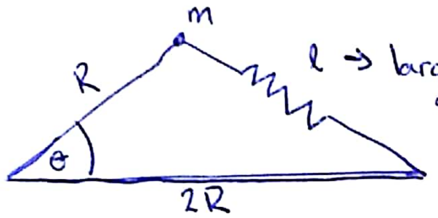
<u>P1</u>

Auxiliar 7

• Un poco de geometría:



$y = R\sin\theta \rightarrow$ altura

$x = R\cos\theta$

• Por el teo. del coseno se obtiene que:

$$\ell^2 = R^2 + (2R)^2 + 2(2R)R\cos\theta \Rightarrow \boxed{\ell = R\sqrt{5-4\cos\theta}}$$

• Con esto, se puede calcular la energía potencial:

$$V = \frac{K}{2}(\ell - \ell_0)^2 + mgy$$

$$\Rightarrow V = \frac{K}{2}\left(R\sqrt{5-4\cos\theta} - \ell_0\right)^2 + mgR\sin\theta$$

$$\Rightarrow \frac{\partial V}{\partial \theta} = K\left(R\sqrt{5-4\cos\theta} - \ell_0\right)\frac{2R\sin\theta}{\sqrt{5-4\cos\theta}} + mgR\cos\theta$$

(a)

• $\theta = \pi/2 \Rightarrow \sin\theta = 1 \wedge \cos\theta = 0$

$$\Rightarrow \left.\frac{\partial V}{\partial\theta}\right|_{\frac{\pi}{2}} = \frac{2KR}{\sqrt{5}}\left(R\sqrt{5} - \ell_0\right) \overset{!}{=} 0 \Rightarrow \boxed{\ell_0 = \sqrt{5}R}$$

(b)

• $\theta = \pi/3 \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \wedge \cos\theta = \frac{1}{2}$

$$\Rightarrow \left.\frac{\partial V}{\partial\theta}\right|_{\frac{\pi}{3}} = \frac{2K\cancel{R}\cancel{\sqrt{3}}}{\cancel{2}}\frac{\left(R\sqrt{3} - \sqrt{5}R\right)}{\cancel{\sqrt{3}}} + \frac{mg\cancel{R}}{2} \overset{!}{=} 0$$

$$\Rightarrow K(\sqrt{5}-\sqrt{3})R = \frac{mg}{2} \Rightarrow \boxed{K = \frac{mg}{2(\sqrt{5}-\sqrt{3})R}}$$

(c)

- Para ver si un equilibrio es estable o inestable hay que analizar el signo de $\frac{\partial^2 V}{\partial \theta^2}$:

$$\frac{\partial V}{\partial \theta} = mgR\cos\theta + 2KR^2\sin\theta - \frac{2KRl_0\sin\theta}{\sqrt{5-4\cos\theta}}$$

$$\Rightarrow \frac{\partial^2 V}{\partial \theta^2} = -mgR\sin\theta + 2KR^2\cos\theta - \frac{2KRl_0\cos\theta}{\sqrt{5-4\cos\theta}} + \frac{4KRl_0\sin^2\theta}{\sqrt{5-4\cos\theta}^3}$$

- Usando $\theta = \pi/2$:

$$\Rightarrow \frac{\partial^2 V}{\partial \theta^2} = -mgR + \frac{KRl_0}{\sqrt{5}^3}$$

$$\Rightarrow \frac{\partial^2 V}{\partial \theta^2} = \frac{mgR}{5(\sqrt{5}-\sqrt{3})} - mgR = mgR\left(\frac{2}{5(\sqrt{5}-\sqrt{3})} - 1\right) < 0$$

