Ponto auxílio 1.

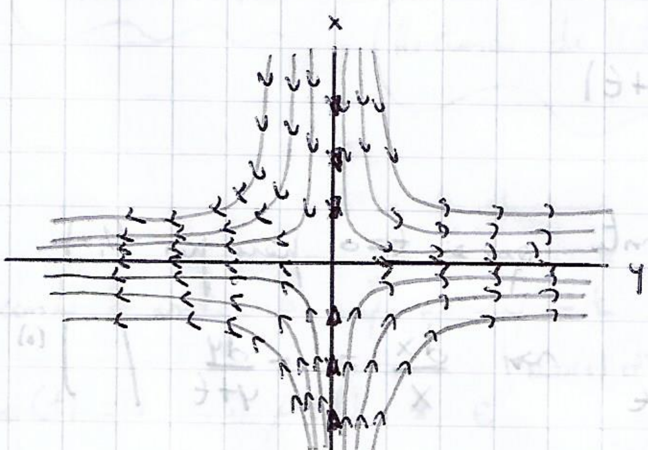
P1) Seu $u = ax$, $v = -ay$, $w = 0$

$$\frac{dx}{dy} = \frac{u}{v} = \frac{ax}{-ay} = -\frac{x}{y}$$

$$\frac{dx}{dy} = -\frac{x}{y} \Big/ \int^{(0)}$$

$\ln(x) = -\ln(y) + c \rightsquigarrow \ln(x \cdot y) = c \;/\; \exp(0)$

$xy = c$ / Ec de uma hyperbole.



Sea $c(x,y,t) = bx^2ye^{-at}$ con $a>0$, p.d.q $\frac{Dc}{Dt} = 0$.

En efecto:

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial x} \cdot \overset{u}{\widetilde{\frac{\partial x}{\partial t}}} + \frac{\partial c}{\partial y} \cdot \overset{v}{\widetilde{\frac{\partial y}{\partial t}}} + \frac{\partial c}{\partial t} \cdot \cancel{\frac{\partial t}{\partial t}}^{\,1}$$

$$\frac{\partial c}{\partial x} = 2bxye^{-at}, \quad \frac{\partial c}{\partial y} = bx^2e^{-at}, \quad \frac{\partial c}{\partial t}\ bx^2y(-a)e^{-at}$$

$$\therefore \frac{Dc}{Dt} = (2bxye^{-at})ax + (bx^2e^{-at})(-ay) - abx^2e^{-at}$$

$$\frac{Dc}{Dt} = \cancel{2abx^2ye^{-at}} - \cancel{2abx^2e^{-at}} = 0$$

P2) Sea $u = x$
$v = -(y+t)$

a) La linea de corriente que en $t=0$ pasa por $(1,1)$

$$\frac{dx}{dy} = \frac{u}{v} = -\frac{x}{y+t} \rightsquigarrow \frac{dx}{x} = -\frac{dy}{y+t} \ \Big/ \int^{(0)}$$

$$\ln(x) = -\ln(y+t) + C$$

$$\ln(x(y+t)) = c$$

$x(y+t) = c_2 \;/\; t=0 \;,\; \vec{x}_0 = (1,1)$

$1(1+0) = c_2 \Rightarrow c_2 = 1$

$\therefore$ la línea de corriente es: $x(y+t) = 1$

b) La línea de trayectoria de una partícula que en $t=0$ pasa por el punto $(1,1)$

En efecto,

$$\frac{dx}{dt} = u = x \;/\; \int(\cdot) \sim \ln(x) = t + c_1$$

$$\frac{dy}{dt} = v = -y-t$$

$$X(t) = c_1 e^t$$

Recuerdo de Rote $\rightarrow$ Sea $y' + \bar{a}_0(t) y = \bar{q}(t)$

$$y(t) = \frac{c}{\mu(t)} + \frac{1}{\mu(t)} \int \mu(t)\, \bar{q}(t)\, dt$$

aplicando lo anterior $\dot{y} = -y - t \sim \dot{y} + y = -t$

$$\bar{a}_0(t) = 1 \quad \therefore \mu(t) = e^{\int \bar{a}_0(t)\, dt} = e^{\int dt} = e^t$$

$$y(t) = \frac{c_2}{e^t} + \frac{1}{e^t} \left\{ \underbrace{\int e^t(-t)\, dt}_{(*)} \right\}$$ (integramos por partes)

$(*) - \int e^t\, t\, dt = e^t t - \left\{ \int e^t dt \right\} = e^t t - e^t$

$u : t, \quad du : 1$
$dv : e^t, \quad dv : e^t$

$(*) = e^t (1 - t)$ luego

$$y(t) = \frac{c_2}{e^t} + \frac{1}{\cancel{e^t}} \cdot \cancel{e^t} (1-t)$$

$$y(t) = \frac{c_2}{e^t} + 1 - t \quad \Big\}$$ Condiciones iniciales

$y(t=0) = 1 \Rightarrow 1 = \frac{c_2}{e^t} + 1 - 0 \Rightarrow c_2 = 0$
$x(t=0) = 1$

$\hookrightarrow 1 = c_1 \Rightarrow c_1 = 1$

luego la línea de trayectoria es: $x = e^t \quad / \ln(\cdot)$
$y = 1 - t$

$\ln(x) = t \Rightarrow y(x) = 1 - \ln(x)$

c) para calcular la línea de emisión (strike line)

debemos sustituir t por τ para considerar a todas las partículas que pasan por $(1,1)$ para todo $t \geq 0$.

