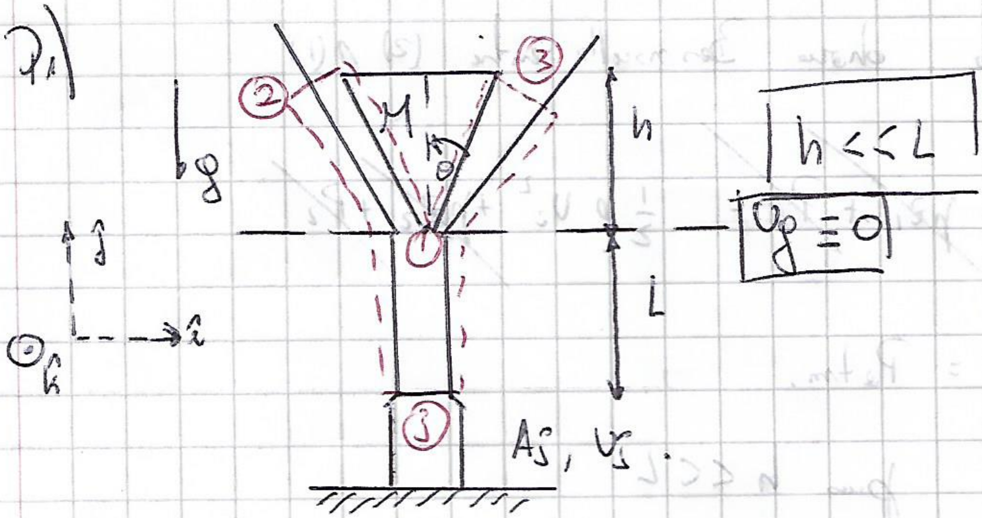
Parte ...ex 5

P1)



Primero calculamos v_1 usando Bernoulli:

$$\frac{1}{2}\rho v_j^2 + \cancel{\rho z_j} + \cancel{P_j} = \frac{1}{2}\rho v_1^2 + \cancel{\rho z_1} + \cancel{P_1}$$

$P_1 = P_j = P_{atm}$ $\Rightarrow$

$z_j = -L$, z_1 0

$$\frac{1}{2}\cancel{\rho} v_j^2 - \cancel{\rho} g L = \frac{1}{2}\cancel{\rho} v_1^2$$

$$v_1 = \sqrt{v_j^2 - 2gL}$$

Aplicando ahora Bernoulli entre (2) y (1)

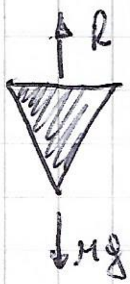
$$\frac{1}{2}\rho V_1^2 + \cancel{\gamma z_1} + \cancel{P_1} = \frac{1}{2}\rho V_2^2 + \cancel{\gamma z_2} + \cancel{P_2}$$

$P_1 = P_2 = P_{atm.}$

$z_1 = z_2$ pues $h << L$

$\Rightarrow \boxed{V_2 = V_1}$, análogamente $\boxed{V_3 = V_1}$

ahora vamos a hacer un DCL para el volumen de control.



$R = Mg$

$\therefore$ Vamos a Aplicar el TTR de momento lineal.

$$\sum (-\dot{m}_{in})(\vec{v}_{in}) + \sum (\dot{m}_{out})(\vec{v}_{out}) = \sum \vec{F}_{ext}.$$

$$\sum \vec{F}_{ext} = -R = -Mg\,\hat{j}.$$

$\dot{m}_{in} = \rho A_j V_j \qquad \vec{v}_{in} = v_j\,\hat{j}$

$\dot{m}_{out\,2} = \rho A_2 V_2 \qquad \vec{v}_{out_2} = -V_2 \operatorname{Sen}\theta\,\hat{i} + V_2 \operatorname{Cos}\theta\,\hat{j}$

$\dot{m}_{out\,3} = \rho A_3 V_3 \qquad \vec{v}_{out_3} = +V_3 \operatorname{Sen}\theta\,\hat{i} + V_3 \operatorname{Cos}\theta\,\hat{j}.$

$$\Rightarrow (-\rho A_j V_j)\,v_j\,\hat{j} + \rho A_2 V_2 (-V_2 \operatorname{Sen}\theta\,\hat{i} + V_2 \operatorname{Cos}\theta\,\hat{j}) + \rho A_3 V_3 (V_3 \operatorname{Sen}\theta\,\hat{i} + V_3 \operatorname{Cos}\theta\,\hat{j}) = -Mg\,\hat{j}$$

$\hat{i})$: $\quad -\rho A_2 V_2^2 \operatorname{Sen}\theta + \rho A_3 V_3^2 \operatorname{Sen}\theta = 0$

Como $\operatorname{Sen}\theta \neq 0 \Rightarrow \boxed{A_2 = A_3}$

$\wedge\ V_2 = V_3 = V_1$

luego como $V_2 = V_3 = V_1 \wedge A_2 = A_3$

usaremos la Ecuacion de Continuidad.

