de_2^3}{12} + (\overline{Y_1} - \overline{Y_T})^2 \cdot 2de_2$$

$$\boxed{\overline{I_1} = 5{,}538 \cdot 10^{-5} \, [m^4]}$$

$$\cancel{\hookrightarrow \overline{I_2} = \left(2\left[\frac{2de_1^3}{12}\right.\right.} +$$

$$* \overline{I_2} = \left(\frac{2de_1^3}{12} + \left(\frac{e_1}{2} - \frac{h}{2}\right)^2 \cdot 2de_1\right) + \left(\frac{2de_1^3}{12} + \left(h - \frac{e_1}{2} - \frac{h}{2}\right)^2 \cdot 2de_1\right)$$

$$+ (\overline{Y_T} - \overline{Y_2})^2 \cdot A_2$$

ó

$$* \overline{I_2} = 2\left(\frac{dh^3}{12} - \frac{(d-e_1)(h-2e_1)^3}{12} + (\overline{Y_T} - \overline{Y_2})^2 \frac{A_2}{2}\right) = 1{,}701 \cdot 10^{-4} \, m^4$$

* $D_e(1) \rightsquigarrow \alpha = \frac{1}{LEI_z}\left[w_0\left(\frac{(L-a)^2}{2} - \frac{(L-x_1)^2}{2}\right) - F\frac{(L-x_2)}{}\right]$

$$\boxed{\alpha = \cancel{3,167\cdot 10^{-4}}} \boxed{= -7,849\cdot 10^{-5}}$$

* $D_e(2)$

$$\gamma = \frac{1}{LEI_z}\left[w_0\left(\frac{(L-a)^4}{24} - \frac{(L-x_1)^4}{24}\right) - \frac{F(L-x_2)^3}{6}\right] - \frac{\alpha L^2}{6}$$

$$\boxed{\gamma = 1,402\cdot 10^{-3}}$$

* Una vez calculadas las ctes., calculamos la deflexión en $x = L/2$

$$\hat{y}(L/2) = -\frac{1}{EI_z}\left[w_0\cdot\frac{(\frac{L}{2}-a)^4}{24}\right] + \frac{\alpha(\frac{L}{2})^3}{6} + \gamma\cdot\frac{L}{2}$$

$$\boxed{\hat{y}(L/2) = 5,363\cdot 10^{-3}\ [m]}$$

* El esf. normal ~~máximo~~ se calcula como:

$$\cancel{\sigma_{max} = -M(x)\cdot y}$$

$$\sigma(x) = -\frac{M(x)\cdot y}{I_z} \quad ; \quad M(x) = EI\frac{d^2\hat{y}}{dx^2}$$

$$\boxed{\therefore \sigma(x) = -\frac{d^2\hat{y}}{dx^2}Ey}$$

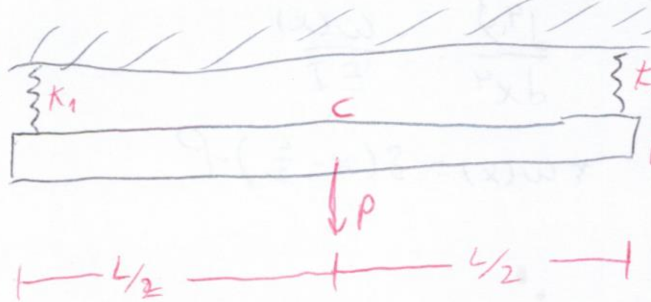
$\hookrightarrow \frac{d^2\hat{y}}{dx^2}\left(\frac{L}{2}\right) = -\frac{w_0}{2EI_z}\left(\frac{L}{2}-a\right)^2 + \alpha\cdot\frac{L}{2} = -4,279\cdot 10^{-4}$

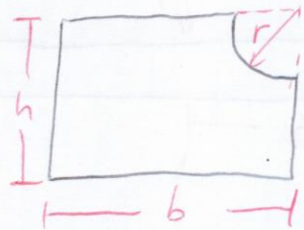
$\hookrightarrow y_{max} = -\bar{y}_T = -0,177$

$$\boxed{\therefore \sigma(L/2) = \cancel{-15147466}\ [Pa]} \boxed{1,515\cdot 10^{7}\ [Pa]}$$

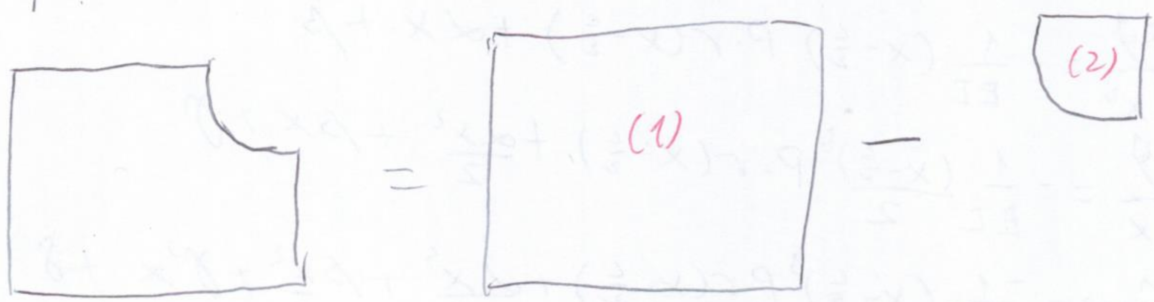
2014

P2]

