35

The Nature of Light and the Laws of Geometric Optics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q35.1 Light travels through a vacuum at a speed of 300 000 km per second. Thus, an image we see from a distant star or galaxy must have been generated some time ago. For example, the star Altair is 16 light-years away; if we look at an image of Altair today, we know only what was happening 16 years ago. This may not initially seem significant, but astronomers who look at other galaxies can gain an idea of what galaxies looked like when they were significantly younger. Thus, it actually makes sense to speak of "looking backward in time."

***Q35.2** 10^4 m/(3×10^8 m/s) is 33 μs. Answer (c).

***Q35.3** We consider the quantity λ/d. The smaller it is, the better the ray approximation works. In (a) it is like 0.34 m/1 m $\approx$ 0.3. In (b) we can have 0.7 μm/2 mm $\approx$ 0.000 3. In (c), 0.4 μm/2 mm $\approx$ 0.000 2. In (d), 300 m/1 m $\approx$ 300. In (e) 1 nm/1 mm $\approx$ 0.000 001. The ranking is then e, c, b, a, d.

Q35.4 With a vertical shop window, streetlights and his own reflection can impede the window shopper's clear view of the display. The tilted shop window can put these reflections out of the way. Windows of airport control towers are also tilted like this, as are automobile windshields.

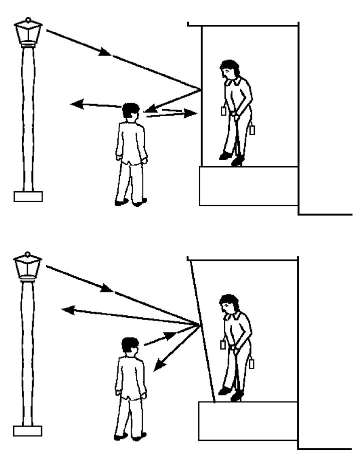


FIG. Q35.4

Q35.5 We assume that you and the child are always standing close together. For a flat wall to make an echo of a sound that you make, you must be standing along a normal to the wall. You must be on the order of 100 m away, to make the transit time sufficiently long that you can hear the echo separately from the original sound. Your sound must be loud enough so that you can hear it even at this considerable range. In the picture, the dashed rectangle represents an area in which you can be standing. The arrows represent rays of sound.

Now suppose two vertical perpendicular walls form an inside corner that you can see. Some of the sound you radiate horizontally will be headed generally toward the corner. It will reflect from both walls with high efficiency to reverse in direction and come back to you. You can stand anywhere reasonably far away to hear a retroreflected echo of sound you produce.

If the two walls are not perpendicular, the inside corner will not produce retroreflection. You will generally hear no echo of your shout or clap.

If two perpendicular walls have a reasonably narrow gap between them at the corner, you can still hear a clear echo. It is not the corner line itself that retroreflects the sound, but the perpendicular walls on both sides of the corner. Diagram (b) applies also in this case.

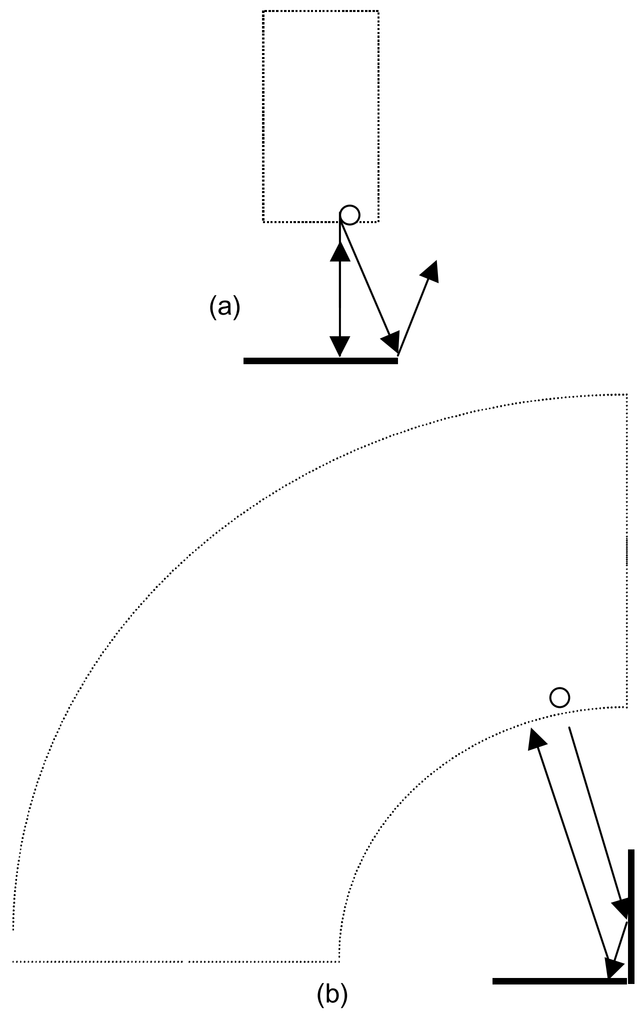


FIG. Q35.5

Q35.6 The stealth fighter is designed so that adjacent panels are not joined at right angles, to prevent any retroreflection of radar signals. This means that radar signals directed at the fighter will not be channeled back toward the detector by reflection. Just as with sound, radar signals can be treated as *diverging* rays, so that any ray that is by chance reflected back to the detector will be too weak in intensity to distinguish from background noise. This author is still waiting for the automotive industry to utilize this technology.

***Q35.7** Snell originally stated his law in terms of cosecants. From $v = c/n$ and $\sin\theta = 1/\csc\theta$ and $\lambda = c/nf$ with c and f constant between media, we conclude that a, b, and c are all correct statements.

Q35.8 An echo is an example of the reflection of sound. Hearing the noise of a distant highway on a cold morning, when you cannot hear it after the ground warms up, is an example of acoustical refraction. You can use a rubber inner tube inflated with helium as an acoustical lens to concentrate sound in the way a lens can focus light. At your next party, see if you can experimentally find the approximate focal point!

