AUX 5 cadena

Recordamos que

$f: \mathbb{R} \to \mathbb{R}$

f es diferenciable $\Leftrightarrow$ f es derivable

y

$Df = f'$

$f: \mathbb{R}^n \to \mathbb{R}$

si f es diferenciable tenemos que

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}$$

y que podemos trabajar con el Df ó el ∇f indistintamente considerando que

$Df(x_0)\cdot(h)$
↓
Producto matrizial

$\nabla f(x_0)\cdot h$
↓
producto escalar

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ejemplo:

$f(x,y)$ es diferenciable

$\Rightarrow Df = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$

$\Downarrow$

matriz de 1 par 2

así $Df(x_0) \cdot h$ con

$h = (u,v)$

$\left(\frac{\partial f(x_0)}{\partial x}, \frac{\partial f(x_0)}{\partial y}\right) \cdot \begin{pmatrix} u \\ v \end{pmatrix}$ → multiplicación matricial

$= \frac{\partial f(x_0)}{\partial x} u + \frac{\partial f(x_0)}{\partial y} v$

ahora

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$$

$\nabla f(x_0) \cdot h$ con $h = (u, v)$

$$= \begin{pmatrix} \frac{\partial f}{\partial x}(x_0) \\ \frac{\partial f}{\partial y}(x_0) \end{pmatrix} \cdot \begin{pmatrix} u \\ v \end{pmatrix}$$

producto escalar

$$= \frac{\partial f}{\partial x}(x_0) \cdot U + \frac{\partial f}{\partial x}(x_0) \cdot V$$

Y por ultimo

si $g = f \circ h$

$$\Rightarrow Dg = D(f \circ h) = Df(h) \circ Dh$$

ahora esta formula esta pensada para funciones si usamos la representación matricial

$$D(f \circ h) = Df(h) \cdot Dh$$

→ producto matrices

P1

Sea $f: \mathbb{R}^3 \to \mathbb{R}$

$$3x^2 \frac{\partial f}{\partial x} + 2y^2 \frac{\partial f}{\partial y} = 3(f(x,y))^2$$

considerando

$$u(x,y) = x \quad \text{i} \quad v(x,y) = \frac{1}{3x} - \frac{1}{2y}$$

Definamos

$$g(u,v) = f(x(u,v), y(u,v))$$

Pruebe que

$$u^2 \frac{\partial g}{\partial u}(u,v) = (g(u,v))^2$$

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calculemos las derivadas en terminos generales

$$\frac{\partial g}{\partial u} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$\frac{\partial g}{\partial v} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial v}$$

por lo que basta calcular

$$\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}$$

ahora las formulas anteriores son una forma analoga de escribir cadena de forma matricial notemos

$$g(u,v) = f(x(u,v), y(u,v))$$

$$= (f \circ h)(u,v)$$

$$h = (x(u,v), y(u,v))$$

así tenemos

$$Dg = Df(h) \circ Dh$$

ahora

$$Df = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

Ya que no sabemos nada de F.

a su vez

$$Dh = \begin{pmatrix} \frac{\partial h}{\partial u} & \frac{\partial h}{\partial v} \end{pmatrix}$$

Tratando a sus derivadas como vectores columna

$$\frac{\partial h}{\partial u} = \left(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u} \right)$$

$$\frac{\partial h}{\partial v} = \left(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v} \right)$$

así

$$Dh = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix}$$

así

$Df \cdot Dh$

$$= \left(\frac{\partial f}{\partial x} , \frac{\partial f}{\partial y} \right) \cdot \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix}$$

$$= \left(\frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial u} , \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial v} \right)$$

de esto identificamos que

$$\frac{\partial g}{\partial u} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial v}$$

$$\frac{\partial g}{\partial v} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial v}$$

entonces notamos

$$u = x \quad , \quad v = \frac{1}{3x} - \frac{1}{2y}$$

$$\rightarrow v = \frac{1}{3u} - \frac{1}{2y}$$

$$\rightarrow \frac{1}{2y} = \frac{1}{3u} - v$$

$$\rightarrow \frac{1}{2y} = \frac{1-3uv}{3u}$$

$$\rightarrow \frac{3u}{1-3uv} = 2y$$

$$\rightarrow Y = \frac{3}{2}\,\frac{U}{1-3UV}$$

así $X(U,V) = U$

$$Y(U,V) = \frac{3}{2}\,\frac{U}{1-3UV}$$

$$\rightarrow \frac{\partial X}{\partial U} = 1 \,, \quad \frac{\partial X}{\partial V} = 0$$

