Desarrollos Auxiliar 4

P1 Notar que $4p(y-y_0) = (x-x_0)^2$ es una parábola vertical, por lo tanto directriz:

$y = y_0 - p$, usando (6) $\Rightarrow -5 = y_0 - p$

$\Rightarrow \boxed{p-5 = y_0}$ ($\star$)

La elipse $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$, por lo tanto $a=5$
$b=3$

Como $a > b \Rightarrow$ Focos $= (x_0 \pm a \cdot e, y_0)$, con $e = \frac{\sqrt{a^2-b^2}}{a}$

Obs: Es una elipse horizontal

$ae = \sqrt{a^2-b^2} = \sqrt{25-9} = \sqrt{16} = 4$, notando que $x_0 = y_0 = 0$

$\Rightarrow$ Focos $= (-4,0)$ y $(4,0)$

Usando esta información (Reemplazando en la ecuación de la parábola)

$$(1) \quad 4p(-y_0) = (-4-x_0)^2 = (x_0+4)^2 = x_0^2 + 8x_0 + 16$$

$$(2) \quad 4p(-y_0) = (4-x_0)^2 = x_0^2 - 8x_0 + 16$$

<u>Restando</u> (1) - (2)

$$\cancel{4p(-y_0)} - \cancel{4p(-y_0)} = \cancel{x_0^2} + 8x_0 + \cancel{16} - (\cancel{x_0^2} - 8x_0 + \cancel{16})$$

$$0 = 16x_0 \implies \boxed{x_0 = 0}$$

$$\implies (y - y_0)4p = x^2$$

, usando que (4,0) está en la parábola

$$\implies -4py_0 = 16 \implies \boxed{y_0 = -\frac{4}{p}} \quad ☺$$

Juntando ☺ con ✪

$$p - 5 = y_0 = -\frac{4}{p} \implies p(p-5) = -4 \implies p^2 - 5p + 4 = 0$$

$$\implies (p-1)(p-4) = 0 \implies p = 1 \lor p = 4$$

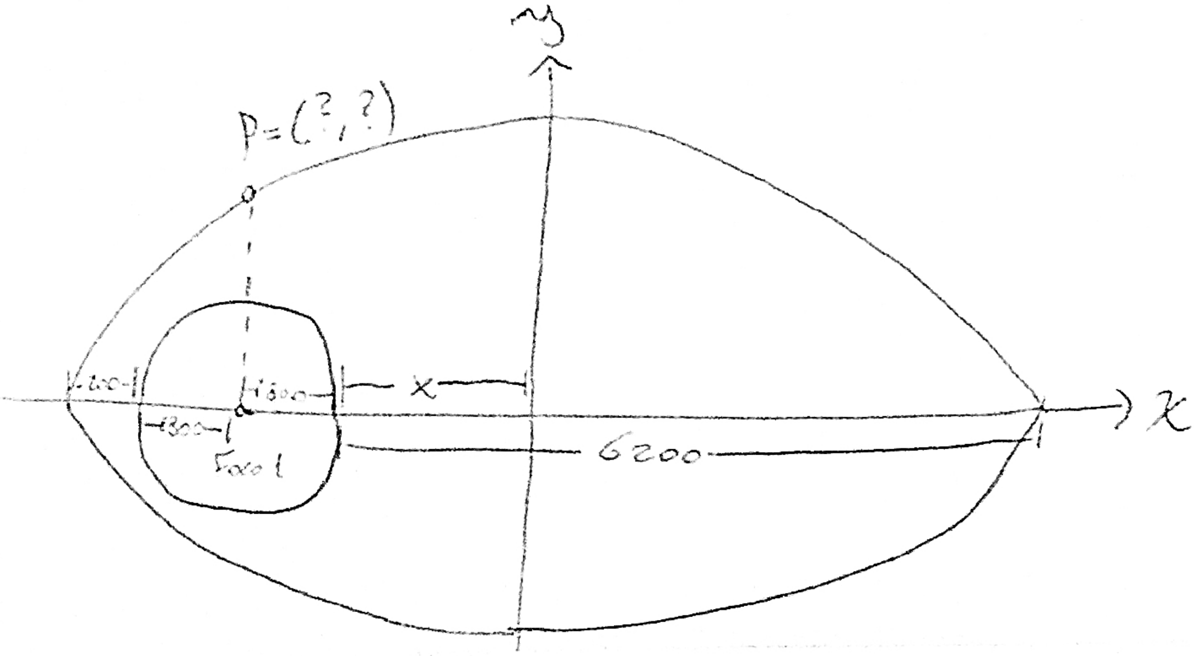
Por (c) $\implies \boxed{p = 4} \implies \boxed{y_0 = -1}$

reemplazando en ☺

$$\implies \boxed{16(y+1) = x^2}$$

$\boxed{P_2}$ Obs: En clases dije que P está en la recta $x = x_c$, con x_c: Coordenada x del centro de MA1001