***Q35.9** (a) Yes. (b) No. (c) Yes. (d) No. If the light moves into a medium of higher refractive index, its wavelength decreases. The frequency remains constant. The speed diminishes by a factor equal to the index of refraction. If its angle of incidence is 0°, it will continue in the same direction.

Q35.10 If a laser beam enters a sugar solution with a concentration gradient (density and index of refraction increasing with depth) then the laser beam will be progressively bent downward (toward the normal) as it passes into regions of greater index of refraction.

***Q35.11** (a) Yes. It must be traveling in the medium in which it moves slower, water, to undergo total internal reflection.

(b) Yes. It must be traveling in the medium in which it moves slower, air, to undergo total internal reflection.

Q35.12 Diamond has higher index of refraction than glass and consequently a smaller critical angle for total internal reflection. A brilliant-cut diamond is shaped to admit light from above, reflect it totally at the converging facets on the underside of the jewel, and let the light escape only at the top. Glass will have less light internally reflected.

Q35.13 Highly silvered mirrors reflect about 98% of the incident light. With a 2-mirror periscope, that results in approximately a 4% decrease in intensity of light as the light passes through the periscope. This may not seem like much, but in low-light conditions, that lost light may mean the difference between being able to distinguish an enemy armada or an iceberg from the sky beyond. Using prisms results in total internal reflection, meaning that 100% of the incident light is reflected through the periscope. That is the “total” in total internal reflection.

***Q35.14** The light with the greater change in speed will have the larger deviation. Since the glass has a higher index than the surrounding air, A travels slower in the glass.

Q35.15 Immediately around the dark shadow of my head, I see a halo brighter than the rest of the dewy grass. It is called the *heiligenschein*. Cellini believed that it was a miraculous sign of divine favor pertaining to him alone. Apparently none of the people to whom he showed it told him that they could see halos around their own shadows but not around Cellini’s. Thoreau knew that each person had his own halo. He did not draw any ray diagrams but assumed that it was entirely natural. Between Cellini’s time and Thoreau’s, the Enlightenment and Newton’s explanation of the rainbow had happened. Today the effect is easy to see whenever your shadow falls on a retroreflecting traffic sign, license plate, or road stripe. When a bicyclist’s shadow falls on a paint stripe marking the edge of the road, her halo races along with her. It is a shame that few people are sufficiently curious observers of the natural world to have noticed the phenomenon.

Q35.16 At the altitude of the plane the surface of the Earth need not block off the lower half of the rainbow. Thus, the full circle can be seen. You can see such a rainbow by climbing on a stepladder above a garden sprinkler in the middle of a sunny day. Set the sprinkler for fine mist. Do not let the slippery children fall from the ladder.

***Q35.17** Light from the lamps along the edges of the sheet enters the plastic. Then it is totally internally reflected by the front and back faces of the plastic, wherever the plastic has an interface with air. If the refractive index of the grease is intermediate between 1.55 and 1.00, some of this light can leave the plastic into the grease and leave the grease into the air. The surface of the grease is rough, so the grease can send out light in all directions. The customer sees the grease shining against a black background. The spotlight method of producing the same effect is much less efficient. With it, much of the light from the spotlight is absorbed by the blackboard. The refractive index of the grease must be less than 1.55. Perhaps the best choice would be $\sqrt{1.55 \times 1.00} = 1.24$.

***Q35.18** Answer (c). We want a big difference between indices of refraction to have total internal reflection under the widest range of conditions.

Q35.19 A mirage occurs when light changes direction as it moves between batches of air having different indices of refraction because they have different densities at different temperatures. When the sun makes a blacktop road hot, an apparent wet spot is bright due to refraction of light from the bright sky. The light, originally headed a little below the horizontal, always bends up as it first enters and then leaves sequentially hotter, lower-density, lower-index layers of air closer to the road surface.

SOLUTIONS TO PROBLEMS

Section 35.1 **The Nature of Light**

Section 35.2 **Measurements of the Speed of Light**

***P35.1** The Moon's radius is 1.74×10^6 m and the Earth's radius is 6.37×10^6 m. The total distance traveled by the light is:

$$d = 2\left(3.84\times10^8\text{ m} - 1.74\times10^6\text{ m} - 6.37\times10^6\text{ m}\right) = 7.52\times10^8\text{ m}$$

This takes 2.51 s, so $v = \dfrac{7.52\times10^8\text{ m}}{2.51\text{ s}} = 2.995\times10^8\text{ m/s} = \boxed{299.5\text{ Mm/s}}$. The sizes of the objects need to be taken into account. Otherwise the answer would be too large by 2%.

P35.2 $\Delta x = ct;\qquad c = \dfrac{\Delta x}{t} = \dfrac{2\left(1.50\times10^8\text{ km}\right)\left(1\,000\text{ m/km}\right)}{(22.0\text{ min})(60.0\text{ s/min})} = 2.27\times10^8\text{ m/s} = \boxed{227\text{ Mm/s}}$

P35.3 The experiment is most convincing if the wheel turns fast enough to pass outgoing light through one notch and returning light through the next: $t = \dfrac{2\ell}{c}$

$$\theta = \omega t = \omega\left(\frac{2\ell}{c}\right)\qquad\text{so}\qquad \omega = \frac{c\theta}{2\ell} = \frac{\left(2.998\times10^8\right)\left[2\pi/(720)\right]}{2\left(11.45\times10^3\right)} = \boxed{114\text{ rad/s}}$$

The returning light would be blocked by a tooth at one-half the angular speed, giving another data point.

Section 35.3 **The Ray Approximation in Geometric Optics**

Section 35.4 **The Wave Under Reflection**

Section 35.5 **The Wave Under Refraction**

P35.4 (a) Let AB be the originally horizontal ceiling, BC its originally vertical normal, AD the new ceiling, and DE its normal. Then angle $BAD = \phi$. By definition DE is perpendicular to AD and BC is perpendicular to AB. Then the angle between DE extended and BC is ϕ because angles are equal when their sides are perpendicular, right side to right side and left side to left side.

