Clase auxiliar.

P1) En efecto.

1° escribimos las Ecuaciones de Navier-Stokes.

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]-\frac{\partial P}{\partial x}+\rho g_x$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right]-\frac{\partial P}{\partial y}+\rho g_y$$

adimensionando las Ecuaciones.

$$u^*=\frac{u}{V},\quad v^*=\frac{v}{V},\quad x^*=\frac{x}{L},\quad y^*=\frac{y}{L}$$

$$t^*=\frac{tV}{L},\quad \frac{\partial()}{\partial x}=\frac{1}{D}\frac{\partial()}{\partial x^*}$$

$$\vec{g}^*=\frac{Lg}{V^2}.\quad p^*=\frac{P}{\rho v^2}.$$

$$\rho\left(\frac{\partial Vu^*}{\partial \frac{Lt^*}{V}}+u^*V\frac{\partial u^*v}{\partial x^*L}+v^*V\frac{\partial u^*v}{\partial y^*L}+w^*V\frac{\partial u^*v}{\partial z^*L}\right)$$

$$=\mu\left[\frac{\partial^2 u^*V}{\partial(x^*L)^2}+\frac{\partial^2 u^*V}{\partial(y^*L)^2}+\frac{\partial^2 u^*V}{\partial(z^*L)^2}\right]-\frac{\partial P^*\rho V^2}{\partial x^*L}+\rho\frac{g^*V^2}{L}$$

$$\Rightarrow \rho \frac{V^2}{L}\left(\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + V^*\frac{\partial u^*}{\partial y^*}\right)$$

$$= \mu \frac{V}{L^2}\left\{\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right\} - \frac{\rho V^2}{L}\frac{\partial P^*}{\partial x^*} + \rho g^* \frac{V^2}{L}$$

$$\Rightarrow Re\left(\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + V^*\frac{\partial u^*}{\partial y^*}\right) = \frac{\partial^2 u^*}{\partial x_*^2} + \frac{\partial^2 u^*}{\partial y_*^2}$$

$$- Re\frac{\partial P_*}{\partial x^*} + Re\,\overline{g_*}\,x$$

Tomando lim $Re \to 0 \Rightarrow$

$$0 = \frac{\partial^2 u^*}{\partial x_*^2} + \frac{\partial^2 u^*}{\partial y_*^2}$$

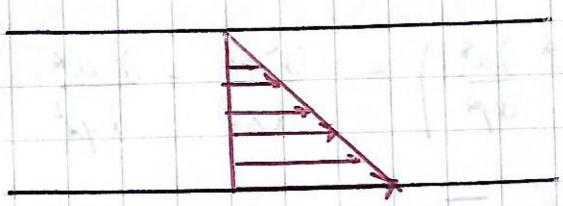
($\frac{\partial^2 u^*}{\partial x_*^2} \to 0$) Por Continuidad.

$$\boxed{0 = \frac{\partial^2 u^*}{\partial y_*^2}}$$

luego, como Tenemos claramente en Fluido Couette

Simplemente.

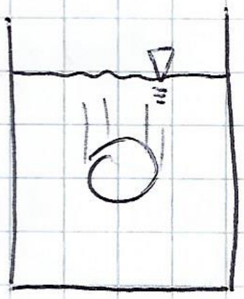
$$u^*(y) = C_1 y + C_2$$



ahora veamos que pasa si $Re \to \infty$

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = g^* - \frac{\partial P^*}{\partial x^*}$$

P2)



$$V_t = h(a, \rho_s, \rho_L, \mu_L)$$

$[a] = L$ $\qquad$ $[\mu] = M/TL$

$[\rho] = \frac{m}{L^3}$ $\qquad$ $[V_t] = L/T$

Tenemos L, M, T como variable.

Luego elegimos $\rho_L$, $a$ y $\mu_L$ como variables repetidas.

$(m = 3)$

Vamos a tener. $5 - 3 = 2$ ($\Pi$'s)

$$\Pi_1 = \frac{L}{T} \left( L^a \right) \left( \frac{m^b}{L^{3b}} \right) \left( \frac{m^c}{T^c L^c} \right) = 0$$

L: $1 + a - 3b - c = 0$

T: $-1 - c = 0$

m: $b + c = 0$

$\Rightarrow$ $c = -1$, $b = 1$

$1 + a - 3 + 1 = 0$ $\quad$ $a = 3 - 1 - 1$ $\quad$ $a = 1$

$$\Pi_1 = \frac{V_t \, \Delta \rho}{\mu}$$

$$\Pi_2 = \frac{\rho_S}{\rho_L}$$

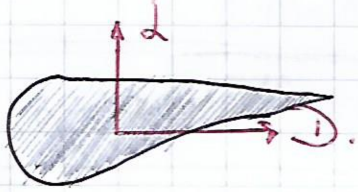
$$\Pi_1 = h(\Pi_2) \Rightarrow$$

$$\boxed{\frac{V_t \, \Delta \rho_L}{\mu} = h\left(\frac{\rho_S}{\rho_L}\right)}$$

