Auxliar 15

P1 a) $x^2+y^2=16$ / Derivando

$$2x+2yy'=0 \Rightarrow \boxed{y'=\frac{-x}{y}}$$

b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ / Derivando

$$\frac{2x}{a^2}+\frac{2y}{b^2}y'=0 \Rightarrow \boxed{y'=\frac{-b^2x}{a^2y}}$$

P2 a) $\lim_{x\to 1}\frac{e^x(x^2-1)^2}{\sin(x^4-1)}=\frac{0}{0}$, por lo tanto por L'H

$$\Rightarrow =\lim_{x\to 1}\frac{e^x(x^2-1)^2+2(x^2-1)(2x)e^x}{\cos(x^4-1)(4x^3)}=\frac{0}{4}=\boxed{0}$$

b) $\lim_{x\to 4}\frac{x^2\ln(x-3)}{\cos\left(\frac{(x-3)\pi}{2}\right)}=\frac{0}{0}\overset{L'H}{\Rightarrow}\lim_{x\to 4}\frac{2x\ln(x-3)+\frac{x^2}{x-3}}{\sin\left((x-3)\frac{\pi}{2}\right)\frac{\pi}{2}}=\frac{0+\frac{16}{1}}{1\,\frac{\pi}{2}}=\boxed{\frac{32}{\pi}}$

P3 $\frac{x^2}{4}+\frac{(y-2)^2}{16}=1 \Rightarrow \frac{2x}{4}+\frac{2(y-2)}{16}y'=0 \Rightarrow \boxed{y'=\frac{-4x}{y-2}}$

En el punto $(x_0,y_0)=(\sqrt{2},2+2\sqrt{2}) \Rightarrow y'=\frac{-4\sqrt{2}}{2\sqrt{2}}=-2$

$$\Rightarrow \boxed{y=-2(x-\sqrt{2})+2+2\sqrt{2}}$$

En el punto $(x_0,y_0)=(\sqrt{2},-2-2\sqrt{2}) \Rightarrow y'=\frac{-4\sqrt{2}}{-4-2\sqrt{2}}=\frac{2\sqrt{2}}{2+\sqrt{2}}$

$$\Rightarrow \boxed{y=\frac{2\sqrt{2}}{2+\sqrt{2}}(x-\sqrt{2})-2-2\sqrt{2}}$$

b) $y=(x-4)^2 \Rightarrow \boxed{y'=2(x-4)}$

Caso $(4,0) \Rightarrow y'=0 \Rightarrow \boxed{y=0(x-4)+0=0}$

Caso $(0,16) \Rightarrow y'=-8 \Rightarrow y=-8(x-0)+16$

$\boxed{y=-8x+16}$

P4 a) $(\text{ArcTan}(x))^{(2)} = \left(\frac{1}{1+x^2}\right)' = \boxed{\frac{2x}{(1+x^2)^2}}$

b) $(xe^{x^2})^{(3)} = \left(e^{x^2} + xe^{x^2}(2x)\right)'' = \left(e^{x^2}(1+2x^2)\right)''$

$= \left(e^{x^2}(2x)(1+2x^2) + e^{x^2}(4x)\right)'$

$= \left(e^{x^2}(6x+4x^3)\right)' =$

$= \left(e^{x^2}(6+12x^2) + e^{x^2}(2x)(6x+4x^3)\right)$

$= \boxed{e^{x^2}(6+24x^2+8x^4)}$

c) $(e^{5x}\ln(x))^{(n)}$, estudiemos $\boxed{(e^{5x})^{(j)} = 5^j e^{5x}}$

y $(\ln(x))^{(j)} = \begin{cases} x^{-1} & j=1 \\ (-1)x^{-2} & j=2 \\ (-1)(-2)x^{-3} & j=3 \\ \vdots & \vdots \\ (-1)(-2)\ldots(-(j-1))x^{-j} & j \end{cases} \Rightarrow \boxed{(-1)^{j-1}(j-1)!\,x^{-j} \quad j\geq 1}$

Ahora reemplazando en la formula

$$\Rightarrow \boxed{\left(e^{5x}\ln(x)\right)^{(n)} = 5^n e^{5x}\ln(x) + \sum_{j=0}^{n}\binom{n}{j}5^{n-j}e^{5x}(-1)^{j-1}(j-1)!\,x^{-j}}$$

d) $\left(x^3\sin(x)\right)^{(n)}$, estudiamos $\left(x^3\right)^{(j)} = \begin{cases} 3x^2 & j=1 \\ 6x & j=2 \\ 6 & j=3 \\ 0 & j\geq 4 \end{cases}$

y $\left(\sin(x)\right)^{j} = \sin\left(x + j\frac{\pi}{2}\right)$

$$\Rightarrow \left(x^3\sin(x)\right)^{(n)} = \sum_{j=0}^{n}\binom{n}{j}\left(x^3\right)^{(j)}\left(\sin(x)\right)^{(n-j)} = \sum_{j=0}^{3}\binom{n}{j}\left(x^3\right)^{(j)}\sin^{(n-j)}(x)$$

$$= \binom{n}{0}x^3\sin\left(x+n\frac{\pi}{2}\right) + \binom{n}{1}3x^2\sin\left(x+(n-1)\frac{\pi}{2}\right)$$

$$+ \binom{n}{2}6x\sin\left(x+(n-2)\frac{\pi}{2}\right) + \binom{n}{3}6\sin\left(x+(n-3)\frac{\pi}{2}\right)$$

$\boxed{P5}$ a) $\text{Taylor}\left(\cos(x)\right) = \cos(x_0) + \cos'(x_0)(x-x_0) + \dfrac{\cos''(x_0)(x-x_0)^2}{2}$

$$= \cos(x_0) - \sin(x_0)(x-x_0) - \frac{\cos(x_0)}{2}(x-x_0)^2$$

b) $\text{Taylor}\left(e^x\right) = e^{x_0} + \left(e^{x_0}\right)'(x-x_0) + \dfrac{\left(e^{x_0}\right)''}{2}(x-x_0)^2$

$$= e^{x_0}\left(1 + (x-x_0) + \frac{(x-x_0)^2}{2}\right)$$

c) $\text{Taylor}(\sinh(x)) = \sinh(x_0) + \sinh'(x_0)(x-x_0) + \sinh''\frac{(x_0)(x-x_0)^2}{2}$

$= \sinh(x_0) + \cosh(x_0)(x-x_0) + \sinh\frac{(x_0)(x-x_0)^2}{2}$

d) $\text{Taylor}(x\sin(x)) = x_0\sin(x_0) + (x_0\sin(x_0))'(x-x_0) + (x_0\sin(x_0))''\frac{(x-x_0)^2}{2}$

$= x_0\sin(x_0) + (\sin(x_0) + x_0\cos(x_0))(x-x_0) + \frac{(2\cos(x_0) - x_0\sin(x_0))(x-x_0)^2}{2}$