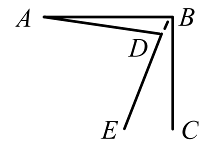


FIG. P35.4 (a)

(b) Now $CBE = \phi$ is the angle of incidence of the vertical light beam. Its angle of reflection is also ϕ. The angle between the vertical incident beam and the reflected beam is 2ϕ.

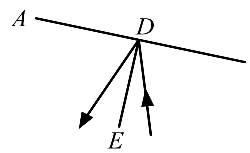


FIG. P35.4 (b)

(c) $\tan 2\phi = \dfrac{1.40\text{ cm}}{720\text{ cm}} = 0.001\,94$ $\quad \boxed{\phi = 0.055\,7°}$

P35.5 (a) From geometry, $1.25\text{ m} = d \sin 40.0°$

so $d = \boxed{1.94\text{ m}}$

(b) $\boxed{\text{50.0° above the horizontal}}$

or parallel to the incident ray.

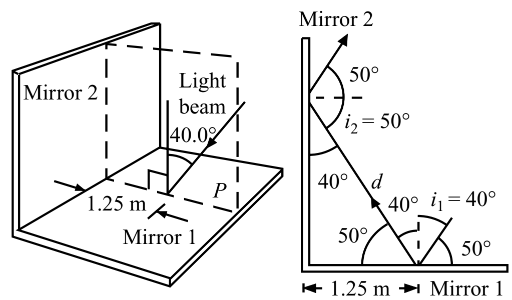


FIG. P35.5

P35.6 (a) **Method One:**
The incident ray makes angle $\alpha = 90° - \theta_1$ with the first mirror. In the picture, the law of reflection implies that

$$\theta_1 = \theta_1'$$

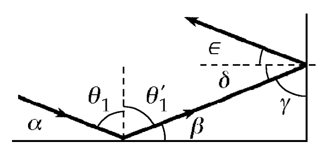


FIG. P35.6

Then $\beta = 90° - \theta_1' = 90 - \theta_1 = \alpha$

In the triangle made by the mirrors and the ray passing between them,

$$\beta + 90° + \gamma = 180°$$
$$\gamma = 90° - \beta$$

Further, $\delta = 90° - \gamma = \beta = \alpha$

and $\in = \delta = \alpha$

Thus the final ray makes the same angle with the first mirror as did the incident ray. Its direction is opposite to the incident ray.

Method Two:
The vector velocity of the incident light has a component v_y perpendicular to the first mirror and a component v_x perpendicular to the second. The v_y component is reversed upon the first

continued on next page

reflection, which leaves v_x unchanged. The second reflection reverses v_x and leaves v_y unchanged. The doubly reflected ray then has velocity opposite to the incident ray.

(b) The incident ray has velocity $v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$. Each reflection reverses one component and leaves the other two unchanged. After all the reflections, the light has velocity $-v_x\hat{\mathbf{i}} - v_y\hat{\mathbf{j}} - v_z\hat{\mathbf{k}}$, opposite to the incident ray.

P35.7 Let d represent the perpendicular distance from the person to the mirror. The distance between lamp and person measured parallel to the mirror can be written in two ways: $2d\tan\theta + d\tan\theta = d\tan\phi$. The condition on the distance traveled by the light is $\dfrac{2d}{\cos\phi} = \dfrac{2d}{\cos\theta} + \dfrac{d}{\cos\theta}$. We have the two equations $3\tan\theta = \tan\phi$ and $2\cos\theta = 3\cos\phi$. To eliminate ϕ we write

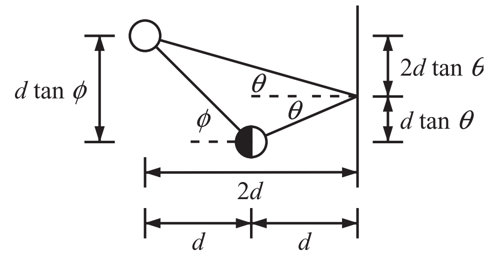


FIG. P35.7

$$\frac{9\sin^2\theta}{\cos^2\theta} = \frac{\sin^2\phi}{\cos^2\phi} \qquad 4\cos^2\theta = 9\cos^2\phi$$

$$9\cos^2\phi\sin^2\theta = \cos^2\theta\left(1-\cos^2\phi\right)$$

$$4\cos^2\theta\sin^2\theta = \cos^2\theta\left(1-\frac{4}{9}\cos^2\theta\right)$$

$$4\sin^2\theta = 1-\frac{4}{9}\left(1-\sin^2\theta\right) \qquad 36\sin^2\theta = 9-4+4\sin^2\theta$$

$$\sin^2\theta = \frac{5}{32} \qquad \theta = \boxed{23.3°}$$

***P35.8** The excess time the second pulse spends in the ice is 6.20 m/[(3.00×10^8 m/s)/1.309] = $\boxed{27.1 \text{ ns}}$

P35.9 Using Snell's law, $\sin\theta_2 = \dfrac{n_1}{n_2}\sin\theta_1$

$$\theta_2 = \boxed{25.5°}$$

$$\lambda_2 = \frac{\lambda_1}{n_1} = \boxed{442 \text{ nm}}$$

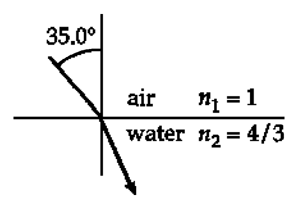


FIG. P35.9

***P35.10** The law of refraction $n_1 \sin\theta_1 = n_2 \sin\theta_2$ can be put into the more general form

$$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$$
$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

In this form it applies to all kinds of waves that move through space.

$$\frac{\sin 3.5^\circ}{343\ \text{m/s}} = \frac{\sin\theta_2}{1\,493\ \text{m/s}}$$
$$\sin\theta_2 = 0.266$$
$$\theta_2 = \boxed{15.4^\circ}$$

The wave keeps constant frequency in

$$f = \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$
$$\lambda_2 = \frac{v_2\lambda_1}{v_1} = \frac{1\,493\ \text{m/s}(0.589\ \text{m})}{343\ \text{m/s}} = \boxed{2.56\ \text{m}}$$

The light wave slows down as it moves from air into water but the sound speeds up by a large factor. The light wave bends toward the normal and its wavelength shortens, but the sound wave bends away from the normal and its wavelength increases.

