Auxiliar 10

Magnetostática

- Ley de Biot - Savart (Campo por definición) ¨

  (Los campos son generados por corrientes)

  Lineales: I — Superficiales: $\vec{K}$ — Volumétricas: $\vec{J}$

  - Para un circuito: $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$

  - Cosas más complicadas

    Distrib superficial: $\vec{B} = \frac{\mu_0}{4\pi} \iint \frac{\vec{K}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')\, ds}{|\vec{r} - \vec{r}\,'|^3}$

    Distrib Volumétrica: $\vec{B} = \frac{\mu_0}{4\pi} \iiint \frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} dV$

  - Recordar que: $\vec{K} = \sigma \vec{v}$, $\vec{J} = \rho \vec{v}$ y usualmente $\vec{v} = \vec{\omega} \times \vec{r}\,'$

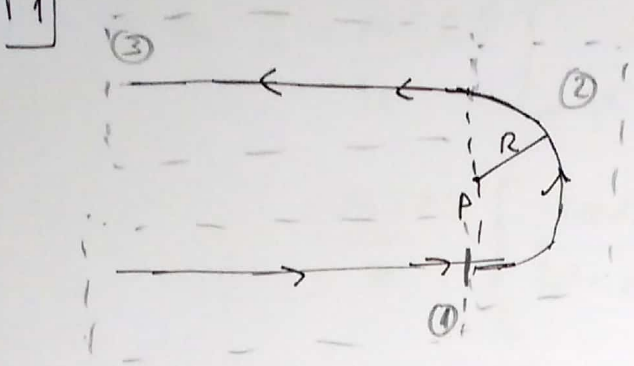
- Ley de Ampere (Forma fácil de sacar campos)

  Forma diferencial: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

  Forma Integral: $\oint_{\Gamma} \vec{B} \cdot d\vec{l} = \mu_0 \underbrace{\iint \vec{J}\, d\vec{s}}_{I}$

Scanned by CamScanner

P_1



→ Calcular el campo magnético en el punto P ubicado en el centro del semicírculo

- Usamos el principio de superposición para el campo magnético

- B por definición $\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{l}\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$

- Calculamos primero el campo del semicírculo

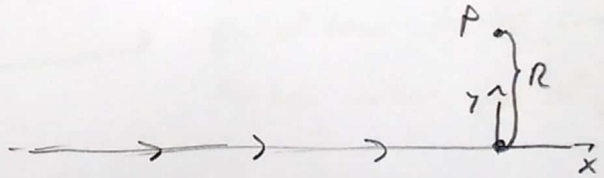
$$\vec{B}_{sc} = \frac{\mu_0}{4\pi}\int \frac{I\,d\hat{l}\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|} \quad ; \quad \vec{r} = 0, \quad \vec{r}' = R\hat{r}, \quad d\vec{l} = R\,d\theta\,\hat{\theta} \quad ; \quad \theta\in[0,\pi]$$

$$= \frac{\mu_0}{4\pi}\int_0^\pi \frac{I\cancel{R}\,d\theta\,\hat{\theta}\times(-\cancel{R}\hat{r})}{R^{\cancel{3}}}$$

$$= \frac{\mu_0}{4\pi}\frac{I}{R}\int_0^\pi d\theta\,(-\overbrace{\hat{\theta}\times\hat{z}}^{-\hat{z}}) \quad ; \quad \hat{z} \text{ no depende de } \theta$$

$$= \frac{\mu_0}{\cancel{4\pi}}\frac{I}{R}\cancel{\pi}\cdot\hat{z} \Rightarrow \boxed{\vec{B}_{sc} = \frac{\mu_0 I}{4R}\hat{z}}$$

- Calculamos el campo de un alambre



$$\vec{r} = R\hat{y}, \quad \vec{r}' = x\hat{x}, \quad d\vec{l} = dx\,\hat{x}; \quad x\in(-\infty,0)$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi}\int_{-\infty}^0 \frac{I\,dx\,\hat{x}\times(R\hat{y}-x\hat{x})}{(R^2+x^2)^{3/2}}$$