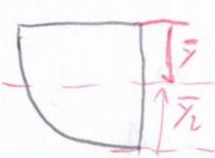




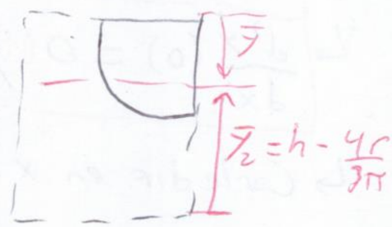
a) Propiedades de área:



(1): $\bar{y}_1 = \frac{h}{2} \;\wedge\; I_1 = \frac{b \cdot h^3}{12}$

(2): 

$\cancel{\bar{y} = \frac{4r}{3\pi}} \;\wedge\; I_2 = \frac{\pi r^4}{16}$



$$\Rightarrow \bar{y}_T = \frac{\bar{y}_1 A_1 - \bar{y}_2 A_2}{A_1 - A_2} = \frac{\frac{h}{2} \cdot hb - \overbrace{\left(h - \frac{4r}{3\pi}\right)}^{\text{Desde la base}} \cdot \frac{1}{4}\pi r^2}{hb - \frac{1}{4}\pi r^2} \cancel{= 4{,}567\,\text{cm}}$$

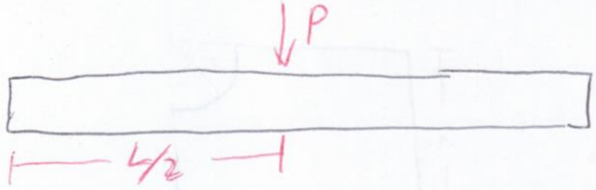
$$\boxed{\bar{y}_T = 4{,}567\,\text{cm}}$$

$$\Rightarrow \bar{I}_1 = I_1 + \delta^2 A = \frac{bh^3}{12} + \left(\frac{h}{2} - \bar{y}_T\right)^2 bh \Rightarrow \boxed{\bar{I}_1 = 1{,}278 \cdot 10^{-5}\,\text{m}^4}$$

$$\bar{I}_2 = I_2 + \delta^2 A = \frac{\pi r^4}{16} + \left(\bar{y}_T - \left(h + \frac{4r}{3\pi}\right)\right)^2 \cdot \frac{1}{4}\pi r^2 \Rightarrow \boxed{\bar{I}_2 = 3{,}38 \cdot 10^{-6}\,\text{m}^4}$$

$$\therefore \bar{I}_T = \bar{I}_1 - \bar{I}_2 \Longrightarrow \boxed{\bar{I}_T = 9{,}401 \cdot 10^{-6}\,\text{m}^4}$$

b) Aplicando la ec. de la curva elástica



$$\frac{d^4 y}{dx^4} = -\frac{\omega(x)}{EI}$$

$$* \ \omega(x) = \delta(x - \tfrac{L}{2}) \cdot P$$

· Integrando:

$$\rightsquigarrow \frac{d^3 y}{dx^3} = -\frac{1}{EI} \cdot P \cdot r(x - \tfrac{L}{2}) + \alpha$$

$$\rightsquigarrow \frac{d^2 y}{dx^2} = -\frac{1}{EI}(x - \tfrac{L}{2}) \cdot P \cdot r(x - \tfrac{L}{2}) + \alpha x + \beta$$

$$\rightsquigarrow \frac{dy}{dx} = -\frac{1}{EI}\frac{(x - \frac{L}{2})^2}{2} P \cdot r(x - \tfrac{L}{2}) + \frac{\alpha x^2}{2} + \beta x + \gamma$$

$$\Rightarrow \hat{y}(x) = \frac{-1}{6EI}(x - \tfrac{L}{2})^3 P\, r(x - \tfrac{L}{2}) + \frac{\alpha x^3}{6} + \beta\frac{x^2}{2} + \gamma x + \delta$$

· Condiciones de Borde:

↳ $\boxed{\frac{d^2 y}{dx^2}(0) = 0 \ (1) \ / \ \frac{d^2 y}{dx^2}(L) = 0 \ (2)}$