P35.11 $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\sin\theta_1 = 1.333 \sin 45^\circ$$
$$\sin\theta_1 = (1.33)(0.707) = 0.943$$
$$\theta_1 = 70.5^\circ \to \boxed{19.5^\circ \text{ above the horizon}}$$

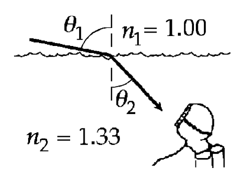


FIG. P35.11

P35.12 (a) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\ \text{m/s}}{6.328\times10^{-7}\ \text{m}} = \boxed{4.74\times10^{14}\ \text{Hz}}$

(b) $\lambda_{\text{glass}} = \dfrac{\lambda_{\text{air}}}{n} = \dfrac{632.8\ \text{nm}}{1.50} = \boxed{422\ \text{nm}}$

(c) $v_{\text{glass}} = \dfrac{c_{\text{air}}}{n} = \dfrac{3.00\times10^{8}\ \text{m/s}}{1.50} = 2.00\times10^{8}\ \text{m/s} = \boxed{200\ \text{Mm/s}}$

P35.13 We find the angle of incidence:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1.333 \sin\theta_1 = 1.52 \sin 19.6^\circ$$
$$\theta_1 = 22.5^\circ$$

The angle of reflection of the beam in water is then also $\boxed{22.5^\circ}$.

***P35.14** (a) As measured from the diagram, the incidence angle is 60°, and the refraction angle is 35°.

From Snell's law, $\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$, then $\frac{\sin 35°}{\sin 60°} = \frac{v_2}{c}$ and the speed of light in the block is $\boxed{2.0\times 10^8 \text{ m/s}}$.

(b) The frequency of the light does not change upon refraction. Knowing the wavelength in a vacuum, we can use the speed of light in a vacuum to determine the frequency: $c = f\lambda$,

thus $3.00\times 10^8 = f\left(632.8\times 10^{-9}\right)$, so the frequency is $\boxed{474.1 \text{ THz}}$.

(c) To find the wavelength of light in the block, we use the same wave speed relation, $v = f\lambda$,

so $2.0\times 10^8 = \left(4.741\times 10^{14}\right)\lambda$, so $\lambda_{\text{glass}} = \boxed{420 \text{ nm}}$.

P35.15 (a) Flint Glass: $v = \frac{c}{n} = \frac{3.00\times 10^8 \text{ m/s}}{1.66} = 1.81\times 10^8 \text{ m/s} = \boxed{181 \text{ Mm/s}}$

(b) Water: $v = \frac{c}{n} = \frac{3.00\times 10^8 \text{ m/s}}{1.333} = 2.25\times 10^8 \text{ m/s} = \boxed{225 \text{ Mm/s}}$

(c) Cubic Zirconia: $v = \frac{c}{n} = \frac{3.00\times 10^8 \text{ m/s}}{2.20} = 1.36\times 10^8 \text{ m/s} = \boxed{136 \text{ Mm/s}}$

***P35.16** From Snell's law, $\sin\theta = \left(\frac{n_{\text{medium}}}{n_{\text{liver}}}\right)\sin 50.0°$

But $\frac{n_{\text{medium}}}{n_{\text{liver}}} = \frac{c/v_{\text{medium}}}{c/v_{\text{liver}}} = \frac{v_{\text{liver}}}{v_{\text{medium}}} = 0.900$

so $\theta = \sin^{-1}\left[(0.900)\sin 50.0°\right] = 43.6°$

From the law of reflection,

$$d = \frac{12.0 \text{ cm}}{2} = 6.00 \text{ cm, and}$$

$$h = \frac{d}{\tan\theta} = \frac{6.00 \text{ cm}}{\tan(43.6°)} = \boxed{6.30 \text{ cm}}$$

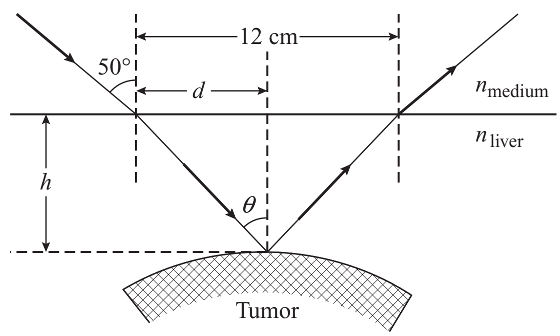


FIG. P35.16

P35.17 $n_1\sin\theta_1 = n_2\sin\theta_2$: $\theta_2 = \sin^{-1}\left(\frac{n_1\sin\theta_1}{n_2}\right)$

$$\theta_2 = \sin^{-1}\left\{\frac{1.00\sin 30°}{1.50}\right\} = \boxed{19.5°}$$

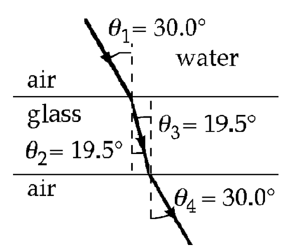


FIG. P35.17

θ_2 and θ_3 are alternate interior angles formed by the ray cutting parallel normals.

