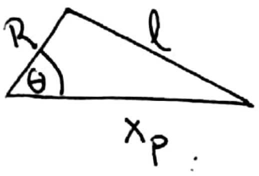
P1)

(a)



• Teorema del coseno:

$$l^2 = R^2 + x_p^2 - 2Rx_p\cos\theta$$

$$\Rightarrow x_p^2 - 2R\cos\theta\, x_p + R^2 - l^2 = 0$$

$$x_p = R\cos\theta \pm \sqrt{R^2\cos^2\theta + l^2 - R^2} \quad / \quad l \gg R \Rightarrow l^2 - R^2 \approx l^2$$

$$\Rightarrow x_p = R\cos\theta \pm \sqrt{R^2\cos^2\theta + l^2} \quad / \quad l \gg R \Rightarrow l^2 \gg R^2 \Rightarrow l^2 \gg R^2\cos\theta$$

$$\Rightarrow x_p = R\cos\theta \pm l \quad / \quad x_p > 0$$

$$\Rightarrow x_p = l + R\cos\theta \Rightarrow \boxed{x_p = l + R\cos(\omega t)}$$

(b)

$$\cancel{U = \frac{1}{2}k(x - x_p - l)^2 = \frac{1}{2}k(x - R\cos(\omega t))^2}$$

$$\cancel{m\ddot{x} = -\frac{\partial U}{\partial x} = -k(x - R\cos(\omega t))}$$

$$\cancel{\Rightarrow \ddot{x} + \omega_0^2 x = R\omega_0^2\cos(\omega t)}$$

$$m\ddot{x} = -k(x - x_p - l) - c\dot{x}$$

$$\Rightarrow \ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = R\omega_0^2\cos(\omega t)$$

$$\Rightarrow \ddot{z} + 2\gamma\dot{z} + \omega_0^2 z = R\omega_0^2 e^{i\omega t} \quad ; \quad x = \mathrm{Re}(z)$$

• Ansatz: $z = A e^{i(\omega t - \delta)}$

$$\Rightarrow (-\omega^2 A + i2\gamma\omega A + \omega_0^2 A)e^{-i\delta} = R\omega_0^2$$

$$\Rightarrow A(\omega_0^2 - \omega^2) + i2\gamma\omega A = R\omega_0^2 e^{i\delta}$$

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$$\Rightarrow \begin{cases} A(\omega_0^2-\omega^2) = R\omega_0^2\cos\delta & (1) \\ A\,2\gamma\omega = R\omega_0^2\sin\delta & (2) \end{cases}$$

$$\frac{(2)}{(1)} \Rightarrow \boxed{\operatorname{tg}\delta = \frac{2\gamma\omega}{\omega_0^2-\omega^2}}$$

- Como $1+\operatorname{tg}^2\delta = \sec^2\delta = \dfrac{1}{\cos^2\delta}$

$$\Rightarrow \cos^2\delta = \frac{1}{1+\operatorname{tg}^2\delta} \Rightarrow \cos^2\delta = \frac{(\omega_0^2-\omega^2)^2}{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}$$

- Reemplazando en (1):

$$\cancel{A(\omega_0^2-\omega^2) = \frac{R\omega_0^2(\omega_0^2-\omega^2)}{\omega_0^2-\omega^2}} \qquad \cancel{A(\omega_0^2-\omega^2) = \frac{R\omega_0^2}{\sqrt{\omega_0^2-\omega^2}}}$$

$$A\,\cancel{(\omega_0^2-\omega^2)} = \frac{R\omega_0^2\,\cancel{(\omega_0^2-\omega^2)}}{\sqrt{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}} \Rightarrow \boxed{A = \frac{R\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}}}$$

$$\Rightarrow z(t) = \frac{R\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}}\, e^{i(\omega t-\delta)}$$

$$\Rightarrow x(t) = \frac{R\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}}\cos(\omega t-\delta)$$

$$\Rightarrow \dot{x}(t) = \frac{-R\omega_0^2\omega}{\sqrt{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}}\sin(\omega t-\delta)$$

- Rapidez máxima: $\boxed{\dfrac{R\omega_0^2\omega}{\sqrt{(\omega_0^2-\omega^2)^2+(2\gamma\omega)^2}}}$

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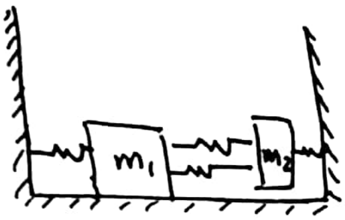
PR1



$$\cancel{m_1\ddot{x}_1 = -k(x_1-l_0) + k((x_2-x_1)-l_0) + k((x_2-x_1)-l_0)}$$

$$\cancel{m_2\ddot{x}_2 = -2k((x_2-x_1)-l_0) + k(x_2-l_0)}$$

$$\cancel{\Rightarrow \begin{cases} \ddot{x}_1 = -\omega_1^2 x_1 + k\omega_1^2(x_2-x_1) \\ \ddot{x}_2 = - \end{cases}}$$

$$m\ddot{x}_1 = -k(x_1-l_0) + 2k((x_2-x_1)-l_0)$$

$$m\ddot{x}_2 = -2k((x_2-x_1)-l_0) + k((\underset{\hookrightarrow\, 3l_0}{H}-x_2)-l_0)$$

$$w_0^2 = \frac{k}{m}$$

$$\Rightarrow \begin{aligned} \ddot{x}_1 &= -3w_0^2 x_1 + 2w_0^2 x_2 - w_0^2 l_0 \\ \ddot{x}_2 &= 2w_0^2 x_1 - 3w_0^2 x_2 + 4w_0^2 l_0 \end{aligned}$$

$$\Rightarrow \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} = -w_0^2 \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - w_0^2 \begin{pmatrix} l_0 \\ -4l_0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} = -w_0^2 \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}\left[\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}^{-1}\begin{pmatrix} l_0 \\ -4l_0 \end{pmatrix}\right]$$

- Esto último se puede hacer ya que

$\det\begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix} = 9 + 4 = 13 \neq 0 \quad \therefore$ es invertible.