## Capa límite.

→ Concepto previo

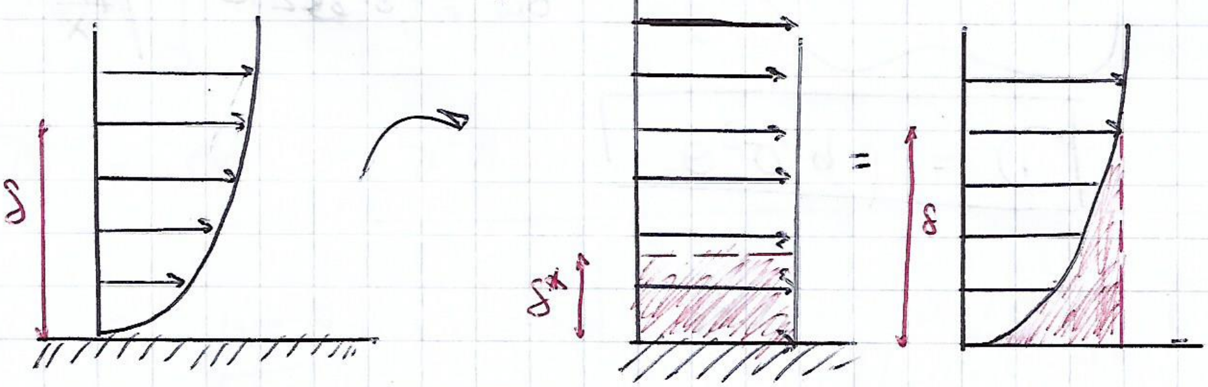
$$C_L = \frac{L}{1/2\, \rho v^2 A} \quad , \quad C_D = \frac{D}{1/2\, \rho v^2 A}$$




Tipo de capa límite laminar e Turbulento.

$Re < 3 \cdot 10^6 \Rightarrow$ Laminar.

$Re > 3 \cdot 10^6 \Rightarrow$ Turbulento.



$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

aproximacion de blasius.

$$\delta = 5\sqrt{\frac{Ux}{\nu}}$$

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}} \quad , \quad \frac{\delta^*}{x} = \frac{1.721}{\sqrt{Re_x}}$$

$$\frac{\theta}{x} = \frac{0.664}{\sqrt{Re_x}} .$$

Caso laminar.

$$\tau_w = 0.332\, U^{3/2} \sqrt{\frac{\rho\mu}{x}}$$

$$D = \rho\, b\, U^2 \theta$$

Capa límite Turbulenta.

$$\frac{\delta}{x} = 0.376/\sqrt[5]{Re}$$

$$\delta^* = 0.0463\left(\frac{U}{\nu}\right)^{1/5} Rex^{4/5}.$$

$$\tau_w = 0.0288\ \frac{\rho \nu^2}{Rex^{1/5}}$$

$$C_D = 0.072\ Re^{-1/5}$$

Ecuación de Von-Karman.

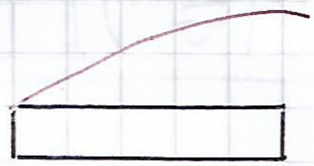
$$\frac{\tau_w}{\rho} = \nu^2\ \frac{d\theta}{dx}$$

$$D_f = \int_0^L b\,\tau_w\, dx.$$

$$D_f = C_D\ \frac{1}{2}\ \rho\, \nu^2 A.$$

$$C_{Df} = \frac{1.328}{\sqrt{Rex}}\ (\text{laminar}),\quad C_{Df} = \frac{0.455}{(\ln Re)^{2.58}}\ (\text{Turbulento})$$

P3)



$v = 2$
$L = 1$
$b = 3$

a) aire → $\rho = 1.23$, $\nu = 1.46 \cdot 10^{5}$

$$Re = \frac{vL}{\nu} = 1.3 \cdot 10^{5}$$

donde $\nu = \frac{\rho}{\mu}$

Como $1.3 \times 10^{5} < 3 \times 10^{6} \Rightarrow$

Capa límite laminar luego usamos las Ecuaciones de Blasius

$$C_D = \frac{1.328}{\sqrt{Re}} = 0.00359$$

$$D = C_D \frac{1}{2} \rho v^2 \underbrace{bL}_{A} = 0.0265 \text{ N}$$

$$\frac{\delta}{L} = \frac{5}{Re^{1/2}} \Rightarrow \delta = 0.0135 \text{ m}$$

$\delta^* = 4.65\,mm$ ∧ $\theta = 1.79\,mm$

Repetimos para el Agua.

$Re = 1.96 \times 10^6 \Rightarrow$ laminar

$C_D = \dfrac{1.328}{\sqrt{Re}} = 0.000948$

$D = 5.7\,N$

$\dfrac{\delta}{L} = \dfrac{5}{\sqrt{Re}} = 0.00357\,m.$

$\delta = 0.00357 \rightarrow 3.57\,mm$

$\delta^* = 1.23\,mm$ ∧ $\theta = 0.48\,mm$