Auxiliar 14

P1 a) $\cosh(x)' = \lim_{h \to 0} \dfrac{\cosh(x+h) - \cosh(x)}{h}$

$$= \lim_{h \to 0} \frac{e^{x+h} + e^{-x-h} - e^{x} - e^{-x}}{2h}$$

$$= \lim_{h \to 0} e^{x}\frac{(e^{h}-1)}{2h} + e^{-x}\frac{(e^{-h}-1)}{2h}$$

$$= \lim_{h \to 0} \frac{e^{x}}{2} \cdot \frac{e^{h}-1}{h} - \frac{e^{-x}}{2} \cdot \frac{e^{-h}-1}{(-h)}$$

Ambos convergen a 1

$$\Rightarrow \quad = \frac{e^{x}}{2} - \frac{e^{-x}}{2} = \sinh(x)$$

b) $(\sinh(x))' = \lim_{h \to 0} \dfrac{e^{x+h} - e^{-x-h} - e^{x} + e^{-x}}{2h}$

$$= \frac{e^{x}}{2} \lim_{h \to 0} \frac{e^{h}-1}{h} + \frac{e^{-x}}{2} \lim_{h \to 0} \frac{e^{-h}-1}{(-h)}$$

$$= \frac{e^{x}}{2} + \frac{e^{-x}}{2} = \cosh(x)$$

c) $\left(x\sin(x)\right)' = \lim_{h\to 0} \frac{(x+h)\sin(x+h) - \sin(x)\,x}{h}$

$= \lim_{h\to 0} \frac{x\left(\sin(x+h) - \sin(x)\right)}{h} + \sin(x+h)$

$= \lim_{h\to 0} x\left(\frac{\sin(x)\,(\cos(h)-1)}{h} + \cos(x)\frac{\sin(h)}{h}\right) + \sin(x+h)$ $\to 0$

$= x\cdot\cos(x) + \sin(x)$

$\boxed{P2}$ a) $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$

$\Rightarrow (\tanh(x))' = \frac{\sinh(x)'\cosh(x) - \cosh(x)'\sinh(x)}{\cosh(x)^2}$

$= \frac{\cosh(x)^2 - \sinh(x)^2}{\cosh(x)^2} = \frac{1}{\cosh(x)^2} = \operatorname{sech}(x)^2$

b) $\left(x^3(x^2-1)^2\right)' = (x^3)'(x^2-1)^2 + x^3\left((x^2-1)^2\right)'$

$= 3x^2(x^2-1)^2 + x^3\,2(x^2-1)\cdot 2x$

$= (x^2-1)x^2(x^2-1+4x^2)$

$= (x^2-1)x^2(5x^2-1)$

c) $$\left(\frac{2x^4}{b^2-x^2}\right)' = \frac{(2x^4)'(b^2-x^2) - (b^2-x^2)'\,2x^4}{(b^2-x^2)^2}$$

$$= 8x^3(b^2-x^2) + 2x(2x^4)$$

$$= 8x^3b^2 - 4x^5 = 4x^3(2b^2-x^2)$$

P3 $$\boxed{(f^{-1})'(y_0) = \frac{1}{f'(f^{-1}(y_0))} \quad \text{(Recordar)}}$$

a) $$(\operatorname{Arccos}(x))' = \frac{1}{-\sin(\operatorname{Arccos}(x))} = \frac{-1}{\sqrt{1-\cos(\operatorname{Arccos}(x))^2}} = \frac{-1}{\sqrt{1-x^2}}$$

b) $$(\operatorname{Artan}(x))' = \frac{1}{\sec^2(\operatorname{Artan}(x))} = \frac{1}{1+\tan^2(\operatorname{Artan}(x))} = \frac{1}{1+x^2}$$

c) Usando que $(2^x)' = \ln(2)\,2^x$

$$(\log_2(x))' = \frac{1}{\ln(2)\,2^{\log_2(x)}} = \frac{1}{\ln(2)\,x}$$

Observación

$$\log_2(x) = \frac{\ln(x)}{\ln(2)} \Rightarrow (\log_2(x))' = \frac{(\ln(x))'}{\ln(2)} = \frac{1}{\ln(2)}\cdot\frac{1}{x}$$

P4) Sea $x \geq m$, Pdq $\frac{1}{\sqrt{x}} \leq \varepsilon$

Sea $m = (\varepsilon^2)^{-1} \Rightarrow$ Si $x \geq m \Rightarrow x \geq (\varepsilon^2)^{-1}$

$\Rightarrow \frac{1}{x} \leq \varepsilon^2$

$\Rightarrow \frac{1}{\sqrt{x}} \leq \varepsilon$

Con lo que se concluye

$\boxed{\text{Idea Para conseguir el } m \text{ partir de } \frac{1}{\sqrt{x}} \leq \varepsilon \Leftrightarrow \dots \Leftrightarrow x \geq (\varepsilon^{-2})}$

P5 $\frac{x e^x}{e^x - x}$ No tiene solucion $(e^x - x > 0)$

$$\lim_{x \to \infty} \frac{e^x}{e^x - x} = \lim_{x \to \infty} \frac{1}{1 - \frac{x}{e^x}} = 1$$

$$\lim_{x \to -\infty} \frac{x e^x}{e^x - x} = \lim_{x \to -\infty} \frac{x}{1 - x e^{-x}} = \lim_{u \to \infty} \frac{-u}{1 + u e^u}$$

$$= \lim_{u \to \infty} \frac{-1}{\frac{1}{u} + e^u} = 0$$

$$\lim_{x \to \infty} \frac{x e^x}{e^x - x} - x = \lim_{x \to \infty} x \left(\frac{e^x}{e^x - x} - 1 \right)$$

$$= \lim_{x \to \infty} x \frac{x}{e^x - x} = \lim_{x \to \infty} \frac{x^2}{e^x - x} = 0$$

$$\Rightarrow \quad y = 0 \quad y \quad y = x$$

$$\boxed{P6} \quad \lim_{x \to 0} \frac{a^x - 1}{x} = \lim_{x \to 0} \frac{e^{\ln(a)x} - 1}{x \ln(a)} \cdot \ln(a)$$

$$= \ln(a)$$

$$\lim_{x \to 4} \frac{2^x - 16}{x - 4} = \lim_{x \to 4} 2^4 \cdot \frac{2^{x-4} - 1}{x - 4}$$

$$= 16 \cdot \lim_{u \to 0} \frac{2^u - 1}{u} = 16 \cdot \ln(2)$$