$$\frac{\partial Y}{\partial U} = \frac{3}{2}\,\frac{1}{(1-3UV)^2}$$

$$\frac{\partial Y}{\partial V} = \frac{9\,U^2}{2(1-3UV)^2}$$

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comesto

$$\frac{\partial g}{\partial v} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial v}$$

$$= \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \cdot \frac{3}{2} \cdot \frac{1}{(1-3v)^2}$$

$$\frac{\partial g}{\partial v} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial v}$$

$$= \cancel{\frac{\partial f}{\partial x} \cdot 0}^{0} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial v}$$

$$= \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial v}$$

$$= \frac{\partial f}{\partial Y} \cdot \frac{9\,U^2}{3(1-3UV)^2}$$

αλλά

$$U^2 \frac{\partial y}{\partial U}$$

$$= U^2\left(\frac{\partial f}{\partial x} + \frac{3}{2}\frac{1}{(1-3UV)^2}\frac{\partial f}{\partial Y}\right)$$

$$= U^2\frac{\partial f}{\partial x} + \frac{3}{2}\left(\frac{U}{1-3UV}\right)^2\frac{\partial f}{\partial Y}$$

$$= x^2\frac{\partial f}{\partial x} + \frac{3}{2}\left(\frac{U}{1-3UV}\right)^2\frac{\partial f}{\partial Y}$$

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Recordamos

$$y = \frac{3}{2}\frac{U}{1-30V}$$

$$\to \frac{U}{1-30V} = \frac{2}{3}y$$

$\to$

$$U^2\frac{\partial g}{\partial U} = x^2\frac{\partial f}{\partial x} + \frac{3}{2}\left(\frac{2}{3}y\right)^2\frac{\partial f}{\partial y}$$

$$= x^2\frac{\partial f}{\partial x} + \frac{3}{2}\cdot\frac{4^2}{9\,3}y^2\frac{\partial f}{\partial y}$$

$$= x^2\frac{\partial f}{\partial x} + \frac{2}{3}y^2\frac{\partial f}{\partial y}$$

$$= \frac{1}{3}\left[3x^2\frac{\partial f}{\partial x} + 2y^2\frac{\partial f}{\partial y}\right]$$

$$= \frac{1}{3} \cdot 3\,(f(x,y))^2$$

$$= (f(x,y))^2$$

$$= (g(u,v))^2$$

muestre que si $h: \mathbb{R} \to \mathbb{R}$ es una foncion de clase C^1 entonces

$$g(u,v) = \frac{u}{1+uh(v)}$$ solociona

La ecuación.

notamos

$$\frac{\partial y}{\partial u} = \frac{1+uh(v) - uh(v)}{(1+uh(v))^2}$$

$$= \frac{1}{(1+uh(v))^2}$$

$$\rightarrow u^2 \frac{\partial y}{\partial u} = \frac{u^2}{(1+uh(v))^2}$$

$$= (g(u,v))^2 \quad //$$

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P2

supongamos
$f,g: \mathbb{R}^n \to \mathbb{R}$ diferenciables
en $x_0 \in \mathbb{R}^n$, $f(x_0) = g(x_0) = 0$
y $\nabla g(x_0) \neq 0$

- suponiendo que $\exists \lambda \in \mathbb{R}$ t.q
  $\nabla f(x_0) = \lambda \cdot \nabla g(x_0)$

muestre que

$$\lim_{\lambda \to 0} \frac{f(x_0 + \lambda e)}{g(x_0 + \lambda e)} = \lambda \quad (\lambda \in \mathbb{R})$$

