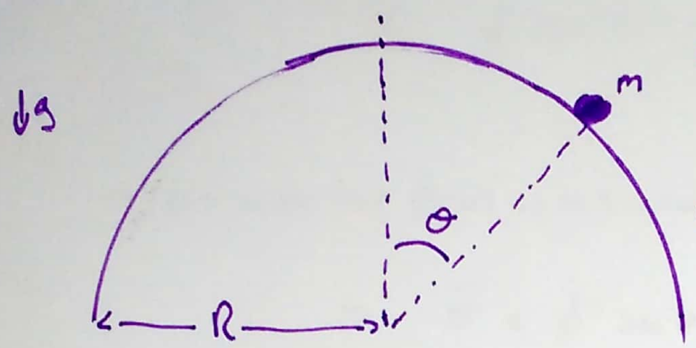
Parte ejercicio 3



$t=0$

$\vec{v} = v_0 \hat{\phi}$

$\theta_0 = \pi/5$

velocidad y aceleración en esféricas

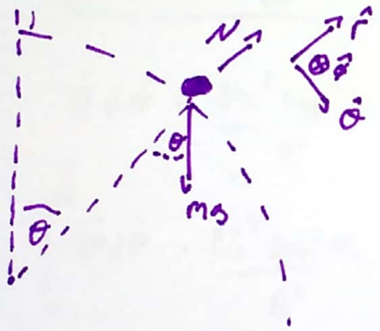
$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + R\dot{\phi}\,\text{sen}\,\theta\,\hat{\phi}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\,\text{sen}\,\theta)\,\hat{r}$$
$$+ (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\,\text{sen}\,\theta\cos\theta)\,\hat{\theta}$$
$$+ \frac{1}{r\,\text{sen}\,\theta}\frac{d}{dt}\left(r^2\dot{\phi}\,\text{sen}^2\theta\right)\hat{\phi}$$

a) Ecuaciones de Movimiento

-> En nuestro caso $r = R \Rightarrow \dot{r} = \ddot{r} = 0$

-> Hacemos el diagrama de fuerzas (DCL)



-> Hacemos $\vec{F} = m\vec{a}$ por componentes

$$\boxed{\begin{aligned} \hat{r}\rfloor \quad & N - mg\cos\theta = m(-R\dot{\theta}^2 - R\dot{\phi}^2\,\text{sen}\,\theta) \\ \hat{\theta}\rfloor \quad & mg\,\text{sen}\,\theta = m(R\ddot{\theta} - R\dot{\phi}^2\,\text{sen}\,\theta\cos\theta) \\ \hat{\phi}\rfloor \quad & 0 = \frac{m}{R\,\text{sen}\,\theta}\frac{d}{dt}(R^2\dot{\phi}\,\text{sen}^2\theta) \end{aligned}}$$

Ecs de Movimiento

b) Encontrar $\vec{v}(\theta)$ y $\vec{a}(\theta)$

-> Viendo las formulas originales, a priori debemos encontrar $\dot{\phi}(\theta)$ y $\dot{\theta}(\theta)$

-> De la ecuación en $\hat{\phi}\rfloor$

Constante

$$\Rightarrow \frac{d}{dt}(\dot{\phi}\,\text{sen}^2\theta) = 0 \Rightarrow \dot{\phi}\,\text{sen}^2\theta = k$$

-> Pero en $t=0$

$$\Rightarrow \vec{v}_0 = v_0\hat{\phi} = R\dot{\phi}_0\,\text{sen}\,\theta_0\,\hat{\phi} \Rightarrow \dot{\phi}_0 = \frac{v_0}{R\,\text{sen}\,\theta_0}$$

$$\Rightarrow \dot{\phi}_0\,\text{sen}^2\theta_0 = k \Rightarrow \boxed{k = \frac{v_0\,\text{sen}\,\theta_0}{R}}$$

→ Despejamos $\dot{\phi}(\theta)$ reemplazando h

$$\Rightarrow \dot{\phi}\, \text{Sen}^2\theta = \frac{v_0\, \text{Sen}\,\theta_0}{R} \Rightarrow \boxed{\dot{\phi}(\theta) = \frac{v_0\, \text{Sen}\,\theta_0}{R\, \text{Sen}^2\theta}} \quad (I)$$

→ Para encontrar $\dot{\theta}(\theta)$ integramos la ecuación en $\hat{\theta}$

$$\Rightarrow \ddot{\theta} = \dot{\phi}^2 \,\text{sen}\,\theta\cos\theta - \frac{g}{R}\,\text{sen}\,\theta$$

