Q2 Feb 2012

$$\max_{c_1,c_2} \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\sigma}}{1-\sigma}$$

s.t.: $c_1 + \frac{c_2}{1+r} = w_1$

$$\mathcal{L} = \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\sigma}}{1-\sigma} + \lambda\left[w_1 - c_1 - \frac{c_2}{1+r}\right]$$

c_1: $c_1^{-\sigma} = \lambda$ $\qquad c_2$: $c_2^{-\sigma} = \frac{\lambda}{1+r} = \frac{c_1^{-\sigma}}{1+r}$ $\qquad c_1 + \frac{c_2}{1+r} = w.$

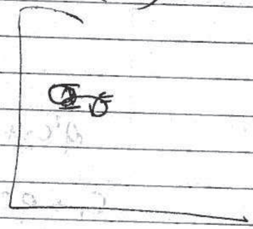
Then $c_1^{-\sigma} = (1+r)\, c_2^{-\sigma} \Rightarrow \left(\frac{c_1}{c_2}\right)^{-\sigma} = 1+r \Rightarrow \left(\frac{c_1}{c_2}\right)^{\sigma} = \frac{1}{1+r}. \Rightarrow \left(\frac{c_2}{c_1}\right)^{\sigma} = 1+r$

$\Rightarrow \frac{c_2}{c_1} = (1+r)^{1/\sigma} \Rightarrow \boxed{c_2 = c_1 (1+r)^{1/\sigma}}$

Budget : $c_1 + c_1(1+r)^{1/\sigma - 1} = w_1$

$c_1\left[1 + (1+r)^{1/\sigma - 1}\right] = w_1$

$$\boxed{c_1 = \frac{w_1}{1+(1+r)^{1/\sigma - 1}}} \qquad \boxed{c_2 = \frac{w_1 (1+r)^{1/\sigma}}{1+(1+r)^{1/\sigma - 1}}}$$



) i) $\sigma < 1$ $\quad r\uparrow$

$\sigma = 1$ $\quad c_1 \downarrow.$

$\sigma > 1.$

	c_1	c_2
IE	+	+
SE.	—	+
WE	0	0
TE	$\sigma<1 \Rightarrow c_1 -$; $\sigma=1 \Rightarrow 0$; $\sigma>1 \Rightarrow c_1 +$	+