$\forall e \in \mathbb{R}^n$ t.q $\nabla g(x_0) \cdot e \neq 0$

notamos que

$$\lim_{\lambda \to 0} \frac{f(x_0+\lambda e)}{g(x_0+\lambda e)} \longrightarrow \frac{f(x_0)}{g(x_0)} = \frac{0}{0}$$

así que nos dan ganas de usar l'Hôpital pero no es válido en $\mathbb{R}^n$

para esto definimos

$$\phi(\lambda) = f(x_0+\lambda e)$$

$$\psi(\lambda) = g(x_0+\lambda e)$$

notamos que

$\phi, \psi : \mathbb{R} \to \mathbb{R}$

$\therefore$ podemos usar l'Hopital así

$$\lim_{\lambda\to 0} \frac{f(x_0+\lambda e)}{g(x_0+\lambda e)} = \lim_{\lambda\to 0} \frac{\phi(\lambda)}{\psi(\lambda)} \overset{LH}{=} \lim_{\lambda\to 0} \frac{\phi'(\lambda)}{\psi'(\lambda)}$$

si esta todo bien definido $\overset{\nabla}{=} \dfrac{\phi'(0)}{\psi'(0)}$

veamos si lo esta

$\phi(\lambda) = f(x_0 + \lambda e)$

si definimos $h(\lambda) = x_0 + \lambda e$

$\Rightarrow \phi(\lambda) = (f \circ h)(\lambda)$

recordamos

$\phi'(\lambda) = D\phi(\lambda)$

$= D(f \circ h)(\lambda)$

$= Df(h(\lambda)) \circ Dh(\lambda)$

ahora

$Dh(\lambda) = e$ [verificar]

y $Df = \nabla f$ [por lo comentado

$\therefore Df(h(\lambda)) \cdot Dh(\lambda)$

$= \nabla f(x_0 + \lambda e) \cdot e$

producto escalar

analogamente

$\phi'(\lambda) = \nabla f(x_0 + \lambda e) \cdot e$

$\psi'(\lambda) = \nabla g(x_0 + \lambda e) \cdot e$

$\therefore\ \phi'(0) = \nabla f(x_0) \cdot e$

$\psi'(0) = \nabla g(x_0) \cdot e \neq 0$

$\therefore\ \dfrac{\phi'(0)}{\psi'(0)}$ esta bien

definido y

$$\lim_{\lambda \to 0} \frac{f(x_0 + \lambda e)}{g(x_0 + \lambda e)}$$

$$= \frac{\phi'(0)}{\psi'(0)} = \frac{\nabla f(x_0) \cdot e}{\nabla g(x_0) \cdot e}$$

$$= \frac{(\lambda \nabla g(x_0) \cdot e)}{\nabla g(x_0) \cdot e}$$ [emv + (cuid)]

$$= \lambda \cdot \frac{\cancel{\nabla g(x_0) \cdot e}}{\cancel{\nabla g(x_0) \cdot e}}$$

$$= \lambda$$

• Muestre que si

$$\lim_{x \to x_0} \frac{f(x)}{g(x)}$$

existe, entonces

$\exists \lambda \in \mathbb{R}$, $\nabla f(x_0) = \lambda \nabla g(x_0)$

el candidato natural a λ es

$$\lambda = \lim_{x \to x_0} \frac{f(x)}{g(x)}$$

ahora el limite existe ! para cualquier camino va a existir y ser igual en particular para $x = x_0 + \lambda e$ con $\lambda \in \mathbb{R}^+$, e fijo t.q $\nabla g(x_0) \cdot e \neq 0$

entonces

$$\lambda = \lim_{\lambda \to 0^+} \frac{f(x_0+\lambda e)}{g(x_0+\lambda e)}$$

y como l'Hôpital se cumple para limites laterales

$$\lambda = \frac{\nabla f(x_0) \cdot e}{\nabla g(x_0) \cdot e}$$

$$\Rightarrow \lambda \nabla g(x_0) \cdot e = \nabla f(x_0) \cdot e$$

$$\Rightarrow [\lambda \nabla g(x_0) - \nabla f(x_0)] \cdot e = 0$$

$$\Rightarrow \lambda \nabla g(x_0) - \nabla f(x_0) = 0$$

$\Downarrow$

ya que $e \neq 0$

$$\Rightarrow \nabla f(x_0) = \lambda \nabla g(x_0)$$

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P3

- Sea $f: \mathbb{R} \to \mathbb{R}$ f de clase C^1.