• Por lo que se puede hacer el cambio de variable:

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}^{-1} \begin{pmatrix} l_0 \\ -4l_0 \end{pmatrix}$$

se obtiene el sistema:

$$\begin{pmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{pmatrix} = -\omega_0^2 \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

• Las frecuencias propias se calculan al suponer $\ddot{u}_i = -\omega^2 u_i$

$$\Rightarrow -\omega^2 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = -\omega_0^2 \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 3\omega_0^2 - \omega^2 & -2\omega_0^2 \\ -2\omega_0^2 & 3\omega_0^2 - \omega^2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0 \quad \Big| \quad \det(A) \stackrel{!}{=} 0$$

$$\Rightarrow (\omega^2 - 3\omega_0^2)^2 - 4\omega_0^4 = 0$$

$$\Rightarrow \omega_\pm^2 - 3\omega_0^2 = \pm 2\omega_0^2$$

$$\Rightarrow \boxed{\omega_\pm^2 = 3\omega_0^2 \pm 2\omega_0^2}$$

Con esto calculamos los vectores propios.

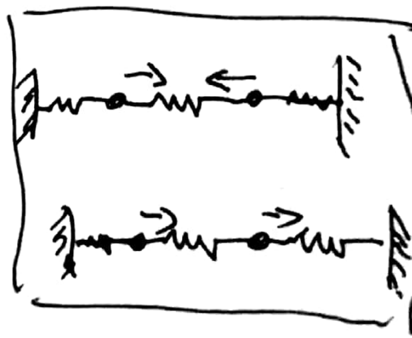
$$(3\omega_0^2 - \omega_\pm^2)u_1 - 2\omega_0^2 u_2 = 0$$

$$\Rightarrow u_2 = \frac{(3\omega_0^2 - \omega_\pm^2)}{2\omega_0^2} u_1$$

$\Rightarrow \boxed{u_2 = \mp [redacted] u_1}$

• $\omega_+^2 = 5\omega_0^2$ está asociado a $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

• $\omega_-^2 = \omega_0^2$ está asociado a $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$



P3)

(a) $|\vec{v}| = R\dot{\phi}$

$$m\vec{a} = \sum \vec{F} \Rightarrow + \cancel{m} R^2 \dot{\phi} = + \frac{GM\cancel{m}}{R^2} \Rightarrow \frac{v^2}{\cancel{R}} = \frac{GM}{R^{\cancel{2}}} \Rightarrow \boxed{v = \sqrt{\frac{GM}{R}}}$$

(b)

- Conservaciones en la elipse:

↳ Energía: $$\frac{\cancel{m} v_A^2}{2} - \frac{GM\cancel{m}}{R_A} = \frac{\cancel{m} v_B^2}{2} - \frac{GM\cancel{m}}{R_B} \qquad (1)$$

↳ Momentum angular: $$R_A \cancel{m} v_A = R_B \cancel{m} v_B \qquad (2)$$

- Conservacion de la energía en la parábola:

$$E = \frac{\cancel{m} v_B^2}{2} - \frac{GM\cancel{m}}{R_A} = 0 \overset{(3)}{\Rightarrow} \boxed{v_B = \sqrt{\frac{2GM}{R_A}}}$$

- Reemplazando v_B en (2):

$$\Rightarrow R_A v_A = \sqrt{\frac{2GMR_B^2}{R_A}} \Rightarrow \boxed{v_A = \sqrt{\frac{2GMR_B^2}{R_A^3}}}$$

- Reemplazando v_A y v_B en (1):

$$\Rightarrow \frac{\cancel{GM} R_B^2}{R_A^3} - \frac{\cancel{GM}}{R_A} = \frac{\cancel{GM}}{R_A} - \frac{\cancel{GM}}{R_B}$$

$$\Rightarrow R_B^3 - R_A^2 R_B = R_A^2 R_B - R_A^3 \Rightarrow R_B(R_B^2 - R_A^2) = R_A^2(R_B - R_A)$$

$$\Rightarrow R_B(R_B + R_A) = R_A^2 \Rightarrow R_A^2 - R_A R_B - R_B^2 = 0$$

$$\Rightarrow R_A = R_B\left(\frac{1 \pm \sqrt{5}}{2}\right) \Rightarrow \boxed{R_A = \left(\frac{1+\sqrt{5}}{2}\right) R_B}$$

- Recordamos que $r_{min} = \dfrac{R}{1+e}$; $r_{max} = \dfrac{R}{1-e}$. Dividiendolas:

$$\Rightarrow \frac{r_{max}}{r_{min}} = \frac{1+e}{1-e} \Rightarrow r_{max} - e r_{max} = r_{min} + e r_{max}$$

$$\Rightarrow r_{max} - r_{min} = e(r_{max} + r_{min}) \Rightarrow \boxed{e = \frac{r_{max} - r_{min}}{r_{max} + r_{min}}}$$

- En este caso $r_{max} = R_A$ y $r_{min} = R_B$

$$\Rightarrow \boxed{e = \frac{\frac{1+\sqrt{5}}{2} - 1}{\frac{1+\sqrt{5}}{2} + 1} = \frac{\sqrt{5} - 1}{\sqrt{5} + 3}}$$

(c)