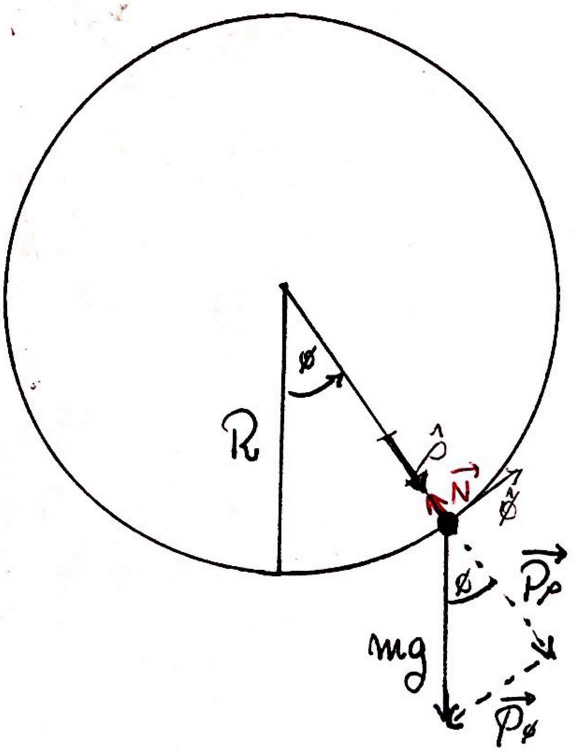

Por geometría:

$\vec{P}_\rho = mg\cos\phi\,\hat{\rho}$

$\vec{P}_\phi = -mg\,\text{sen}\,\phi\,\hat{\phi}$

$\vec{N} = -N\hat{\rho}$

$\vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})\hat{\phi}$

Pero $\rho = R \rightarrow \dot{\rho} = \ddot{\rho} = 0$.

$\therefore\ \vec{a} = -\rho\dot{\phi}^2\hat{\rho} + \rho\ddot{\phi}\hat{\phi}$

Ecuaciones de movimiento

$\hat{\rho}|\quad mg\cos\phi - N = -m\rho\dot{\phi}^2 = -mR\dot{\phi}^2$

$\hat{\phi}|\quad -mg\,\text{sen}\,\phi = m\rho\ddot{\phi} = mR\ddot{\phi}$

Usando $\hat{\phi}|$:

$\dot{\phi} = \frac{v}{R}$

$\dot{\phi}_0 = \frac{\sqrt{4gR}}{R} = \sqrt{\frac{4g}{R}}$

$$-mg\,\text{sen}\,\phi = mR\dot{\phi}\frac{d\dot{\phi}}{d\phi}$$

$$-\frac{g}{R}\,\text{sen}\,\phi\,d\phi = \dot{\phi}\,d\dot{\phi} \qquad \int_0^{\phi}\int_{\sqrt{\frac{4g}{R}}}^{\dot{\phi}}$$

$$\frac{g}{R}(\cos\phi - 1) = \frac{\dot{\phi}^2 - \frac{4g}{R}}{2}$$

$$\frac{2g}{R}(\cos\phi - 1) = \dot{\phi}^2 - \frac{4g}{R}$$

$$\frac{4g}{R} + \frac{2g}{R}(\cos\phi - 1) = \dot{\phi}^2$$

$$\frac{2g}{R}\left(2+\cos\phi-1\right)=\dot{\phi}^2$$

$$\sqrt{\frac{2g}{R}}\sqrt{1+\cos\phi}=\dot{\phi}$$

$$\sqrt{1+\cos\phi}=\sqrt{1+\cos\phi}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$
$$=\sqrt{\frac{1+\cos\phi}{2}}\sqrt{2}$$
$$=\cos\left(\frac{\phi}{2}\right)\cdot\sqrt{2}$$

$$\sqrt{\frac{2g}{R}}\cdot\sqrt{2}\cos\frac{\phi}{2}=\frac{d\phi}{dt}$$

$$2\sqrt{\frac{g}{R}}\,dt=\frac{d\phi}{\cos\left(\frac{\phi}{2}\right)}$$

sea $u=\frac{\phi}{2}$

$du=\frac{1}{2}d\phi \rightarrow d\phi=2du$

$$2\sqrt{\frac{g}{R}}\,dt=2\frac{du}{\cos(u)}$$

$$\sqrt{\frac{g}{R}}\,dt=\sec(u)\,du \qquad \int_0^t \quad \int_0^u$$

$$\sqrt{\frac{g}{R}}\,t=\ln\left(\frac{tg\frac{\phi}{2}+\sec\frac{\phi}{2}}{tg\,0+\sec 0}\right)$$

$$\therefore\ t=\sqrt{\frac{R}{g}}\ln\left[tg\frac{\phi}{2}+\sec\frac{\phi}{2}\right].$$