Se define $F: \mathbb{R}^3 \to \mathbb{R}^2$

$F(x,y,z) = (z - x f^2(y+z),\ x + y f(xz^2))$

~~entonces~~ calcule el Jacobiano de F.

entonces calculamos

$\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}$

$$\frac{\partial F}{\partial x} = \left(\frac{\partial}{\partial x}\left(z - x f^2(y+z)\right), \frac{\partial}{\partial x}\left(x + y f(xz^2)\right) \right)$$

$$= \left(-f^2(y+z), 1 + z^2 y f'(xz^2) \right)$$

$$\frac{\partial F}{\partial y} = \left(\frac{\partial}{\partial y}\left(z - x f^2(y+z)\right), \frac{\partial}{\partial y}\left(x + y f(xz^2)\right) \right)$$

$$= \left(-2x f(y+z) f'(y+z), f(xz^2) \right)$$

$$\frac{\partial F}{\partial z} = \left(\frac{\partial}{\partial z}\left(z - x f^2(y+z)\right), \frac{\partial}{\partial z}\left(x + y f(xz^2)\right) \right)$$

$$= \left(1 - 2x f(y+z) f'(y+z), 2yxz f'(xz^2) \right)$$

$DF = JF$

$$= \begin{pmatrix} -f^2(y+z) & -2xf(y+z)f'(y+z) & 1-2xf(y+z)f'(y+z) \\ 1+z^2yf'(xz^2) & f(xz^2) & 2yxzf'(xz^2) \end{pmatrix}$$

• $f: \mathbb{R}^2 \to \mathbb{R}^2$

$f(x,y) = (x^2y, xy^2)$

→ Calcule el jacobiano de f. ¿es diferenciable?

$$J_f = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{pmatrix}$$

$$\frac{\partial f}{\partial x} = (2xy, y^2)$$

$$\frac{\partial f}{\partial y} = (x^2, 2xy)$$

$\to J_f(x,y) = \begin{pmatrix} 2xy & y^2 \\ x^2 & 2xy \end{pmatrix}$

Todas sus derivadas parciales son continuas

$\therefore$ f es diferenciable //

Sea $f_n = f \circ f \circ \cdots \circ f$ n veces

$f_0 = \mathrm{Id} \to f_1 = f$

pdq

$J f_n(1,1) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^n$

par inducción

$\boxed{n=0}$

$Jf_0(1,1) = J\,id(1,1)$

$id(x,y) = (x,y)$

$\frac{\partial}{\partial x}(id(x,y)) = (1,0)$

$\frac{\partial}{\partial y}(id(x,y)) = (0,1)$

$\Rightarrow J\,id(1,1) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^0$

$$\Delta \quad J f_0(1,1) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^0$$

$n \rightarrow n+1$

$$J f_{n+1} = J[f_n \circ f]$$

cadena ⇒

$$= J f_n(f(1,1)) \cdot J f(1,1)$$

H-I ⇒

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^3 \cdot \left.\begin{bmatrix} 2xy & y^2 \\ x^2 & 2xy \end{bmatrix}\right|_{(1,1)}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^3 \cdot \begin{bmatrix} 2 \cdot 1 \cdot 1 & 1^2 \\ 1^2 & 2 \cdot 1 \cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^3 \cdot \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

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$$z\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{n-1}$$

P(4) $f, g: \mathbb{R} \to \mathbb{R}$ dos funciones derivables y $f' \neq 0$, $g' \neq 0$ definimos

$$z = f(x\,g(y))$$

- Calcule

$$\frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} = f'(xg(y)) \cdot g(y)$$

$$\frac{\partial z}{\partial y} = f'(xg(y)) \cdot x \cdot g'(y)$$

- Suponiendo que

$$x \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

$$\Rightarrow g(y) - eg'(y) = 0$$

$$x \cdot f'(xg(y)) \cdot g(y) + f'(xg(y)) \cdot x \cdot g(y)$$

$$= f'(xg(y)) \cdot x\,[g(y) - eg'(y)]$$

$$= 0$$

$\Rightarrow g(y) + g'(y) = 0$ //

$\Downarrow$

$f(x, g(y)) \neq 0 \quad \wedge \quad x \neq 0 \quad \forall x \neq 0 \; [1, 1]$

P5) Considere la superficie definida por la ecuación

$$z = \sin(\pi x^2)\, e^{x - y^3}$$

Encuentre la ecuación del plano tangente a esta superficie en el punto $(1, 1, 0)$

