Problem set 9
Macroeconomics - Professor Elton Dusha

1. Consider a consumer solving the following work/leisure problem:

$$\max_{c_1,c_2,l_1,l_2} u(c_1,l_1) + \beta u(c_2,l_2)$$

subject to:

$$c_1 + \frac{c_2}{1+r} = w_1(1-l_1) + \frac{w_2(1-l_2)}{1+r}$$

where l_i is leisure in period i and $1-l_i$ is labour in period i.

a) Solve the optimization problem for this consumer assuming that $u(c,l) = \alpha \ln c + (1-\alpha) \ln l$.

b) Obtain the FOC's (first order conditions) for the problem assuming that $u(c,l) = \frac{\left(c^\alpha l^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}$. Show that these FOC's become the same FOC's as the ones in part a when $\sigma = 1$. Solve for c_1, c_2, l_1, l_2 as functions of parameter values for $u(c,l) = \frac{\left(c^\alpha l^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}$ and $w_1 = w_2$.

2) Suppose a consumer solving the following maximization problem:

$$\max_{c_1,c_2} \frac{(c_1)^{1-\sigma}}{1-\sigma} + \frac{(c_2)^{1-\sigma}}{1-\sigma}$$

subject to:

$$c_1 + \frac{c_2}{1+r} = w_1$$

a) Find the optimal consumption bundles

b) Write down the income, substitution and wealth effects for both bundles. Find the total effects for $\sigma < 1$, $\sigma = 1$ and $\sigma > 1$.

3) Consider a three period economy. Preferences are given by $u(c_1)+u(c_2)+u(c_3)$; where c_t is consumption in period t. The real interest rate is $r = 0$. Before tax incomes are given by $e_1 = 200$, $e_2 = 1000$ and $e_3 = 200$. The consumer is not allowed to borrow, i.e. there is a constraint on borrowing. The government uses lump sum taxes to finance government expenditures.

a) Assuming that lump-sum taxes are given by $T_1 = 100, T_2 = 100$, and $T_3 = 100$, solve the consumer's optimal consumption and end-of-period net asset position in each period.

b) Assume that the government reduces current tax T_1 to zero without changing its expenditures. T_3 also remains at 100. Solve the consumer's optimal consumption and end-of-period net asset position in each period.