Ultima parte de la derivación de la ecuación de onda de gravedad interna

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)^2 \left(\frac{\partial w'}{\partial x} - \frac{\partial u'}{\partial z}\right) + N^2 \frac{\partial w'}{\partial x} = 0$$

Se sabe que por ecuacion de continuidad

$$\frac{\partial u'}{\partial x} = -\frac{\partial w'}{\partial z}$$

$$\frac{\partial u'}{\partial x} = -\frac{\partial w'}{\partial z}/\frac{\partial}{\partial z}$$

$$\frac{\partial^2 u'}{\partial z \partial x} = -\frac{\partial^2 w'}{\partial z^2}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u'}{\partial z}\right) = -\frac{\partial^2 w'}{\partial z^2}$$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)^2 \left(\frac{\partial w'}{\partial x} - \frac{\partial u'}{\partial z}\right) + N^2 \frac{\partial w'}{\partial x} = 0/\frac{\partial}{\partial x}$$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\left(\frac{\partial w'}{\partial x} - \frac{\partial u'}{\partial z}\right) + N^2 \frac{\partial w'}{\partial x} = 0/\frac{\partial}{\partial x}$$

$$\left(\frac{\partial^2}{\partial t^2} + 2\bar{u}\frac{\partial}{\partial x}\frac{\partial}{\partial t} + \bar{u}^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial w'}{\partial x} - \frac{\partial u'}{\partial z}\right) + N^2 \frac{\partial w'}{\partial x} = 0/\frac{\partial}{\partial x}$$

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial w'}{\partial x}\right) - \frac{\partial^2}{\partial t^2}\left(\frac{\partial u'}{\partial z}\right) + 2\bar{u}\frac{\partial}{\partial t}\frac{\partial}{\partial x}\left(\frac{\partial w'}{\partial x}\right) - 2\bar{u}\frac{\partial}{\partial x}\frac{\partial}{\partial t}\left(\frac{\partial u'}{\partial z}\right) + \bar{u}^2\frac{\partial^2}{\partial x^2}\left(\frac{\partial w'}{\partial x}\right) - \bar{u}^2\frac{\partial^2}{\partial x^2}\frac{\partial u'}{\partial z}$$

$$+ N^2\frac{\partial w'}{\partial x} = 0/\frac{\partial}{\partial x}$$

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w'}{\partial x^2}\right) - \frac{\partial^2}{\partial t^2}\left(\boldsymbol{\frac{\partial^2 u'}{\partial x \partial z}}\right) + 2\bar{u}\frac{\partial}{\partial t}\frac{\partial}{\partial x}\left(\frac{\partial^2 w'}{\partial x^2}\right) - 2\bar{u}\frac{\partial}{\partial x}\frac{\partial}{\partial t}\left(\boldsymbol{\frac{\partial^2 u'}{\partial x \partial z}}\right) + \bar{u}^2\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w'}{\partial x^2}\right)$$

$$- \bar{u}^2\frac{\partial^2}{\partial x^2}\left(\boldsymbol{\frac{\partial^2 u'}{\partial x \partial z}}\right) + N^2\frac{\partial^2 w'}{\partial x^2} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u'}{\partial z}\right) = -\frac{\partial^2 w'}{\partial z^2}$$

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w'}{\partial x^2}\right) - \frac{\partial^2}{\partial t^2}\left(-\frac{\partial^2 w'}{\partial z^2}\right) + 2\bar{u}\frac{\partial}{\partial t}\frac{\partial}{\partial x}\left(\frac{\partial^2 w'}{\partial x^2}\right) - 2\bar{u}\frac{\partial}{\partial x}\frac{\partial}{\partial t}\left(-\frac{\partial^2 w'}{\partial z^2}\right) + \bar{u}^2\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w'}{\partial x^2}\right)$$

$$- \bar{u}^2\frac{\partial^2}{\partial x^2}\left(-\frac{\partial^2 w'}{\partial z^2}\right) + N^2\frac{\partial^2 w'}{\partial x^2} = 0$$

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w'}{\partial x^2}\right) + \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w'}{\partial z^2}\right) + 2\bar{u}\frac{\partial}{\partial t}\frac{\partial}{\partial x}\left(\frac{\partial^2 w'}{\partial x^2}\right) + 2\bar{u}\frac{\partial}{\partial x}\frac{\partial}{\partial t}\left(\frac{\partial^2 w'}{\partial z^2}\right) + \bar{u}^2\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w'}{\partial x^2}\right)$$

$$+ \bar{u}^2\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w'}{\partial z^2}\right) + N^2\frac{\partial^2 w'}{\partial x^2} = 0$$

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w'}{\partial x^2}\right) + \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w'}{\partial z^2}\right) + 2\bar{u}\frac{\partial}{\partial t}\frac{\partial}{\partial x}\left(\frac{\partial^2 w'}{\partial x^2}\right) + 2\bar{u}\frac{\partial}{\partial x}\frac{\partial}{\partial t}\left(\frac{\partial^2 w'}{\partial z^2}\right) + \bar{u}^2\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w'}{\partial x^2}\right)$$

$$+ \bar{u}^2\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w'}{\partial z^2}\right) + N^2\frac{\partial^2 w'}{\partial x^2} = 0$$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)^2\left(\frac{\partial^2 w'}{\partial x^2} + \frac{\partial^2 w'}{\partial z^2}\right) + N^2\frac{\partial^2 w'}{\partial x^2} = 0$$