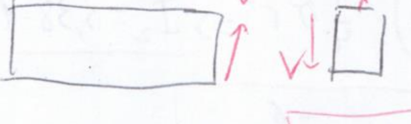
↳ Corte dif. en $x = 0 \rightsquigarrow$

$$\left.\begin{aligned} &V(0) + A = 0 \\ &\Rightarrow V(0) = -A \end{aligned}\right\}$$

$$* \ V(x) = -EI\frac{d^3 y}{dx^3}(x) \quad y \quad A = -k_1 \cdot \hat{y}(0)$$

$$\boxed{\therefore -EI\frac{d^3 y}{dx^3}(0) = k_1 \hat{y}(0) \ (3)}$$

↳ Corte dif. en $x = L$



$$\left.\begin{aligned} &V(L) - B = 0 \Rightarrow V(L) = B \\ &* \ B = -k_2 \hat{y}(L) \end{aligned}\right\}$$

$$\boxed{\therefore EI\frac{d^3 y}{dx^3}(L) = k_2 \hat{y}(L) \ (4)}$$

• Aplicando las cond. de Borde

(1) $\frac{d^2y}{dx^2}(0)=0 \Rightarrow \boxed{\beta=0}$

(2) $\frac{d^2y}{dx^2}(L)=0 \rightsquigarrow -\frac{1}{EI}(L-\frac{L}{2})\cdot P+\alpha L=0$

$\Rightarrow \boxed{\alpha=\frac{P}{2EI}}$

(3) $-EI\frac{d^3y}{dx^3}(0)=k_1\hat{y}(0)$

$-\alpha EI=k_1\delta \rightsquigarrow \delta=-\frac{\alpha EI}{k_1}=-\frac{P}{2\cancel{EI}}\cdot\frac{\cancel{EI}}{k_1} \Rightarrow \boxed{\delta=\frac{-P}{2k_1}}$

(4) $EI\frac{d^3y}{dx^3}(L)=k_2\hat{y}(L)$

$-P+\alpha EI=k_2\left(-\frac{1}{6EI}(L-\frac{L}{2})^3P+\frac{\alpha L^3}{6}+\gamma L+\delta\right)$

~~Despejando~~

$-P+\frac{P}{2}=k_2\left(-\frac{L^3P}{48EI}+\frac{\overset{\frac{P}{2EI}}{\alpha}L^3}{6}+\frac{P}{2k_1}\right)+k_2L\gamma$

$$\boxed{\therefore \gamma=\frac{1}{k_2L}\left(-\frac{P}{2}+k_2P\left(-\frac{L^3}{48EI}+\frac{L^3}{12EI}-\frac{1}{2k_1}\right)\right)}$$

• Una vez calculadas todas las constantes, evaluemos $\hat{y}(\frac{L}{2})$ para encontrar el desplazamiento en el punto C.

$\hat{y}(\frac{L}{2})=\alpha\frac{L^3}{6}+\cancel{\beta\frac{L^2}{2}}+\gamma L+\delta=\frac{\alpha(\frac{L}{2})^3}{6}+\gamma\frac{L}{2}+\delta$

• Calculando las ctes: $\alpha=1{,}4\cdot10^{-3}$

$\gamma=-7{,}4\cdot10^{-3}$

$\delta=-8{,}3\cdot10^{-3}$

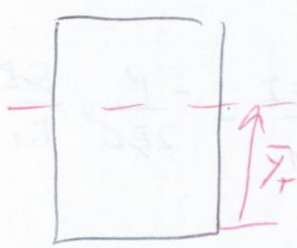
$$\boxed{\therefore \hat{y}(\tfrac{L}{2})=-0{,}0241\text{ m}=-2{,}41\text{ cm}}$$

~~a) $M = EI \dfrac{d^2y}{dx^2} \Rightarrow M_{max} \rightsquigarrow$ derivamos $M(x)$ y obtenemos x para luego evaluar en $M(x)$~~

~~$\Rightarrow \dfrac{d^3y}{dx^3} = 0 \rightsquigarrow -\dfrac{1}{EI} P \cdot r\left(x - \dfrac{L}{2}\right) + \dfrac{P}{2EI}$~~

c) $M_{max} \rightsquigarrow \left(\dfrac{d^2y}{dx^2}\right)_{max} \rightsquigarrow x = \dfrac{L}{2}$

$Y_{max} \rightsquigarrow$



$* \; y = -\overline{Y}_T = -4,867$ cm

$* \; y = h - \overline{Y}_T = 5,433$ cm $\rightarrow$ Máximo (Extr. sup)

$\therefore \sigma_{max} = -\dfrac{M}{I} y \quad \wedge \quad M = EI \dfrac{d^2y}{dx^2}$

$\Rightarrow \sigma_{max} = -E\, y_{max} \left(\dfrac{d^2y}{dx^2}\right)_{max} \Bigg\} \quad \dfrac{d^2y}{dx^2}(L/2) = \dfrac{PL}{4EI}$

$\boxed{\therefore \sigma_{max} = -43,342 \text{ MPa}}$