$\Rightarrow$ $\boxed{\theta = \frac{\pi}{2} \text{ es un punto inestable.}}$

- Usando $\theta = \pi/3$:

$$\frac{\partial^2 V}{\partial \theta^2} = -mgR\frac{\sqrt{3}}{2} + KR^2 - \frac{KRl_0}{\sqrt{3}} + \frac{\cancel{4}KRl_0}{\cancel{8}\sqrt{3}}$$

$$\Rightarrow \frac{\partial^2 V}{\partial \theta^2} = mgR\left(-\frac{\sqrt{3}}{2} + \frac{1}{2(\sqrt{5}-\sqrt{3})} - \cancel{\frac{\sqrt{5}}{2(\sqrt{15}-3)}} + \cancel{\frac{\sqrt{5}}{2(\sqrt{15}-3)}}\right)$$

$$\Rightarrow \frac{\partial^2 V}{\partial \theta^2} = \frac{mgR}{2(\sqrt{5}-\sqrt{3})}\left(1 - \sqrt{15} + 3\right) > 0$$

$\Rightarrow \theta = \frac{\pi}{3}$ es un punto estable

- Para encontrar la frec. de pequeñas oscilaciones, hacemos Taylor:

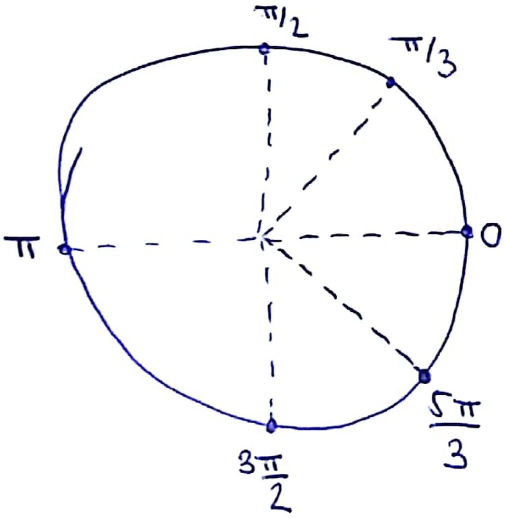
$$V(\theta) \approx V(\pi/3) + \cancelto{0}{\frac{\partial V}{\partial \theta}}\Big|_{\frac{\pi}{3}} (\theta - \pi/3) + \frac{\partial^2 V}{\partial \theta^2}\Big|_{\frac{\pi}{3}} (\theta - \pi/3)^2$$

$$\Rightarrow V(\theta) \approx \frac{KR^2}{2}\left(\sqrt{3} - \sqrt{5}\right)^2 + mgR\frac{\sqrt{3}}{2} + \frac{mgR\left(4 - \sqrt{15}\right)}{2\left(\sqrt{5} - \sqrt{3}\right)}\left(\theta - \frac{\pi}{3}\right)^2$$

$$\omega^2 = \frac{1}{mR^2} \cdot \frac{\partial^2 V}{\partial \theta^2} \Rightarrow \boxed{\omega^2 = \frac{g}{2R}\left(\frac{1}{\sqrt{5} - \sqrt{3}} - \sqrt{3}\right)}$$

(d)

- Por simetría:



Candidatas a equilibrio

↳ Hay que comprobar si $\frac{\partial V}{\partial \theta} = 0$ para cada caso.

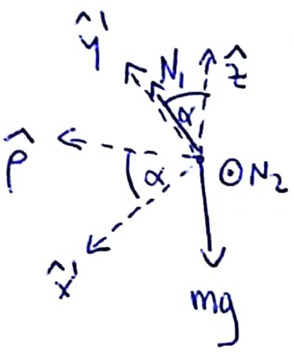
y ver el signo de $\frac{\partial^2 V}{\partial \theta^2}$ si corresponde.

P2 |

Auxiliar 7

(a)

$$m\vec{a}' = \vec{F} - m\ddot{\vec{R}} - m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}\,') - 2m\vec{\Omega}\times\vec{v}\,' - m\overset{0}{\cancel{\dot{\vec{\Omega}}}}\times\vec{r}\,'$$



$$\vec{R} = 0$$

$$\vec{\Omega} = \Omega\hat{z} = \Omega(-\sin\alpha\,\hat{x}' + \cos\alpha\,\hat{y}')$$

$$\vec{r}\,' = x\hat{x}' \Rightarrow \vec{v}\,' = \dot{x}\hat{x}'$$

- $\Omega\times\vec{v}\,' = \Omega\cos\alpha\,\dot{x}\,(-\hat{z}')$
- $\vec{\Omega}\times(\vec{\Omega}\times\vec{r}\,') = -\Omega^2\cos\alpha\, x\left(\sin\alpha\,\hat{y}' + \cos\alpha\,\hat{x}'\right)$
- Así, las ec. de mov. son:

$\hat{x}'$ | $m\ddot{x} = mg\sin\alpha + m\Omega^2\cos^2\alpha\, X$

$\hat{y}'$ | $0 = N_1 - mg\cos\alpha + m\Omega^2\sin\alpha\cos\alpha\, X$

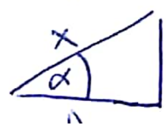
$\hat{z}'$ | $0 = -N_2 + m\Omega\cos\alpha\,\dot{x}$

(b)

- La partícula se separa cuando $N_1 = 0$

$$\Rightarrow \cancel{m}\Omega^2\sin\alpha\, X = \cancel{m}g \Rightarrow X = \frac{g}{\Omega^2\sin\alpha}$$

- La separación con $\hat{z}$ es:



$$\boxed{D = \frac{g}{\Omega^2}\cdot\frac{\cos\alpha}{\sin\alpha}}$$

(c)

• Por la ec. en $(\hat{z}')$ se tiene que:

$$N_2 = m\Omega \cos\alpha \ \dot{x} \rightarrow \text{Fa a encontrar } \dot{x};$$

• A partir de la ec. de $(\hat{x}')$:

$$m\ddot{x} = mg\sin\alpha + m\Omega^2\cos^2\alpha \, x \quad \Big/ \ \dot{x}$$

$$\Rightarrow \frac{m}{2}\frac{d}{dt}(\dot{x}^2) = mg\sin\alpha \ \dot{x} + \frac{m\Omega^2\cos^2\alpha}{2}\frac{d}{dt}(x^2) \quad \Big/ \int_0^{t_f} dt$$

$$\Rightarrow \dot{x}^2 = 2\cancel{m}g\sin\alpha \, x + \Omega^2\cos^2\alpha \, x^2 \quad \Big| \ x = \frac{g}{\Omega^2\sin\alpha}$$

$$\Rightarrow \dot{x}^2 = \frac{2g^2}{\Omega^2} + \frac{g^2\cos^2\alpha}{\Omega^2\sin^2\alpha} \Rightarrow \boxed{\dot{x} = \frac{g}{\Omega}\sqrt{2+\cot\tan\alpha}}$$

$$\Rightarrow \boxed{N_2 = mg\cos\alpha\sqrt{2+\cot\tan\alpha}}$$

P3 |

Auxiliar 7

(a)

- Podemos notar que $V_1(r) = \frac{A r^3}{3}$ es una función tal que

$$\vec{F}_1 = -\nabla V_1 \Rightarrow \vec{F}_1 \text{ es conservativa y su potencial es } V_1$$

(b) $\frac{\partial F_x}{\partial y} \neq \frac{\partial F_y}{\partial x} \Rightarrow \vec{F}_2$ es no conservativa

$$W_2 = \int \vec{F}_2 \cdot d\vec{r} = B \int (y^2 dx - x^2 dy) \quad \Big| \quad y = \frac{x^2}{L} \Rightarrow dy = \frac{2x}{L} dx$$

$$\Rightarrow W_2 = B \int_L^0 dx \, \frac{x^4}{L^2} - \frac{2x^3}{L} = -B \left(\frac{1}{5} \frac{L^5}{L^2} - \frac{1}{2} \frac{L^4}{L^2} \right)$$

$$\Rightarrow \boxed{W_2 = \frac{3BL^3}{10}}$$

(c)

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 - \frac{A}{3} 2\sqrt{2} L^3 = \frac{3BL^3}{10}$$

$$\Rightarrow \boxed{v_f^2 = v_0^2 + \frac{2L^3}{m} \left(\frac{3B}{10} + \frac{2\sqrt{2} A}{3} \right)}$$