So, $\theta_3 = \theta_2 = \boxed{19.5°}$

$1.50\sin\theta_3 = 1.00\sin\theta_4$

$\theta_4 = \boxed{30.0°}$

P35.18 $\sin\theta_1 = n_w \sin\theta_2$

$$\sin\theta_2 = \frac{1}{1.333}\sin\theta_1 = \frac{1}{1.333}\sin(90.0° - 28.0°) = 0.662$$

$$\theta_2 = \sin^{-1}(0.662) = 41.5°$$

$$h = \frac{d}{\tan\theta_2} = \frac{3.00 \text{ m}}{\tan 41.5°} = \boxed{3.39 \text{ m}}$$

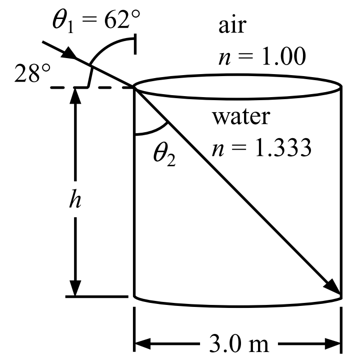


FIG. P35.18

P35.19 At entry, $n_1 \sin\theta_1 = n_2 \sin\theta_2$

or $1.00 \sin 30.0° = 1.50 \sin\theta_2$

$\theta_2 = 19.5°$

The distance h the light travels in the medium is given by

$$\cos\theta_2 = \frac{2.00 \text{ cm}}{h}$$

or $$h = \frac{2.00 \text{ cm}}{\cos 19.5°} = 2.12 \text{ cm}$$

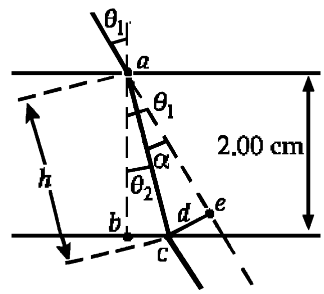


FIG. P35.19

The angle of deviation upon entry is $\alpha = \theta_1 - \theta_2 = 30.0° - 19.5° = 10.5°$

The offset distance comes from $\sin\alpha = \dfrac{d}{h}$: $d = (2.21 \text{ cm})\sin 10.5° = \boxed{0.388 \text{ cm}}$

P35.20 The distance h traveled by the light is $h = \dfrac{2.00 \text{ cm}}{\cos 19.5°} = 2.12 \text{ cm}$

The speed of light in the material is $v = \dfrac{c}{n} = \dfrac{3.00 \times 10^8 \text{ m/s}}{1.50} = 2.00 \times 10^8 \text{ m/s}$

Therefore, $t = \dfrac{h}{v} = \dfrac{2.12 \times 10^{-2} \text{ m}}{2.00 \times 10^8 \text{ m/s}} = 1.06 \times 10^{-10} \text{ s} = \boxed{106 \text{ ps}}$

P35.21 Applying Snell's law at the air-oil interface,

$$n_{\text{air}} \sin\theta = n_{\text{oil}} \sin 20.0°$$

yields $\boxed{\theta = 30.4°}$

Applying Snell's law at the oil-water interface

$$n_w \sin\theta' = n_{\text{oil}} \sin 20.0°$$

yields $\boxed{\theta' = 22.3°}$

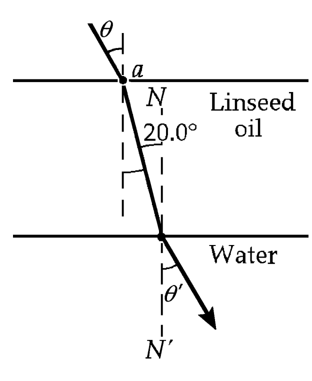


FIG. P35.21

P35.22 For sheets 1 and 2 as described,

$n_1 \sin 26.5° = n_2 \sin 31.7°$

$0.849 n_1 = n_2$

For the trial with sheets 3 and 2,

$n_3 \sin 26.5° = n_2 \sin 36.7°$

$0.747 n_3 = n_2$

Now

$0.747 n_3 = 0.849 n_1$

$n_3 = 1.14 n_1$

For the third trial,

$n_1 \sin 26.5° = n_3 \sin \theta_3 = 1.14 n_1 \sin \theta_3$

$\theta_3 = \boxed{23.1°}$

***P35.23** Refraction proceeds according to $(1.00)\sin\theta_1 = (1.66)\sin\theta_2$ (1)

(a) For the normal component of velocity to be constant, $v_1 \cos\theta_1 = v_2 \cos\theta_2$

or $(c)\cos\theta_1 = \left(\dfrac{c}{1.66}\right)\cos\theta_2$ (2)

We multiply Equations (1) and (2), obtaining: $\sin\theta_1 \cos\theta_1 = \sin\theta_2 \cos\theta_2$

or $\sin 2\theta_1 = \sin 2\theta_2$

The solution $\theta_1 = \theta_2 = 0$ does not satisfy Equation (2) and must be rejected. The physical solution is $2\theta_1 = 180° - 2\theta_2$ or $\theta_2 = 90.0° - \theta_1$. Then Equation (1) becomes:

$\sin\theta_1 = 1.66\cos\theta_1$, or $\tan\theta_1 = 1.66$

which yields $\boxed{\theta_1 = 58.9°}$

In this case, $\boxed{\text{yes}}$, the perpendicular velocity component does remain constant.

(b) Light entering the glass slows down and makes a smaller angle with the normal. Both effects reduce the velocity component parallel to the surface of the glass.
Then

$\boxed{\text{no, the parallel velocity component cannot remain constant, or will remain constant only in the trivial case } \theta_1 = \theta_2 = 0}$.

