Considere $f: \mathbb{R} \to \mathbb{R}$ def

$f(x) = (1 + \text{sen}(x))/(1 - \cos(x))$ 1) Dom, ceros, signos, periodicidad, inyectividad

$-\frac{\pi}{2}$ $\pi + \frac{\pi}{2}$ Dom $1 - \cos(x) \neq 0$

2π $1 \neq \cos(x)$

$x = 2k\pi \pm 0 = 2k\pi \quad , k \in \mathbb{N}$

1) $\Rightarrow$ Dom $f(x) = \mathbb{R} - \{x / x = 2k\pi, k \in \mathbb{N}\}$

2) Ceros $Z(f)$: $1 + \text{sen}(x) = 0$

$\text{sen}(x) = -1$

$\Rightarrow x = k\pi + (-1)^k \frac{3}{2}\pi \,;\, \pi + -\frac{3\pi}{2} < -\frac{\pi}{2}$

$\Rightarrow Z(f) = \{x / x = k\pi + (-1)^k \frac{3}{2}\pi, k \in \mathbb{N}\}$

3) signos $\to$ $f(x) = \dfrac{1 + \text{sen}(x)}{1 - \cos(x)}$ | $-1 \leq \text{sen}(x) \leq 1$ | $\cos(x) \leq 1 \wedge -1 \leq \cos(x)$

$\boxed{0 \leq \text{sen}(x) + 1 \leq 2}$ | $-1 \leq -\cos(x) \wedge -\cos(x) \leq 1$

$\Rightarrow -1 \leq -\cos(x) \leq 1$

$1 + \text{sin}(x) \geq 0 \quad \forall x \in \text{Dom}$ | $\boxed{0 \leq 1 - \cos(x) \leq 2}$

$1 - \cos(x) > 0 \quad \forall x \in \text{Dom}$

$\Rightarrow f(x) \geq 0 \quad \forall x \in \text{Dom}(f(x))$

4) Paridad Dado $x = \frac{3}{2}\pi \Rightarrow f(x) = 0$ / $\nexists x \in \text{Dom } f(x) < 0$

$x = -\frac{3\pi}{2} \Rightarrow f(x) \neq 0$

$$\frac{1 + \text{sen}(x)}{1 - \cos(x)} \neq \frac{1 + -\text{sin}(x)}{1 - \cos(x)} \neq \frac{-1 + \text{sin}(x)}{1 - \cos(x)}$$

Periodo

Debe ser al menos 2π periódica, cociente 2π periódica.

Periodo $f(x+p) = f(x)$, $\forall$ menor $p \in \mathbb{R}$

Por contradicción

si $p \in (0, 2\pi)$

$f(x) = f(x+p)$, $x = \frac{3\pi}{2}$

$$0 = \frac{1 + \sin\left(\frac{3\pi}{2} + p\right)}{1 - \cos\left(\frac{3\pi}{2} + p\right)} \Rightarrow -1 = \sin\left(\frac{3\pi}{2} + p\right)$$

$$\frac{3\pi}{2} + p = k\pi + (-1)^k \frac{3\pi}{2}$$

$\forall k$, si $k=0$ $\quad p = k\pi + (-1)^k \frac{3\pi}{2} - \frac{3\pi}{2}$

$p = 0$

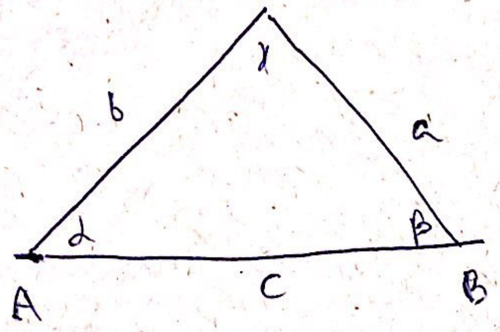
Inyectividad $\forall x, y$ $\frac{1+\sin(x)}{1-\cos x} = \frac{1+\sin(y)}{1-\cos(y)} \Rightarrow x = y$

pero $x = \frac{3\pi}{2}$ $\wedge$ $x = \frac{7\pi}{2}$ $\Rightarrow$

$\Rightarrow f(x) = 0$

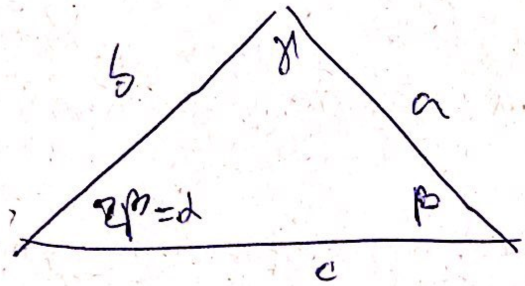
a)

P1)



PDQ $\underline{\alpha = 2\beta}$

$\Rightarrow a^2 = b(c+b)$



$\sin(\alpha) = \sin(2\beta) = 2\sin\beta\cos\beta$

$\alpha, \beta \neq 0 \quad \frac{a}{b} = \frac{\sin\alpha}{\sin(\beta)} = \cos\beta \cdot 2$

$b^2 = a^2 + c^2 - 2ac\cos(\beta)$

$b^2 = a^2 + c^2 - \frac{a^2 c}{b} \Rightarrow b^2 - c^2 = a^2\left(1 - \frac{c}{b}\right)$

$b^2 - c^2 = a^2\left(\frac{b-c}{b}\right)$

$\boxed{(b+c)b = a^2}$

$A, B, C \in \mathbb{R} \quad A > B$

$h(x) = (A-B)\sin(x)\cos(x) + C(\cos^2(x) - \sin^2(x))$

a) $C = 0 \Rightarrow h(x) = \frac{(A-B)}{2} 2\sin\cos(x)$

$s^2 + c^2 = 1$

$s = \sqrt{1-c^2}$

$= \left(\frac{A-B}{2}\right)\sin(2\alpha) \rightarrow$

$2\alpha = \frac{\pi}{2} + 2k\pi$

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$$\tan(\alpha) - \tan(\beta) = \frac{\operatorname{sen}(\alpha - \beta)}{\cos(\alpha)\cos(\beta)}$$

$$= \frac{\operatorname{sen}(\alpha)\cos(\beta) - \operatorname{sen}(\beta)\cos(\alpha)}{\cos(\alpha)\cos(\beta)}$$

$$= \frac{\operatorname{sen}(\alpha)\cancel{\cos(\beta)}}{\cos(\alpha)\cancel{\cos(\beta)}} - \frac{\operatorname{sen}(\beta)\cancel{\cos(\alpha)}}{\cancel{\cos(\alpha)}\cos(\beta)}$$

$$= \frac{\operatorname{sen}(\alpha)}{\cos(\alpha)} - \frac{\operatorname{sen}(\beta)}{\cos(\beta)}$$

$$= \tan(\alpha) - \tan(\beta) \quad \blacksquare$$

Tomando un $a > b$ con $a, b \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\tan(\alpha) - \tan(\beta) = \frac{\operatorname{sen}(\alpha - \beta)}{\cos(\alpha)\cos(\beta)}$$

$\cos(\alpha)$ y $\cos(\beta)$ son mayores que cero en este intervalo

$\therefore$ denominador > 0

y num > 0

$$\tan(\alpha) - \tan(\beta) > 0$$

y por lo tanto creciente

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