Dibujemos reemplazando el radio del planeta



Notamos que el eje mayor es:

$$2a = 200 + 1800 + 1800 + 6200 = 10.000$$

$$\Rightarrow a = 5000$$

$$\Rightarrow x + 1800 + 1800 + 200 = 5000 \Rightarrow x = 1200$$

$$\Rightarrow \text{Foco } 1 = (\underset{-(1800+x)}{\underset{\uparrow}{-3000}}, 0) \Rightarrow -ae = -3000 \Rightarrow 5000e = 3000$$

$$\Rightarrow e = \frac{3}{5}$$

Usando que $e = \dfrac{\sqrt{a^2-b^2}}{a}$

$$\Rightarrow b^2 = a^2 - [ae]^2 = 5000^2 - 3000^2$$
$$= 4000^2$$

$$\Rightarrow \frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1 \Leftrightarrow \boxed{\underline{\text{Ec. órb. } T_1} \quad \frac{x^2}{5000^2} + \frac{y^2}{4000^2} = 1}$$

En este caso

Para obtener la altura hay que evaluar en

$\bar{x} = -ae = -3000$

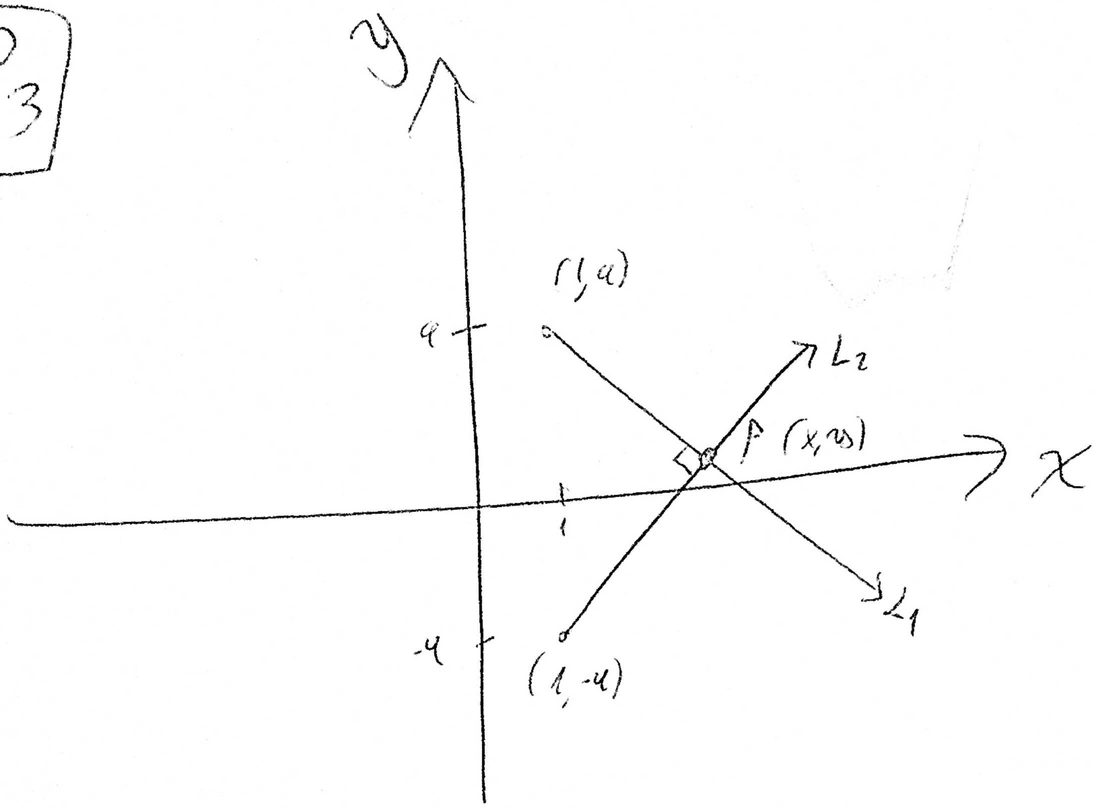
$$\Rightarrow \frac{(-3000)^2}{5000^2} + \frac{\bar{y}^2}{4000^2} = 1$$

$$\Rightarrow \frac{\bar{y}^2}{4000^2} = \frac{5000^2 - 3000^2}{5000^2}$$

$$\Rightarrow \bar{y}^2 = \frac{4000^2 \cdot 4000^2}{5000^2}$$

$$\boxed{\bar{y} = \frac{4000 \cdot 4000}{5000} = 3200}$$

P3



$$m_{L_1} = \frac{y-4}{x-1}$$

$$m_{L_2} = \frac{y+4}{x-1} \Rightarrow m_{L_1} \cdot m_{L_2} = -1 = \frac{y^2-16}{(x-1)^2}$$