la formula del plano tangente

$$Z = f(x,y)$$ ~~es~~

en el punto (x_0, y_0, Z_0) es:

$$Z - \cancel{Z_0} = f(x_0, y_0) + \nabla f(x_0, y_0) \cdot \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}$$

donde $Z_0 = f(x_0, y_0)$

entonces $f(x,y) = \sin(\pi x^2)\, e^{x^2 - y^3}$

$$Z \cancel{+0} = f(1,1) + \nabla f(1,1) \begin{pmatrix} x - 1 \\ y - 1 \end{pmatrix}$$

$$Z = 0 + \nabla f(1,1) \begin{pmatrix} x - 1 \\ y - 1 \end{pmatrix}$$

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} \quad y \; f(x,y) = \sin(\pi x^2) e^{x-y^3}$$

$$\frac{\partial f}{\partial x} = 2\pi x \cos(\pi x^2) e^{x-y^3} + \sin(\pi x^2) e^{x-y^3}$$

$$\frac{\partial f}{\partial x}(1,1) = -2\pi$$

$$\frac{\partial f}{\partial y} = -3y^2 \sin(\pi x^2) e^{x-y^3}$$

$$\frac{\partial f}{\partial x}(1,1) = 0$$

$$Z = \begin{pmatrix} -2\pi \\ 0 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-1 \end{pmatrix}$$

$$Z = -2\pi(x-1)$$

Sea $f: \mathbb{R}^2 \to \mathbb{R}^3$ definida por

$$f(x,y) = (x^2 y + e^{xy}, x + y^2, \sin(xy))$$

Demuestre que f es diferenciable y encuentre su matriz jacobiana en ese punto.

$$\frac{\partial f}{\partial x} = (2xy + ye^{xy}, 1, y\cos(xy))$$

$$\frac{\partial f}{\partial y} = (x^2 + xe^{xy}, 2y, x\cos(xy))$$

∴ sus derivadas parciales son continuas en todo $\mathbb{R}^2$

$\Rightarrow$ f es diferenciable en $\mathbb{R}^2$

$\Rightarrow$ f es diferenciable en $(1,0)$

$$J_f = \begin{pmatrix} 2xy + ye^{xy} & x^2 + xe^{xy} \\ 1 & 2y \\ y\cos(xy) & x\cos(xy) \end{pmatrix}$$

$$J_f(1,0) = \begin{pmatrix} 0 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

//

P6) sea $f: \mathbb{R}^3 \to \mathbb{R}$

diferenciable

$f = f(x, y, z)$ y definimos

$F(r, \theta, \phi)$

$= f(r\cos(\theta)\sin(\phi), r\sin(\theta)\sin(\phi),$
$r\cos(\phi))$

$= f(x(r,\theta,\phi), y(r,\theta,\phi), z(r,\theta,\phi))$

con

$x(r,\theta,\phi) = r\cos(\theta)\sin(\phi)$

$y(r,\theta,\phi) = r\sin(\theta)\sin(\phi)$

$z(r,\theta,\phi) = r\cos(\phi)$

ahora

$F = (f \circ h)$

$h = (x, y, z)$

$\therefore DF = Df \circ Dh$

$$Df = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$Dh = \left(\frac{\partial h}{\partial r}, \frac{\partial h}{\partial \theta}, \frac{\partial h}{\partial \phi} \right.$$

$$\frac{\partial h}{\partial r} = \left(\cos(\theta)\,\sin(\phi),\ \sin(\theta)\,\sin(\phi),\ \cos(\phi) \right)$$

$$\frac{\partial h}{\partial \theta} = \left(-\operatorname{sen}(\theta)\sin(\phi)\, r,\ r\cos(\theta)\sin(\phi),\ 0 \right)$$

$$\frac{\partial h}{\partial \phi} = \left(r\cos(\theta)\cos(\phi),\ r\sin(\theta)\cos(\phi),\ -r\sin(\phi) \right)$$

$$\rightarrow Dh = \begin{pmatrix} \cos(\theta)\sin(\phi) & -r\operatorname{sen}(\theta)\sin(\phi) & r\cos(\theta)\cos(\phi) \\ \operatorname{sen}(\theta)\operatorname{sen}(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) \\ \cos(\phi) & 0 & -r\sin(\phi) \end{pmatrix}$$