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}.$$

Con $v_2 = v_1$

$\Rightarrow \cancel{\rho} A_j v_j = 2 \cancel{\rho} A_2 v_2$

$$\therefore \boxed{A_2 = A_3 = \frac{A_j v_j}{\sqrt{v_j^2 - 2gL}}}$$

usando ahora la componente en $\hat{j}$:

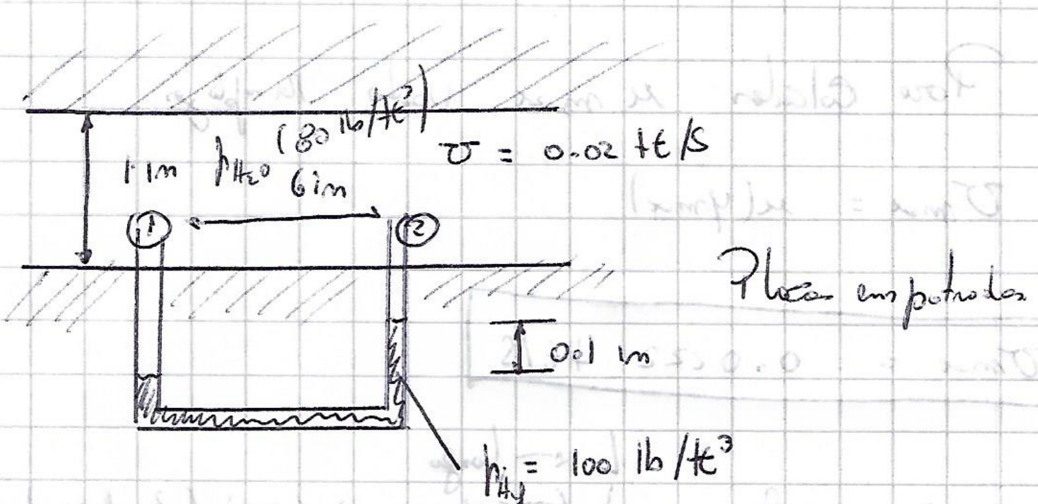
$$\hat{j}) : \quad -\rho A_j v_j^2 + 2\rho A_2 v_1^2 \cos\theta = -Mg$$

$$\rho A_j v_j^2 - Mg = 2\rho \frac{A_j v_j}{\cancel{\sqrt{v_j^2 - 2gL}}} \cdot \cancel{(v_j^2 - 2gL)} \cos\theta$$

$$\frac{\rho A_j v_j^2 - Mg}{2\rho \cos\theta A_j v_j} = \sqrt{v_j^2 - 2gL} \quad / ()^2$$

$$\frac{v_j^2 - \left(\frac{\rho A_j v_j^2 - Mg}{2\rho \cos\theta A_j v_j}\right)^2}{2g} = L$$

P2)



En efecto, usando las Ecuaciones de N-S, claramente estamos ante un flujo poiseuille. luego

$$u(y) = v\frac{y}{L} + \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)\left(y^2 - Ly\right)$$

La velocidad máxima ocurre cuando $\frac{\partial u}{\partial y} \overset{!}{=} 0 \Rightarrow$

$$y_{max} = -\frac{\mu v}{L\left(\frac{\partial P}{\partial x}\right)} + \frac{L}{2} \qquad (L = 1 in)$$

$$P_1 - P_2 = (\rho_{Hg} - \rho_{H_2O})\,\Delta h = 0.167 \ lb/ft^2$$

$$-\frac{\partial P}{\partial x} = \frac{P_1 - P_2}{l} = \boxed{-0.334 \ lb/ft^3}$$

$$\Rightarrow \boxed{y_{max} = 0.759 \ in}$$

Para calcular u_{max}, basta Reemplazar

$$v_{max} = u(y_{max})$$

$$\boxed{v_{max} = 0.0222 \ ft/s}$$

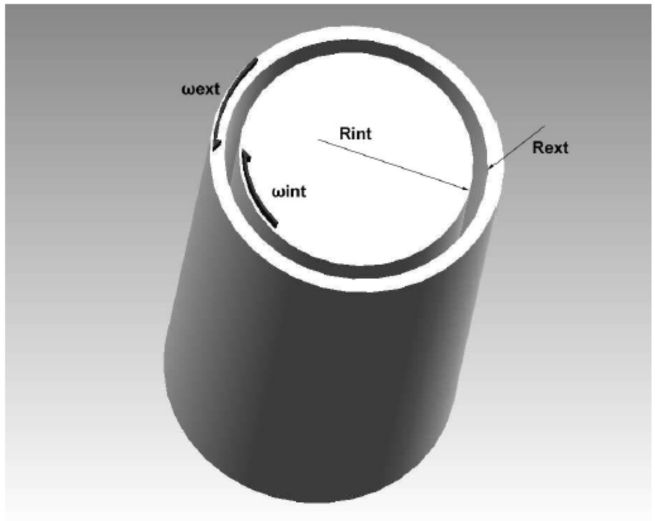
$$q = \frac{Q}{\underset{\text{ancho}}{L}} = \int_0^{\overset{\text{largo}}{L}} \left(v \frac{y}{\cancel{b}} + \frac{1}{2\mu} \left(\frac{\partial P}{\partial x} \right) (y^2 - \cancel{b}y) \right) dy$$