P35.24 Consider glass with an index of refraction of 1.5, which is 3 mm thick. The speed of light in the glass is

$$\frac{3\times10^{8}\text{ m/s}}{1.5}=2\times10^{8}\text{ m/s}$$

The extra travel time is $\dfrac{3\times10^{-3}\text{ m}}{2\times10^{8}\text{ m/s}}-\dfrac{3\times10^{-3}\text{ m}}{3\times10^{8}\text{ m/s}}\boxed{\sim10^{-11}\text{ s}}$

For light of wavelength 600 nm in vacuum and wavelength $\dfrac{600\text{ nm}}{1.5}=400$ nm in glass,

the extra optical path, in wavelengths, is $\dfrac{3\times10^{-3}\text{ m}}{4\times10^{-7}\text{ m}}-\dfrac{3\times10^{-3}\text{ m}}{6\times10^{-7}\text{ m}}\boxed{\sim10^{3}\text{ wavelengths}}$

P35.25 Taking Φ to be the apex angle and $\delta_{\min}$ to be the angle of minimum deviation, from Equation 35.9, the index of refraction of the prism material is

$$n=\frac{\sin\left[\left(\Phi+\delta_{\min}\right)/2\right]}{\sin\left(\Phi/2\right)}$$

Solving for $\delta_{\min}$, $\delta_{\min}=2\sin^{-1}\left(n\sin\dfrac{\Phi}{2}\right)-\Phi=2\sin^{-1}\left[(2.20)\sin(25.0°)\right]-50.0°=\boxed{86.8°}$

P35.26 $n(700\text{ nm})=1.458$

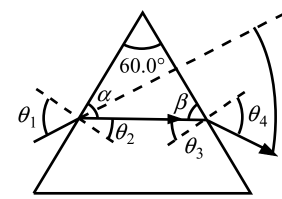


FIG. P35.26

(a) $(1.00)\sin75.0°=1.458\sin\theta_2;\ \theta_2=\boxed{41.5°}$

(b) Let $\theta_3+\beta=90.0°,\ \theta_2+\alpha=90.0°$ then $\alpha+\beta+60.0°=180°$

So $60.0°-\theta_2-\theta_3=0\Rightarrow60.0°-41.5°=\theta_3=\boxed{18.5°}$

(c) $1.458\sin18.5°=1.00\sin\theta_4 \qquad \theta_4=\boxed{27.6°}$

(d) $\gamma=(\theta_1-\theta_2)+\left[\beta-(90.0°-\theta_4)\right]$

$\gamma=75.0°-41.5°+(90.0°-18.5°)-(90.0°-27.6°)=\boxed{42.6°}$

P35.27 At the first refraction, $1.00\sin\theta_1=n\sin\theta_2$

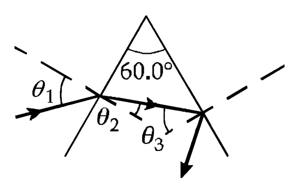


FIG. P35.27

The critical angle at the second surface is given by $n\sin\theta_3=1.00$:

or $\theta_3=\sin^{-1}\left(\dfrac{1.00}{1.50}\right)=41.8°$

But, $\theta_2=60.0°-\theta_3$

Thus, to avoid total internal reflection at the second surface (i.e., have $\theta_3<41.8°$)

it is necessary that $\theta_2>18.2°$

Since $\sin\theta_1=n\sin\theta_2$, this becomes $\sin\theta_1>1.50\sin18.2°=0.468$

or $\theta_1>\boxed{27.9°}$

P35.28 At the first refraction, $1.00\sin\theta_1 = n\sin\theta_2$

The critical angle at the second surface is given by

$n\sin\theta_3 = 1.00$, or $\theta_3 = \sin^{-1}\left(\frac{1.00}{n}\right)$

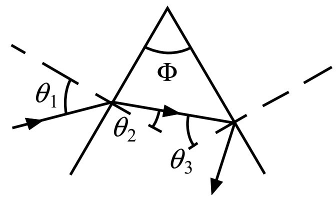


FIG. P35.28

But $(90.0° - \theta_2) + (90.0° - \theta_3) + \Phi = 180°$

which gives $\theta_2 = \Phi - \theta_3$

Thus, to have $\theta_3 < \sin^{-1}\left(\frac{1.00}{n}\right)$ and avoid total internal reflection at the second surface,

it is necessary that $\theta_2 > \Phi - \sin^{-1}\left(\frac{1.00}{n}\right)$

Since $\sin\theta_1 = n\sin\theta_2$, this requirement becomes $\sin\theta_1 > n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right]$

or $\theta_1 > \boxed{\sin^{-1}\left(n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right]\right)}$

Through the application of trigonometric identities, $\theta_1 > \boxed{\sin^{-1}\left(\sqrt{n^2-1}\sin\Phi - \cos\Phi\right)}$

P35.29 Note for use in every part: $\Phi + (90.0° - \theta_2) + (90.0° - \theta_3) = 180°$

so $\theta_3 = \Phi - \theta_2$

At the first surface the deviation is $\alpha = \theta_1 - \theta_2$

At exit, the deviation is $\beta = \theta_4 - \theta_3$

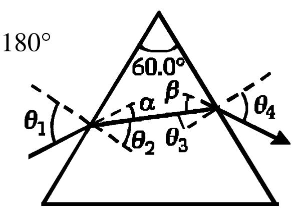


FIG. P35.29

The total deviation is therefore $\delta = \alpha + \beta = \theta_1 + \theta_4 - \theta_2 - \theta_3 = \theta_1 + \theta_4 - \Phi$

(a) At entry: $n_1\sin\theta_1 = n_2\sin\theta_2$ or $\theta_2 = \sin^{-1}\left(\frac{\sin 48.6°}{1.50}\right) = 30.0°$

Thus, $\theta_3 = 60.0° - 30.0° = 30.0°$

At exit: $1.50\sin 30.0° = 1.00\sin\theta_4$ or $\theta_4 = \sin^{-1}\left[1.50\sin(30.0°)\right] = 48.6°$

so the path through the prism is symmetric when $\theta_1 = 48.6°$.

