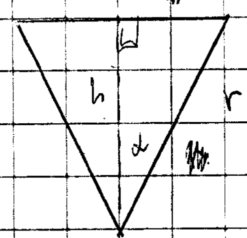
FECHA: / /

P1)



Coordenadas esfericas,

$\phi = \alpha$, $\theta = [0, 2\pi)$ $\quad \|\vec r - \vec r'\| = \vec r'$

$r \in (0, h/\cos\alpha)$ $\quad \vec r = 0$

$$d\vec s = r \sin\phi \, dr \, d\theta \, \hat\phi .$$

$$d\vec s = r \sin\alpha \, dr \, d\theta .$$

$$V_c(\vec r) = \frac{1}{4\pi\epsilon_0} \int_0^{h/\cos\alpha} \int_0^{2\pi} \frac{\sigma \cancel{r'} \sin\alpha \, dr' \, d\theta}{\cancel{r'}}$$

$$V_c(\vec r) = \frac{\sigma \sin\alpha}{4\pi\epsilon_0} \frac{h}{\cos\alpha} 2\pi = \boxed{\frac{\sigma h \tan(\alpha)}{2\epsilon_0}}$$

FECHA: / /

análogamente, para el anillo de radio a

$\vec{r}' = a\hat{r} - d\hat{k} \qquad \theta \in [0, 2\pi)$

$\vec{r} = 0$

$d\lambda(\vec{r}') = \lambda a \, d\theta$

$$V_a(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{\lambda a \, d\theta}{\sqrt{a^2+d^2}}$$

$$V_a(\vec{r}) = \frac{1}{\cancel{4}\cancel{\pi}\varepsilon_0} \frac{\lambda a}{\sqrt{a^2+d^2}} \cdot \cancel{2\pi}$$

$$V_a(\vec{r}) = \frac{\lambda a}{2\varepsilon_0 \sqrt{a^2+d^2}}$$

luego $V(0) = V_c + V_a = \dfrac{1}{2\varepsilon_0}\left(\sigma h \, tg(\alpha) + \dfrac{\lambda a}{\sqrt{d^2+a^2}}\right)$

FECHA: / /

La energía Total se expresa como.

$$\frac{1}{2} m v_i^2 + U_i = \frac{1}{2} m v_f^2 + U_f$$

$U_i = q V(0)$ $\qquad \frac{q}{2\varepsilon_0}\left(\sigma h \, tg(\alpha) + \frac{\lambda a}{\sqrt{d^2+a^2}}\right).$

$U_f = q \, |V(\infty)| = 0$

$v_f \overset{!}{=} 0.$

$v_0 = 0 \quad \Rightarrow$

$$\frac{q}{2\varepsilon_0}\left(\sigma h \, tg(\alpha) + \frac{\lambda a}{\sqrt{d^2+a^2}}\right) = 0$$

FECHA: / /

P2) $U = -Q\, V(r)$

$$V(r) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{\|\hat{r} - \hat{r}_i'\|}$$

en nuestro caso $n = 8$.

$q_i = -e$, $Q = e$ además

$\|r - r_i'\| = \dfrac{\sqrt{3}a}{2} \quad \forall i$

$$V(r) = \frac{1}{4\pi\varepsilon_0} \frac{-e}{\sqrt{3}a/2} \cdot 8$$

$$V(r) = \frac{-4e}{\pi\varepsilon_0\sqrt{3}a}$$

$$U = \frac{4e^2}{\pi\varepsilon_0\sqrt{3}a}$$

FECHA: / /

P3) Encontrar la diferencia de potencial entre el polo sur y el centro.

Usaremos coordenadas esféricas.

primero en el centro.

$\vec{r} = 0$

$\vec{r}' = R\hat{r}$. $\quad ($ ~~$r \in [0, R]$~~ $, \theta \in [0, 2\pi], \phi \in [0, \pi/2])$

$$ds = R^2 \sin\phi \, d\theta \, d\phi$$

$$V_0 = \frac{1}{4\pi\varepsilon_0} \int_0^{\pi/2} \int_0^{2\pi} \frac{\sigma R^2 \sin\phi \, d\phi \, d\theta}{R}$$

$$V_0 = \frac{1}{4\pi\varepsilon_0} R \, 2\pi \sigma = \boxed{\frac{\sigma R}{2\varepsilon_0}}$$

FECHA: / /

ahora en el polo sur.

$$\vec{r}^{*} = -R\hat{k}, \qquad \vec{r}' = R\hat{r}.$$

$$\|r - \vec{r}'\| =$$

$$R\hat{r} = R \operatorname{sen}\phi \left(\cos\theta\, \hat{\imath} + \operatorname{sen}\theta\, \hat{\jmath} \right) + R\cos\phi\, \hat{k}.$$

$$\|r - \vec{r}\| = \left(R^2 \operatorname{sen}^2\phi \cos^2\theta + R^2 \operatorname{sen}^2\phi \operatorname{sen}^2\theta + R^2 (1 + \cos\phi)^2 \right)^{1/2}$$

$$\|r - \vec{r}\| = R\sqrt{\operatorname{sen}^2\phi + (1 + \cos\phi)^2}$$

$$\|r - \vec{r}\| = R\sqrt{\operatorname{sen}^2\phi + 1 + \cos^2\phi + 2\cos\phi}$$

$$\|r - \vec{r}'\| = \sqrt{2}\, R \sqrt{1 + \cos\phi}$$

$$V_{polo\,sur} = \int_0^{2\pi}\int_0^{\pi/2} \frac{1}{4\pi\varepsilon_0}\,\frac{1}{\sqrt{2}\,R}\,\frac{\sigma R^2 \sin\phi\, d\phi\, d\theta}{\sqrt{1+\cos\phi}}$$

$$V_{polo\,sur} = \frac{1}{4\cancel{\pi}\varepsilon_0}\,\frac{1}{\sqrt{2}}\,\sigma R\, 2\cancel{\pi}\int_0^{\pi/2}\frac{\sin\phi}{\sqrt{1+\cos\phi}}\,d\phi$$

$$\frac{\partial}{\partial\phi}\left(\sqrt{1+\cos\phi}\right) = \frac{1}{2}\,\frac{-\sin\phi}{\sqrt{1+\cos\phi}}$$

$$V_{polo\,sur} = \frac{\sigma R}{2\varepsilon_0\sqrt{2}}\cdot -2\left\{\sqrt{1+\cos\phi}\right\}\Big|_0^{\pi/2}$$

$$V_{polo\,sur} = \frac{\sigma R}{2\sqrt{2}\,\varepsilon_0}\left\{2\sqrt{2} - 2\right\}$$

$$\boxed{V_{polo\,sur} = \frac{\sigma R}{2\varepsilon_0}\left\{2 - \sqrt{2}\right\}}$$

FECHA: / /

Finalmente.

$$V_{pared} - V_0 = \frac{R\sigma}{2\varepsilon_0}\left\{1-\sqrt{2}\right\}$$