$$\boxed{q = v \frac{y^2}{2L} + \frac{1}{2\mu} \left(\frac{\partial P_x}{\partial x} \right) \left(\frac{y^3}{\cancel{b}} - \frac{L y^2}{2} \right)}$$

$$q = \frac{vL}{2} + \frac{1}{2\mu} \left(\frac{\partial P_x}{\partial x} \right) \left(\frac{L^3}{6} - \frac{L^3}{2} \right)$$

35.1 Enunciado

Sean dos cilindros concéntricos de longitud unitaria, con radios R_{ext} y R_{int}, respectivamente, separados por una película de aceite de viscosidad μ. El cilindro exterior gira a una velocidad angular ω_{int} (sentido horario), mientras que el exterior gira a una velocidad angular ω_{ext} (sentido antihorario).



Halle las ecuaciones que definen:

1. La distribución de velocidades entre cilindros.
2. La distribución de presiones entre cilindros.
3. El par necesario en el cilindro exterior para que se produzca el giro.

35.2 Resolución

Cálculos previos

- Las condiciones de contorno que definen este problema són:

$$r = R_{ext} \Rightarrow V_\theta = \omega_{ext}\, R_{ext}$$

$$r = R_{int} \Rightarrow V_\theta = \omega_{int} R_{int} (-1)$$

- La ecuación de continuidad, en coordenadas cilíndricas, establece:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \times \frac{\partial}{\partial r}(\rho r V_r) + \frac{1}{r} \times \frac{\partial}{\partial \theta}(\rho V_\theta) + \frac{\partial}{\partial z}(\rho V_z) = 0$$

- La ecuación de Navier-Stokes, en cilíndricas, se enuncia:

$$\rho \left(\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + V_\theta \frac{1}{r}\frac{\partial V_r}{\partial \theta} + V_z \frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r} \right) =$$
$$\rho\, g_r - \frac{\partial P}{\partial r} + \mu \left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, V_r) \right) + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} + \frac{\partial^2 V_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta} \right]$$

$$\rho \left(\frac{\partial V_\theta}{\partial t} + V_r \frac{\partial V_\theta}{\partial r} + V_\theta \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + V_z \frac{\partial V_\theta}{\partial z} - \frac{V_\theta\, V_r}{r} \right) =$$
$$\rho\, g_\theta - \frac{\partial P}{r\partial \theta} + \mu \left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, V_\theta) \right) + \frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{\partial^2 V_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta} \right]$$

$$\rho \left(\frac{\partial V_z}{\partial t} + V_r \frac{\partial V_z}{\partial r} + V_\theta \frac{1}{r}\frac{\partial V_z}{\partial \theta} + V_z \frac{\partial V_z}{\partial z} \right) =$$
$$\rho\ g_z - \frac{\partial P}{\partial z} + \mu \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\ \frac{\partial V_z}{\partial r} \right) + \frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2} \right]$$

Únicamente existe variación de velocidad V_θ en dirección radial, con lo que se tiene:

- La ecuación de continuidad:

$$\frac{1}{r}\ \frac{\partial}{\partial \theta}(\rho V_\theta) = 0 \Rightarrow \frac{\partial}{\partial \theta}(\rho V_\theta) = 0 \Rightarrow \rho \frac{\partial V_\theta}{\partial \theta} = 0 \Rightarrow \frac{\partial V_\theta}{\partial \theta} = 0$$
$$\rho = \text{constante}$$

- La ecuación de Navier-Stokes:

  La presión reducida variará únicamente en la dirección radial

$$-\rho\ \frac{V_\theta^2}{r} = \rho\, g_r - \frac{\partial P}{\partial r} = -\rho\, g \frac{\partial h}{\partial r} - \frac{\partial P}{\partial r} = -\frac{\partial P^*}{\partial r}$$

$$0 = \mu\ \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\ \ V_\theta) \right) \Rightarrow \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\ \ V_\theta) \right) = 0$$

Se considera que no existen fuerzas másicas en la dirección z.

$$0 = \rho\ g_z - \frac{\partial P}{\partial z} \Rightarrow \frac{\partial P^*}{\partial z} = 0$$

No hay gradiente de presión reducida en la dirección z.