$$c)\ DF^{\bar{\lambda}} = \frac{\partial f}{\partial x}\cdot \cos(\theta)\sin(\phi) + \frac{\partial f}{\partial y}\operatorname{sen}(\theta)\sin(\phi) + \frac{\partial f}{\partial z}\cos(\phi)$$

$$-\pi\frac{\partial f}{\partial x}\operatorname{sen}(\theta)\sin(\phi) + \pi\frac{\partial f}{\partial y}\cos(\theta)\sin(\phi) + \cancel{\frac{\partial f}{\partial z}\cos(\phi)}$$

$$\frac{\partial f}{\partial x}\pi\cos(\theta)\cos(\phi) + \frac{\partial f}{\partial y}\operatorname{sen}(\theta)\cos(\phi) - \pi\frac{\partial f}{\partial z}\cos(\phi)$$

asi identificamos

$$\frac{\partial F}{\partial r} = \frac{\partial f}{\partial x}\cos(\theta)\sin(\phi) + \frac{\partial f}{\partial y}\operatorname{sen}(\theta)\sin(\phi)$$
$$+ \frac{\partial f}{\partial z}\cos(\phi)$$

$$\frac{\partial F}{\partial \theta} = -r\frac{\partial f}{\partial x}\sin(\theta)\sin(\phi) + r\frac{\partial f}{\partial y}\cos(\theta)\sin(\phi)$$

$$\frac{\partial F}{\partial \phi} = \frac{\partial f}{\partial x} r\cos(\theta)\cos(\phi)$$
$$+ \frac{\partial f}{\partial y} r\operatorname{sen}(\theta)\cos(\phi)$$
$$- \frac{\partial f}{\partial z} r\operatorname{sen}(\phi)$$

$$\left(\frac{\partial f}{\partial r}\right)^2$$

$$= \left(\frac{\partial f}{\partial x} \cdot \cos(\theta)\sin(\phi) + \frac{\partial f}{\partial y}\,\mathrm{sen}(\theta)\sin(\phi) + \frac{\partial f}{\partial z}\cos(\phi)\right)^2$$

$$= \left[\frac{\partial f}{\partial x}\cos(\theta)\sin(\phi) + \frac{\partial f}{\partial y}\,\mathrm{sen}(\theta)\sin(\phi)\right]^2$$

$$+ 2\frac{\partial f}{\partial z}\cos(\phi)\left[\frac{\partial f}{\partial x}\cos(\theta)\sin(\phi) + \frac{\partial f}{\partial y}\,\mathrm{sen}(\theta)\sin(\phi)\right]$$

$$+ \left(\frac{\partial f}{\partial z}\right)^2\cos^2(\phi)$$

$$\left(\frac{1}{r\sin\theta}\frac{\partial F}{\partial \theta}\right)^2$$

$$=\left(-\frac{\partial F}{\partial x}\sin(\theta)+\frac{\partial F}{\partial y}\cos(\theta)\right)^2$$

$$=\left(\frac{\partial F}{\partial y}\right)^2\cos^2(\theta)\cancel{-2\sin(\theta)\cos(\theta)}$$

$$-2\sin(\theta)\cos(\theta)\frac{\partial F}{\partial x}\cdot\frac{\partial F}{\partial y}$$

$$+\left(\frac{\partial F}{\partial x}\right)^2\sin^2(\theta)$$

$$\left(\frac{1}{r}\frac{\partial F}{\partial \phi}\right)^2$$

$$= \left(\frac{\partial F}{\partial x}\cos(\theta)\cos(\phi) + \frac{\partial F}{\partial y}\operatorname{sen}(\theta)\cos(\phi) - \frac{\partial F}{\partial z}\operatorname{sen}(\phi)\right)^2$$

$$= \left[\frac{\partial F}{\partial x}\cos(\theta)\cos(\phi) + \frac{\partial F}{\partial y}\operatorname{sen}(\theta)\cos(\phi)\right]^2$$

$$-2\frac{\partial F}{\partial z}\operatorname{sen}(\phi)\cdot\left[\frac{\partial F}{\partial x}\cos(\theta)\cos(\phi) + \frac{\partial F}{\partial y}\operatorname{sen}(\theta)\cos(\phi)\right] + \left(\frac{\partial F}{\partial z}\right)^2\operatorname{sen}^2(\phi)$$