$\therefore \quad x = c_1 e^{t}$ (si sustituimos t por τ y x por 1),

$1 = c_1 e^{\tau} \sim c_1 = e^{-\tau}$

$y = c_2 e^{-t} + 1 - t$ (si subt. t por τ y y por 1)

$\cancel{1} = c_2 e^{-\tau} + \cancel{1} - \tau \sim \tau = c_2 e^{-\tau} \sim c_2 = \tau e^{\tau}$

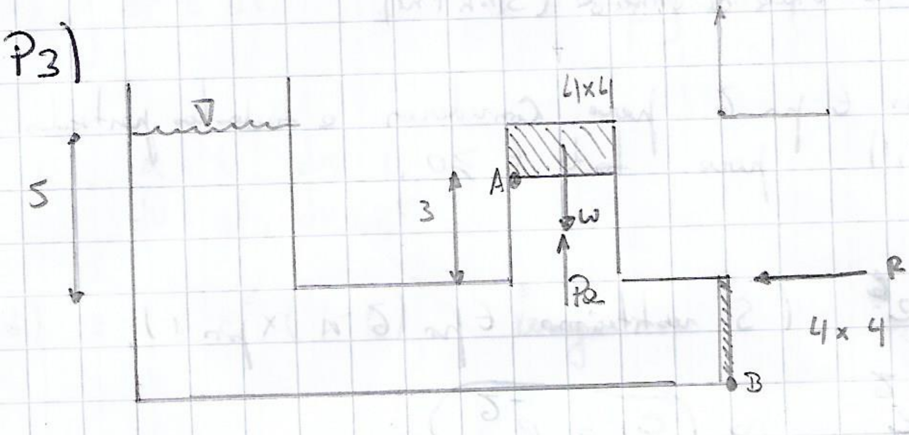
$$x = e^{-\tau} e^{t}, \quad y = \tau e^{\tau} e^{-t} + 1 - t$$

evaluando con $t = 0$

$$x = e^{-\tau}, \quad y = \tau e^{\tau} + 1$$

$$y = -\frac{\ln(x)}{x} + 1$$

P3)



Determinar w y R

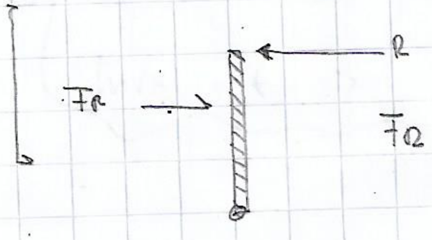
$\sum M_A : -2 \cdot w + 2 \cdot P_a \cdot 16 = 0$

$P_a = \rho_{H_2O}\, h = 1'000 \cdot 9{,}8 \cdot (5-3) = 19'600$

$2w = 2 \cdot 16 \cdot 19'600 \rightsquigarrow w = 313600 [N]$

$w = 313{,}6 [kN]$

ahora resolveremos en DCl para la segunda compuerta.



$F_R = (5+2)\,\rho \cdot 16 = 1097600\ N$

$\sum M_B = 4 \cdot R - l \cdot F_R = 0$

l? , $y_R = \dfrac{I_{xc}}{y_c A} + y_c$

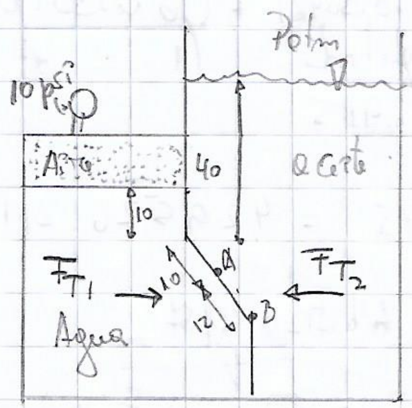
$$y_R = \frac{\frac{1}{12} \cdot \cancel{4}^4}{7 \cdot \cancel{4}^2} + 7$$

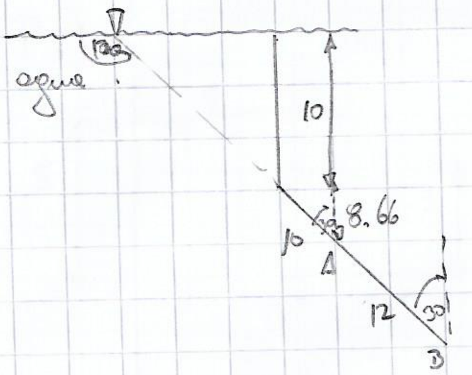
$9 - y_R = l = 1.8$ ∴

$\boxed{\text{luego } R = 4965 33.34\ [N]}$

P4) En efecto

1 psi = 6894.76 [Pa]
1 ft = 12 in = 0.3048 m





$F_{R_1} = \rho_{H_2O} \cdot h_{c_1} A \quad , \quad h_{c_1} = 10 + 8.66 + 6 \cos 30 = 23.85 [ft]$

$h_{c_1} = 7.271 [mt]$

$A = 4.45 [mt^2]$

$F_{R_1} = 317088.31 [N]$

$F_{T_1} = F_R + A \underbrace{P_{atm}}_{10^5} + A \cdot 10 \cdot 6894.76 = 1068905.13 [Pa]$

$F_{R_2} = \rho_{oil} \cdot h_{c2} A, \quad h_{c2} = (40 \cdot 0.3048) + (16 \cos 30 \cdot 0.3048)$

$h_{c2} = 16.415$

$F_{R2} = 16.415 \cdot 0.6 \cdot 9.8 \cdot 1000 \cdot 4.45 = 429526.2117$

$F_{T2} = 10^5 \cdot 4.45 + F_{R2} = 874526.217$

$F_T = \| F_{R1} - F_{R2} \| = 194378.91 [N]$