Pero $\dot{\phi}$ lo conocemos

$$\Rightarrow \ddot{\theta} = \frac{v_0^2\, \text{Sen}^2\theta_0}{R^2\, \text{Sen}^{\cancel{4}3}\theta}\, \cancel{\text{sen}\,\theta}\cos\theta - \frac{g}{R}\,\text{sen}\,\theta$$

$$(II) \quad \boxed{\ddot{\theta}(\theta) = \frac{v_0^2\, \text{Sen}^2\theta_0}{R^2} \cdot \frac{\cos\theta}{\text{Sen}^3\theta} - \frac{g}{R}\,\text{sen}\,\theta}$$

Pero $\ddot{\theta} = \dot{\theta}\dfrac{d\dot{\theta}}{d\theta}$

$$\Rightarrow \dot{\theta}\, d\dot{\theta} = \frac{v_0^2\, \text{Sen}^2\theta_0}{R^2} \cdot \frac{\cos\theta}{\text{Sen}^3\theta}\, d\theta - \frac{g}{R}\,\text{sen}\,\theta\, d\theta \quad / \int$$

$$\Rightarrow \int_0^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = \frac{v_0^2\, \text{Sen}^2\theta_0}{R^2} \underbrace{\int_{\pi/3}^{\theta} \frac{\cos\theta}{\text{sen}^3\theta}\, d\theta}_{I} - \frac{g}{R}\int_{\pi/3}^{\theta} \text{Sen}\,\theta\, d\theta$$

→ Resolvemos I haciendo el cambio de variables $u = \text{sen}\,\theta \Rightarrow du = \cos\theta\, d\theta$

$$\Rightarrow I = \int_{\sqrt{3}/2}^{\text{Sen}\,\theta} \frac{du}{u^3} = \left.\frac{u^{-2}}{-2}\right|_{\sqrt{3}/2}^{\text{sen}\,\theta} = \frac{-1}{2}\left[\frac{4}{3} - \frac{1}{\text{Sen}^2\theta}\right]$$

→ Reemplazamos I e integramos el resto

$$\Rightarrow \frac{\dot{\theta}^2}{2} = -\frac{v_0^2\, \text{sen}^2\theta_0}{2R^2}\left[\frac{4}{3} - \frac{1}{\text{Sen}^2\theta}\right] + \frac{g}{R}\cos\theta\Big|_{\pi/3}^{\theta}$$

$$\Rightarrow \boxed{\dot{\theta}^2(\theta) = -\frac{v_0^2\, \text{sen}^2\theta_0}{R^2}\left[\frac{4}{3} - \frac{1}{\text{Sen}^2\theta}\right] + \frac{2g}{R}\left[\cos\theta - \frac{1}{2}\right]} \quad (III)$$

-) Con las ecuaciones (I), (II) y (III) nos queda reemplazar

$$\boxed{\vec{v}(\theta) = R\dot{\theta}\hat{\theta} + R\dot{\phi}\,\text{sen}\,\theta\,\hat{\phi}}$$ con $\dot{\theta}$ y $\dot{\phi}$ conocidos

$$\boxed{\vec{a}(\theta) = (-R\dot{\theta}^2 - R\dot{\phi}^2\,\text{sen}\,\theta)\hat{r} + (R\ddot{\theta} - R\dot{\phi}^2\,\text{sen}\,\theta\cos\theta)\hat{\theta} + \frac{1}{R\,\text{sen}\,\theta}\frac{d}{dt}(R^2\dot{\phi}\,\text{sen}^2\theta)\hat{\phi}}$$

Con $\dot{\theta}$, $\dot{\phi}$ y $\ddot{\theta}$ conocidos.

c) Determinar θ' ángulo de despegue

-) De la ecuación en $\hat{r}$ hacemos $N \overset{!}{=} 0$

$$\Rightarrow -\cancel{m}g\cos\theta^* = \cancel{m}(-R\dot{\theta}^2(\theta^*) - R\dot{\phi}^2(\theta^*)\,\text{sen}\,\theta)$$

$$\Rightarrow \frac{g}{\cancel{R}}\cos\theta^* = -\frac{R v_0^2\,\text{sen}^2\theta_0}{R^3}\left[\frac{4}{3} - \frac{1}{\text{sen}^2\theta^*}\right] + \frac{2g\cancel{R}}{\cancel{R}}\left[\cos\theta^* - \frac{1}{2}\right] + \cancel{R}\frac{v_0^2\,\text{sen}^2\theta_0}{\cancel{R^3}\,\text{sen}^{\cancel{4}\,3}\theta^*}\cancel{\text{sen}\,\theta'}$$

$$\Rightarrow \boxed{g\cos\theta' = -v_0^2\,\text{sen}^2\theta_0\left[\frac{4}{3} - \frac{1}{\text{sen}^2\theta^*}\right] + 2gR\left[\cos\theta^* - \frac{1}{2}\right] + \frac{v_0^2\,\text{sen}^2\theta_0}{\text{sen}^3\theta'}}$$

Ecuación para encontrar θ^*