$$\left(\frac{\partial F}{\partial r}\right)^2 + \left(\frac{1}{r\sin(\phi)}\frac{\partial F}{\partial \theta}\right)^2 + \left(\frac{1}{r}\frac{\partial F}{\partial \phi}\right)^2$$

$$= \left[\frac{\partial f}{\partial x}\cos(\theta)\sin(\phi) + \frac{\partial f}{\partial y}\operatorname{sen}(\theta)\sin(\phi)\right]^2$$

$$+ 2\frac{\partial f}{\partial z}\cos(\phi)\left[\frac{\partial f}{\partial x}\cos(\theta)\sin(\phi) + \frac{\partial f}{\partial x}\operatorname{sen}(\theta)\sin(\phi)\right]$$

$$+ \left(\frac{\partial f}{\partial z}\right)^2\cos^2(\phi)$$

$$+ \left(\frac{\partial f}{\partial x}\right)^2\sin^2(\theta) + \left(\frac{\partial f}{\partial y}\right)^2\cos^2(\theta)$$

$$- 2\sin(\theta)\cos(\theta)\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial x}$$

$$+ \left[\frac{\partial f}{\partial x}\cos(\theta)\cos(\phi) + \frac{\partial f}{\partial y}\operatorname{sen}(\theta)\operatorname{sen}(\phi)\right]^2$$

$$- 2\frac{\partial f}{\partial z}\operatorname{sen}(\theta)\left[\frac{\partial f}{\partial x}\cos(\theta)\cos(\phi) + \frac{\partial f}{\partial y}\operatorname{sen}(\theta)\cos(\phi)\right]$$

$$+ \left(\frac{\partial f}{\partial z}\right)^2\operatorname{sen}^2(\phi)$$

$$= \operatorname{sen}^2(\theta)\left[\cos(\theta)\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\operatorname{sen}(\theta)\right]^2$$

$$+\cos^2(\theta)\left[\cos(\theta)\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\operatorname{sen}(\theta)\right]^2$$

$$+2\frac{\partial f}{\partial z}\cos(\theta)\operatorname{sin}(\theta)\left[\frac{\partial f}{\partial x}\cos(\theta) + \frac{\partial f}{\partial y}\operatorname{sen}(\theta)\right]$$

$$-2\frac{\partial f}{\partial z}\operatorname{sen}(\theta)\cos(\theta)\left[\frac{\partial f}{\partial x}\cos(\theta) + \frac{\partial f}{\partial y}\operatorname{sen}(\theta)\right]$$

$$+\left(\frac{\partial f}{\partial z}\right)^2\left[\cos^2(\theta) - \operatorname{sen}^2(\theta)\right]$$

$$\left(\frac{\partial f}{\partial x}\right)^2\operatorname{sen}^2(\theta) - 2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\cos(\theta)\operatorname{sen}(\theta)$$

$$+\left(\frac{\partial f}{\partial y}\right)^2\cos^2(\theta)$$

$$= \left[\cos(\theta)\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\operatorname{sen}(\theta)\right]^2$$

$$+\left(\frac{\partial f}{\partial z}\right)^2 + \left(\frac{\partial f}{\partial x}\right)^2 \operatorname{sen}^2(\theta)$$

$$-2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}(\cos(\theta))\operatorname{sen}(\theta)$$

$$+\left(\frac{\partial f}{\partial y}\right)^2 \cos^2(\theta)$$

$$= \left(\frac{\partial f}{\partial z}\right)^2 + \left(\frac{\partial f}{\partial x}\right)^2 \cos^2(\theta) +$$

$$\left(\frac{\partial f}{\partial x}\right)^2 \operatorname{sen}^2(\theta) + \left(\frac{\partial f}{\partial y}\right)^2 \cos^2(\theta)$$

$$+\left(\frac{\partial f}{\partial y}\right)^2 \operatorname{sen}^2(\theta)$$

$$\cancel{-2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\cos(\theta)\operatorname{sen}(\theta)} \quad 0$$

$$\cancel{+2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\cos(\theta)\operatorname{sen}(\theta)}$$

$$= \left(\frac{\partial f}{\partial z}\right)^2 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$$

$$= \|\nabla f\|^2$$

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