(b) $\delta = 48.6° + 48.6° - 60.0° = \boxed{37.2°}$

(c) At entry: $\sin\theta_2 = \frac{\sin 45.6°}{1.50} \Rightarrow \theta_2 = 28.4°$ $\theta_3 = 60.0° - 28.4° = 31.6°$

At exit: $\sin\theta_4 = 1.50\sin(31.6°) \Rightarrow \theta_4 = 51.7°$ $\delta = 45.6° + 51.7° - 60.0° = \boxed{37.3°}$

continued on next page

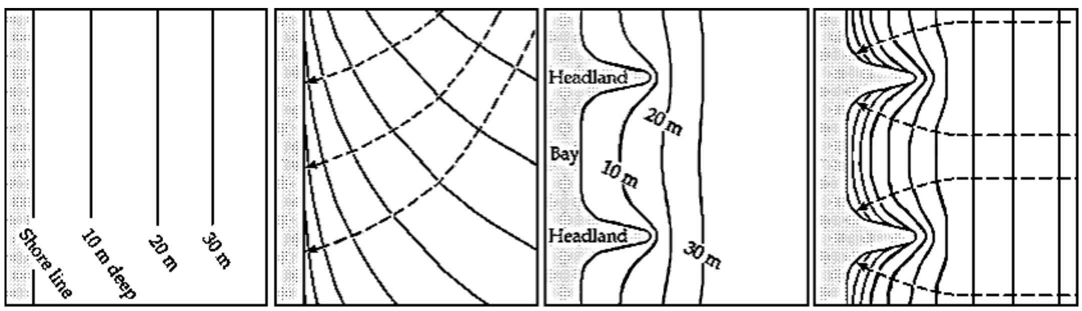
(d) At entry: $\sin\theta_2 = \dfrac{\sin 51.6°}{1.50} \Rightarrow \theta_2 = 31.5°$ $\theta_3 = 60.0° - 31.5° = 28.5°$

At exit: $\sin\theta_4 = 1.50\sin(28.5°) \Rightarrow \theta_4 = 45.7°$ $\delta = 51.6° + 45.7° - 60.0° = \boxed{37.3°}$

Section 35.6 **Huygens's Principle**

P35.30 (a) For the diagrams of contour lines and wave fronts and rays, see Figures (a) and (b) below. As the waves move to shallower water, the wave fronts bend to become more nearly parallel to the contour lines.

(b) For the diagrams of contour lines and wave fronts and rays, see Figures (c) and (d) below. We suppose that the headlands are steep underwater, as they are above water. The rays are everywhere perpendicular to the wave fronts of the incoming refracting waves. As shown, the rays bend toward the headlands and deliver more energy per length at the headlands.



(a) Contour lines (b) Wave fronts and rays (c) Contour lines (d) Wave fronts and rays

FIG. P35.30

Section 35.7 **Dispersion**

P35.31 For the incoming ray, $\sin\theta_2 = \dfrac{\sin\theta_1}{n}$

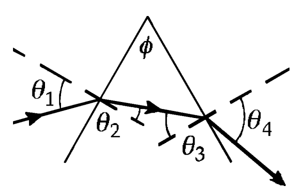


FIG. P35.31

Using the figure to the right,

$$(\theta_2)_{\text{violet}} = \sin^{-1}\left(\frac{\sin 50.0°}{1.66}\right) = 27.48°$$

$$(\theta_2)_{\text{red}} = \sin^{-1}\left(\frac{\sin 50.0°}{1.62}\right) = 28.22°$$

For the outgoing ray, $\theta_3 = 60.0° - \theta_2$

and $\sin\theta_4 = n\sin\theta_3$: $(\theta_4)_{\text{violet}} = \sin^{-1}[1.66\sin 32.52°] = 63.17°$

$$(\theta_4)_{\text{red}} = \sin^{-1}[1.62\sin 31.78°] = 58.56°$$

The angular dispersion is the difference $\Delta\theta_4 = (\theta_4)_{\text{violet}} - (\theta_4)_{\text{red}} = 63.17° - 58.56° = \boxed{4.61°}$

P35.32 From Fig 35.21 $n_v = 1.470$ at 400 nm and $n_r = 1.458$ at 700 nm

Then $1.00 \sin\theta = 1.470 \sin\theta_v$ and $1.00 \sin\theta = 1.458 \sin\theta_r$

$$\delta_r - \delta_v = \theta_r - \theta_v = \sin^{-1}\left(\frac{\sin\theta}{1.458}\right) - \sin^{-1}\left(\frac{\sin\theta}{1.470}\right)$$

$$\Delta\delta = \sin^{-1}\left(\frac{\sin 30.0°}{1.458}\right) - \sin^{-1}\left(\frac{\sin 30.0°}{1.470}\right) = \boxed{0.171°}$$

Section 35.8 **Total Internal Reflection**

P35.33 $n\sin\theta = 1$. From Table 35.1,

(a) $\theta = \sin^{-1}\left(\frac{1}{2.419}\right) = \boxed{24.4°}$

(b) $\theta = \sin^{-1}\left(\frac{1}{1.66}\right) = \boxed{37.0°}$

(c) $\theta = \sin^{-1}\left(\frac{1}{1.309}\right) = \boxed{49.8°}$

***P35.34** For total internal reflection, $n_1 \sin\theta_1 = n_2 \sin 90.0°$

$1.50 \sin\theta_1 = 1.33(1.00)$ or $\theta_1 = \boxed{62.5°}$

P35.35 $\sin\theta_c = \frac{n_2}{n_1}$

$n_2 = n_1 \sin 88.8° = (1.000\ 3)(0.999\ 8) = \boxed{1.000\ 08}$

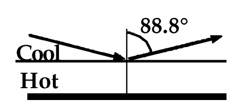


FIG. P35.35

P35.36 $\sin\theta_c = \frac{n_{\text{air}}}{n_{\text{pipe}}} = \frac{1.00}{1.36} = 0.735$ $\theta_c = 47.3°$

Geometry shows that the angle of refraction at the end is

$\phi = 90.0° - \theta_c = 90.0° - 47.3° = 42.7°$

Then, Snell's law at the end, $1.00 \sin\theta = 1.36 \sin 42.7°$

gives $\boxed{\theta = 67.2°}$

The 2-μm diameter is unnecessary information.