$$\Rightarrow -(x-1)^2 = y^2 - 16 \Rightarrow \boxed{(x-1)^2 + y^2 = 16}$$

$$\Rightarrow \frac{(x-1)^2}{4^2} + \frac{y^2}{4^2} = 1$$

es una circunferencia

centro (1,0) y radio 4

P4 i) $(-a)b^{-1} = -ab^{-1}$

Por unicidad del inverso hay que probar que

$$ab^{-1} + (-a)b^{-1} = 0$$

<u>Demostración</u>

$$ab^{-1} + (-a)b^{-1} = [a + (-a)]\,b^{-1} \quad \text{/ Ax distributividad}$$

$$= 0 \cdot b^{-1} \quad \text{/ Inverso Aditivo } \cancel{\text{Neutro}}$$

$$= 0 \quad \text{/ Prop (A) [Hay que probarla]}$$

<u>Prop (A)</u>: $\forall x \in \mathbb{R} \quad 0 \cdot x = 0$

$$0 \cdot x = 0x + 0 \quad \text{/ Neutro Aditivo}$$

$$= 0x + [x + (-x)] \quad \text{/ Inverso Aditivo}$$

$$= [0x + x] + (-x) \quad \text{/ Ax Asociatividad}$$

$$= [0x + 1 \cdot x] + (-x) \quad \text{/ Neutro multiplicativo}$$

$$= [0 + 1]\,x + (-x) \quad \text{/ Ax Asociatividad}$$

$$= 1 \cdot x + (-x) \quad \text{/ Neutro Aditivo}$$

$= x + (-x)$ / Neutro Multiplicativo

$= 0$ / Inverso Aditivo

ii) Probar que

$$x^2 + xy + y^2 \geq |xy|$$

es equivalente a probar que

$$\underset{(1)}{x^2 + xy + y^2 \geq xy} \wedge \underset{(2)}{x^2 + xy + y^2 \geq -xy}$$

Probar (1)

$$x^2 + xy + y^2 \geq xy \iff x^2 + y^2 \geq 0 \iff V$$

Probar (2)

$$x^2 + xy + y^2 \geq -xy \iff x^2 + 2xy + y^2 \geq 0$$

$$\iff (x+y)^2 \geq 0 \iff V$$

Como ambas son verdaderas

$$\Rightarrow x^2 + xy + y^2 \geq |xy| \iff V$$

$$\frac{3-2x}{x^2-|2x+3|} \leq 0$$

<u>Caso $x < -\frac{3}{2}$</u>

$$\Rightarrow \frac{3-2x}{x^2+2x+3} \leq 0 \iff 3-2x \leq 0$$

$$\iff 3 \leq 2x$$

$$\iff \frac{3}{2} \leq x \Rightarrow S_1 = \emptyset$$

$b^2-4ac = 4-12=-8$
Es siempre positivo

<u>Caso $x \geq -\frac{3}{2}$</u>

$$\frac{3-2x}{x^2-2x-3} \leq 0$$

$$\Leftrightarrow \frac{3-2x}{(x-3)(x+1)} \leq 0$$

	$-\infty$	-1	$\frac{3}{2}$	3	∞
$3-2x$	$+$	$+$	$-$	$-$	
$x+1$	$-$	$+$	$+$	$+$	
$x-3$	$-$	$-$	$-$	$+$	
	$+$	$-$	$+$	$-$	

$$S' = (-1, \tfrac{3}{2}] \cup (3, \infty)$$

$$S = S' \cap \text{Caso} = (-1, \tfrac{3}{2}] \cup (3, \infty)$$

$\boxed{P5}$ $\left|\dfrac{x^2-2x-3}{x^4-4x^3+8x^2-4x+7}\right| \leq \dfrac{1}{x^2+1}$

$\Leftrightarrow \dfrac{|x^2-2x-3|}{\cancel{(x^2+1)}|(x^2-4x+7)|} \leq \dfrac{1}{\cancel{x^2+1}}$

$\Leftrightarrow \dfrac{|x^2-2x-3|}{(x^2-4x+7)} \leq 1$ (Pues $x^2-4x+7>0$)

$b^2-4ac<0$

$16-28=-12$

$\Leftrightarrow |(x-3)(x+1)| \leq x^2-4x+7$

Caso $x \in (-\infty,-1] \cup [3,\infty)$ ($x^2-2x-3 \geq 0$)

$\cancel{x^2}-2x-3 \leq \cancel{x^2}-4x+7$

$\Leftrightarrow 2x \leq 10$

$x \leq 5$ $\Rightarrow S_1 = (-\infty,-1] \cup [3,5]$

Caso $x \in (-1,3)$ ($x^2-2x-3<0$)

$\boxed{S = (-\infty,1] \cup [2,5]}$

$-x^2+2x+3 \leq x^2-4x+7$

$\Leftrightarrow 0 \leq 2x^2-6x+4 \Leftrightarrow 0 \leq x^2-3x+2$

$\Leftrightarrow 0 \leq (x-2)(x-1) \Rightarrow x \in (-\infty,1] \cup [2,\infty) \Rightarrow S_2 = (-1,1] \cup [2,3)$