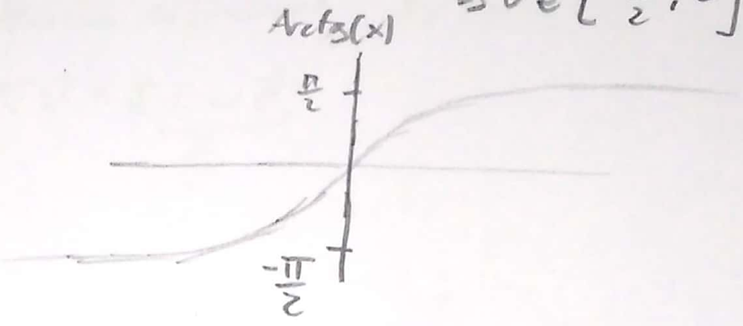
$$= \frac{\mu_0 I}{4\pi}\int_{-\infty}^0 \frac{[R\,dx\,\overbrace{\hat{x}\times\hat{y}}^{\hat{z}} - x\,dx\,\overbrace{\hat{x}\times\hat{x}}^{0}]}{(R^2+x^2)^{3/2}}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I R \hat{z}}{4\pi}\int_{-\infty}^{0}\frac{dx}{(R^2+x^2)^{3/2}}$$

Hacemos el cambio de variables

$$x = R\, tg\,\theta \iff \theta = Arctg\left(\frac{x}{R}\right)$$

$$dx = R\,sec^2(\theta)\,d\theta \qquad \Rightarrow \theta \in \left[-\frac{\pi}{2}, 0\right]$$



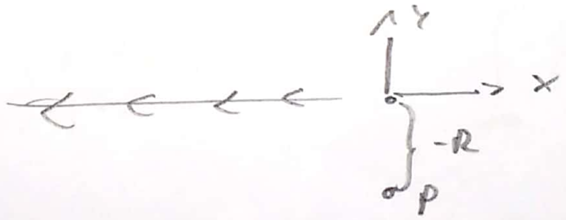
$$= \frac{\mu_0 I R \hat{z}}{4\pi}\int_{-\frac{\pi}{2}}^{0}\frac{R\,sec^2(\theta)\,d\theta}{(R^2+R^2 tg^2(\theta))^{3/2}}$$

$$= \frac{\mu_0 I \cancel{R} \hat{z}}{4\pi}\int_{-\frac{\pi}{2}}^{0}\frac{\cancel{R}\,sec^2(\theta)\,d\theta}{R^{\cancel{3}}(sec^2(\theta))^{3/2}}$$

$$= \frac{\mu_0 I \hat{z}}{4\pi R}\int_{-\frac{\pi}{2}}^{0}\cos\theta\,d\theta = \frac{\mu_0 I \hat{z}}{4\pi R}\overbrace{\sin\theta\Big|_{-\pi/2}^{0}}^{1} = \frac{\mu_0 I \hat{z}}{4\pi R}$$

$$\Rightarrow \boxed{B_{cable} = \frac{\mu_0 I \hat{z}}{4\pi R}}$$

Para el alambre de arriba



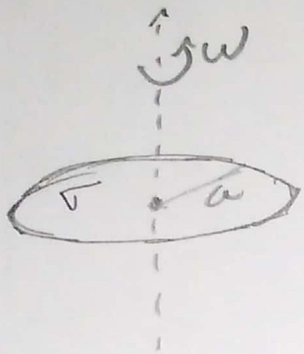
$$r = -R\hat{y}, \quad \vec{r}' = x\hat{x}, \quad d\vec{l} = -dx\,\hat{x}$$

En el caso de los cables el sentido viene por el $\underline{\underline{d\vec{l}}}$

$$\Rightarrow B_{cable} = \frac{\mu_0 I \hat{z}}{4\pi R}$$

$$\therefore \boxed{B_{tot} = \frac{\mu_0 I}{R}\left(\frac{1}{2\pi} + \frac{1}{4}\right)\hat{z}}$$

P2)



→ Calcular el campo magnético en el eje

• Usamos coordenadas cilíndricas, $\hat{r}, \hat{\theta}, \hat{z}$

$$\vec{J} = \sigma \vec{v} = \sigma \underbrace{r \omega \hat{\theta}}_{\vec{v}}$$

• Usamos el campo por definición: $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} ds$