1. Así, se tiene que:

De la primera ecuación de Navier-Stokes:

$$P^* = \int \rho \frac{V_\theta^2}{r} dr$$

Será necesario conocer la distribución de velocidades en la dirección θ, ya que esta dependerá de r.

De la segunda ecuación de Navier-Stokes:

$$0 = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\ V_\theta\right)\right)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\ V_\theta\right) = C_1$$

Las constantes C1 y C2 son constantes de integración

$$\frac{\partial}{\partial r}\left(r\ V_\theta\right) = r\ C_1$$

$$r\ V_\theta = C_1\ \frac{r^2}{2} + C_2$$

$$V_\theta = C_1\ \frac{r}{2} + \frac{C_2}{r}$$

Con las condiciones de contorno:

$$r = R_{ext} \Rightarrow V_\theta = \omega_{ext}\ R_{ext}$$

$$r = R_{int} \Rightarrow V_\theta = \omega_{int}\ R_{int}\ (-1)$$

$$(1) \qquad \omega_{ext}\ R_{ext} = C_1\ \frac{R_{ext}}{2} + \frac{C_2}{R_{ext}}$$

$$(2) \qquad -\omega_{int}\ R_{int} = C_1\ \frac{R_{int}}{2} + \frac{C_2}{R_{int}}$$

$$C_2 = \omega_{ext}\ R_{ext}^2 - C_1\ \frac{R_{ext}^2}{2} \Rightarrow -\omega_{int}\ R_{int} = C_1\ \frac{R_{int}}{2} + \frac{1}{R_{int}}\left[\omega_{ext}\ R_{ext}^2 - C_1\ \frac{R_{ext}^2}{2}\right]$$

$$-\omega_{ext}\ R_{ext}^2 - \omega_{int}\ R_{int}^2 = C_1\left(\frac{R_{int}^2}{2} - \frac{R_{ext}^2}{2}\right) \Rightarrow C_1 = \frac{2\left(\omega_{int}\ R_{int}^2 + \omega_{ext}\ R_{ext}^2\right)}{R_{ext}^2 - R_{int}^2}$$

$$C_2 = \omega_{ext}\ R_{ext}^2 - \frac{2\ \left(\omega_{int}\ R_{int}^2 + \omega_{ext}\ R_{ext}^2\right)}{R_{ext}^2 - R_{int}^2}\ \frac{R_{ext}^2}{2}$$

$$C_2 = \omega_{ext}\ R_{ext}^2 - R_{ext}^2\ \frac{\left(\omega_{int}\ R_{int}^2 + \omega_{ext}\ R_{ext}^2\right)}{R_{ext}^2 - R_{int}^2}$$

Entonces:

$$V_\theta = \frac{2\ \left(\omega_{int}\ R_{int}^2 + \omega_{ext}\ R_{ext}^2\right)}{R_{ext}^2 - R_{int}^2}\ \frac{r}{2} + \frac{1}{r}\left(\omega_{ext}\ R_{ext}^2 - R_{ext}^2\ \frac{\left(\omega_{int}\ R_{int}^2 + \omega_{ext}\ R_{ext}^2\right)}{R_{ext}^2 - R_{int}^2}\right)$$

2. De la primera ecuación de Navier-Stokes:

$$P^* = \int \rho \frac{V_\theta^2}{r} dr$$

Introduciendo la ecuación de V_θ en la integral, se tiene:

$$P^* = \int_{R_{int}}^{R_{ext}} \rho \, \frac{1}{r} \left(C_1^2 \, \frac{r^2}{4} + \frac{C_2^2}{r^2} + C_1 \, C_2 \right) dr$$

y se halla:

$$P^* = \rho \left(C_1^2 \, \frac{R_{ext}^2 - R_{int}^2}{8} + \frac{C_2^2}{2} \, \frac{1}{R_{int}^2 - R_{ext}^2} + C_1 \, C_2 \, Ln \frac{R_{ext}}{R_{int}} \right)$$

3. Los esfuerzos cortantes en cilíndricas se pueden dar:

$$\tau_{r\theta} = r \;\mu \; \frac{\partial}{\partial r}\left(\frac{V_\theta}{r} \right) \Bigg|_{r=R_{ext}}$$

puesto que

$$V_\theta = C_1 \; \frac{r}{2} + \frac{C_2}{r}$$

$$\frac{\partial}{\partial r}\left(\frac{V_\theta}{r} \right) = - \frac{C_2}{r}$$

$$\tau_{r\theta} = - \mu \, C_2$$

$$M_{R_{ext}} = \tau_{r\theta} \; 2 \;\pi \; R_{ext} \; R_{ext}$$

así, queda:

$$M_{R_{ext}} = -2 \;\pi \;\mu \; C_2 \, R_{ext}^2$$