Pauta ejercicio 2

a) → Tenemos que la velocidad del enunciado es $\vec{v} = v_2\hat{\phi} + v_3\hat{k}$

- Pero la velocidad en cilíndricas es $\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{k}$

- Igualamos cada término de la velocidad

(1) $\dot{\rho} = 0 \Rightarrow \rho = \rho_0 \rightarrow$ Cte por determinar

(2) $v_2 = \rho\dot{\phi} \Rightarrow \dot{\phi} = \dfrac{v_2}{\rho_0} \Rightarrow \dfrac{d\phi}{dt} = \dfrac{v_2}{\rho_0} \quad / \int$

$$\Rightarrow \int_{\phi_0=0}^{\phi} d\phi = \int_0^t \frac{v_2}{\rho_0}\,dt \Rightarrow \boxed{\phi = \frac{v_2 t}{\rho_0}}$$

→ En particular para $t = \tau$

$$\Rightarrow \phi(\tau) = \underset{\text{Dato}}{\omega_0 \cancel{\tau}} = \frac{v_2\cancel{\tau}}{\rho_0} \Rightarrow \boxed{\rho_0 = \frac{v_2}{\omega_0}}$$

$$\therefore \boxed{\phi(t) = \omega_0 t}$$

(3) $\dot{z} = v_3 \Rightarrow \displaystyle\int_0^z dz = v_3 \int_0^t dt \Rightarrow \boxed{z(t) = v_3 t}$

- Luego reemplazamos para la posición y aceleración

$$\rightarrow \vec{r} = \rho\hat{\rho} + z\hat{z} \Rightarrow \boxed{\vec{r} = \frac{v_2}{\omega_0}\hat{\rho} + v_3 t\,\hat{k}}$$

$$\rightarrow \vec{a} = \frac{d}{dt}\vec{v} = v_2\dot{\hat{\phi}} = -v_2\dot{\phi}\hat{\rho} \quad ; \quad \text{pero } \dot{\phi} = \omega_0$$

$$\Rightarrow \boxed{\vec{a} = -v_2\omega_0\hat{\rho}}\,_{//}$$

b) Usamos la forma directamente

$$\vec{\omega} = \frac{\left[\frac{v_2}{\omega_0}\hat{\rho} + v_3 t\,\hat{k}\right] \times \left[v_2\hat{\phi} + v_3\hat{k}\right]}{\left(\frac{v_2^2}{\omega_0^2} + v_3^2 t^2\right)} = \frac{\left(\frac{v_2^2}{\omega_0}\overbrace{(\hat{\rho}\times\hat{\phi})}^{\hat{k}} + \frac{v_2 v_3}{\omega_0}\overbrace{(\hat{\rho}\times\hat{k})}^{-\hat{\phi}} + v_3 v_2 t\overbrace{(\hat{k}\times\hat{\phi})}^{-\hat{\rho}}\right)}{\left(\frac{v_2^2}{\omega_0^2} + v_3^2 t^2\right)}$$

$$\Rightarrow \boxed{\vec{\omega} = \frac{\frac{v_2^2}{\omega_0}\hat{k} - \frac{v_2 v_3}{\omega_0}\hat{\phi} - v_2 v_3 t\,\hat{\rho}}{\left(\frac{v_2^2}{\omega_0^2} + v_3^2 t^2\right)}}$$

- Si $v_3 = 0$

$$\Rightarrow \vec{\omega} = \frac{\frac{\cancel{v_2^2}}{\cancel{\omega_0}}\hat{k}}{\frac{\cancel{v_2^2}}{\omega_0^{\cancel{2}}}} \Rightarrow \boxed{\vec{\omega} = \omega_0\hat{k}}$$

- En $t \to \infty$, el denominador grand

$$\therefore \boxed{\omega \to 0}$$