Donde $\vec{r} = z\hat{k}$, $\vec{r}' = r\hat{r}$, $ds = r dr d\theta$, reemplazamos

$$B(z) = \frac{\mu_0}{4\pi} \int_0^a \int_0^{2\pi} \frac{\sigma r \omega \hat{\theta} \times (z\hat{k} - r\hat{r}) r dr d\theta}{(z^2 + r^2)^{3/2}} \quad ; \hat{\theta} \times \hat{k} = \hat{r} \quad ; \hat{\theta} \times \hat{r} = -\hat{k}$$

$$= \frac{\mu_0 \sigma \omega}{4\pi} \int_0^{2\pi} \int_0^a \frac{(r^2 z \hat{r} + r^3 \hat{k}) dr d\theta}{(z^2 + r^2)^{3/2}} \quad ; \text{ Pero } \int_0^{2\pi} \hat{r} \, d\theta = 0$$

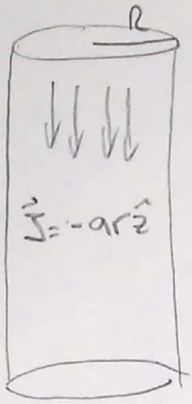
$$= \frac{\mu_0 \sigma \omega \hat{k}}{4\pi} \int_0^{2\pi} \int_0^a \frac{r^3 dr d\theta}{(z^2 + r^2)^{3/2}} = \frac{\mu_0 \sigma \omega}{\cancel{4\pi}\, 2} \hat{k} \cancel{2\pi} \int_0^a \frac{r^3}{(z^2 + r^2)^{3/2}} dr$$

$$= \frac{\mu_0 \sigma \omega}{2} \hat{k} \int_0^a \frac{r^3}{(z^2 + r^2)} dr \xrightarrow{\text{Por tablas}} = \frac{a^2 + 2z^2 - 2|z|(a^2 + z^2)^{1/2}}{(a^2 + z^2)^{1/2}}$$

$$\Rightarrow B(z) = \frac{\mu_0 \sigma \omega}{2} \left[\frac{a^2 + 2z^2 - 2|z|(a^2 + z^2)^{1/2}}{(a^2 + z^2)^{1/2}} \right] \hat{k}$$

P3

→ Calcular el campo magnético en el espacio.

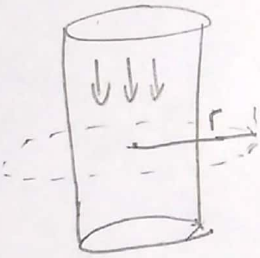


∘ Usamos ley de Ampere.

$$\oint B\,dl = \mu_0 \iint \vec{J}$$

Circuito cerrado paralelo a donde suponemos el campo

$r \geq R$



$$\Rightarrow \oint B\,dl = B\,\underbrace{2\pi r}_{\text{Perímetro de la circunferencia}}$$

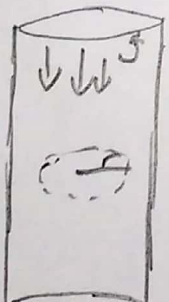
$$\Rightarrow \mu_0 \iint \vec{J}\,ds = \mu_0 \int_0^R \int_0^{2\pi} -\alpha r \cdot \underbrace{r\,d\theta\,dr}_{ds}$$

$$= -\alpha \mu_0 2\pi \frac{R^3}{3}$$

Juntamos

$$\Rightarrow \cancel{2\pi} r B = -\alpha \mu_0 \cdot \cancel{2\pi} \frac{R^3}{3} \Rightarrow \boxed{\vec{B} = \frac{-\alpha \mu_0 R^3}{3r}\hat{\theta}}$$

$r < R$



$$\Rightarrow \oint B\,dl = B \cdot 2\pi r$$

$$\Rightarrow \mu_0 \iint \vec{J}\,ds = \mu_0 \int_0^r \int_0^{2\pi} -\alpha r^2\,d\theta\,dr = -\alpha \mu_0 2\pi \frac{r^3}{3}$$

$$\Rightarrow \cancel{2\pi r} B = -\alpha \mu_0 \cancel{2\pi} \frac{r^{\cancel{3}\,2}}{3} \Rightarrow \boxed{\vec{B} = -\frac{\alpha \mu_0 r^2}{3}\hat{\theta}}$$