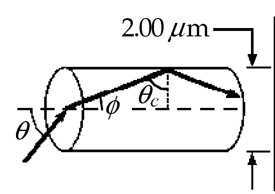


FIG. P35.36

P35.37 (a) A ray along the inner edge will escape if any ray escapes. Its angle of incidence is described by $\sin\theta = \dfrac{R-d}{R}$ and by $n\sin\theta > 1\sin 90°$. Then

$$\frac{n(R-d)}{R} > 1 \qquad nR - nd > R \qquad nR - R > nd \qquad R > \boxed{\frac{nd}{n-1}}$$

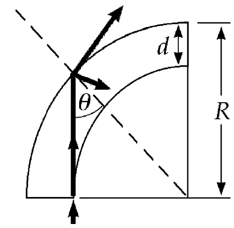


FIG. P35.37

(b) As $d \to 0$, $R_{\min} \to 0$. This is reasonable.

As n increases, $R_{\min}$ decreases. This is reasonable.

As n decreases toward 1, $R_{\min}$ increases. This is reasonable.

(c) $$R_{\min} = \frac{1.40\left(100\times10^{-6}\text{ m}\right)}{0.40} = \boxed{350\times10^{-6}\text{ m}}$$

P35.38 (a) $\dfrac{\sin\theta_2}{\sin\theta_1} = \dfrac{v_2}{v_1}$ and $\theta_2 = 90.0°$ at the critical angle

$\dfrac{\sin 90.0°}{\sin\theta_c} = \dfrac{1\,850\text{ m/s}}{343\text{ m/s}}$ so $\theta_c = \sin^{-1}(0.185) = \boxed{10.7°}$

(b) Sound can be totally reflected if it is traveling in the medium where it travels slower: $\boxed{\text{air}}$.

(c) $\boxed{\text{Sound in air falling on the wall from most directions is 100\% reflected}}$, so the wall is a good mirror.

P35.39 For plastic with index of refraction $\boxed{n \ge 1.42}$ surrounded by air, the critical angle for total internal reflection is given by

$$\theta_c = \sin^{-1}\left(\frac{1}{n}\right) \le \sin^{-1}\left(\frac{1}{1.42}\right) = 44.8°$$

In the gasoline gauge, skylight from above travels down the plastic. The rays close to the vertical are totally reflected from the sides of the slab and from both facets at the lower end of the plastic, where it is not immersed in gasoline. This light returns up inside the plastic and makes it look bright. Where the plastic is immersed in gasoline, with index of refraction about 1.50, total internal reflection should not happen. The light passes out of the lower end of the plastic with little reflected, making this part of the gauge look dark. To frustrate total internal reflection in the gasoline, the index of refraction of the plastic should be $\boxed{n < 2.12}$.

since $\theta_c = \sin^{-1}\left(\dfrac{1.50}{2.12}\right) = 45.0°$

Additional Problems

***P35.40** From the textbook figure we have $w = 2b + a$

so $$b = \frac{w-a}{2} = \frac{700\ \mu\text{m} - 1\ \mu\text{m}}{2} = 349.5\ \mu\text{m}$$

$$\tan\theta_2 = \frac{b}{t} = \frac{349.5\ \mu\text{m}}{1\,200\ \mu\text{m}} = 0.291 \qquad \theta_2 = 16.2°$$

For refraction at entry, $$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\theta_1 = \sin^{-1}\frac{n_2\sin\theta_2}{n_1} = \sin^{-1}\frac{1.55\sin 16.2°}{1.00} = \sin^{-1}0.433 = \boxed{25.7°}$$

P35.41 Scattered light leaves the center of the photograph (a) in all horizontal directions between $\theta_1 = 0°$ and 90° from the normal. When it immediately enters the water (b), it is gathered into a fan between 0° and $\theta_{2\,\max}$ given by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1.00 \sin 90 = 1.333 \sin\theta_{2\,\max}$$
$$\theta_{2\,\max} = 48.6°$$

The light leaves the cylinder without deviation, so the viewer only receives light from the center of the photograph when he has turned by an angle less than 48.6°. When the paperweight is turned farther, light at the back surface undergoes total internal reflection (c). The viewer sees things outside the globe on the far side.

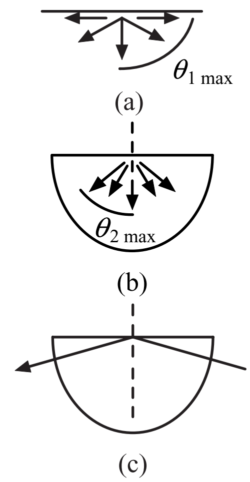


FIG. P35.41

P35.42 Let $n(x)$ be the index of refraction at distance x below the top of the atmosphere and $n(x = h) = n$ be its value at the planet surface.

Then, $$n(x) = 1.000 + \left(\frac{n - 1.000}{h}\right)x$$

(a) The total time interval required to traverse the atmosphere is

$$\Delta t = \int_0^h \frac{dx}{v} = \int_0^h \frac{n(x)}{c}\,dx: \qquad \Delta t = \frac{1}{c}\int_0^h \left[1.000 + \left(\frac{n-1.000}{h}\right)x\right]dx$$

$$\Delta t = \frac{h}{c} + \frac{(n-1.000)}{ch}\left(\frac{h^2}{2}\right) = \boxed{\frac{h}{c}\left(\frac{n+1.000}{2}\right)}$$

(b) The travel time in the absence of an atmosphere would be $\frac{h}{c}$.

Thus, the time in the presence of an atmosphere is $\boxed{\left(\frac{n+1.000}{2}\right)\text{ times larger}}$.

P35.43 Let the air and glass be medium 1 and 2, respectively. By Snell's law, $n_2 \sin\theta_2 = n_1 \sin\theta_1$

or $1.56 \sin\theta_2 = \sin\theta_1$

But the conditions of the problem are such that $\theta_1 = 2\theta_2$. $1.56 \sin\theta_2 = \sin 2\theta_2$

We now use the double-angle trig identity suggested. $1.56 \sin\theta_2 = 2\sin\theta_2 \cos\theta_2$

or $\cos\theta_2 = \frac{1.56}{2} = 0.780$

Thus, $\theta_2 = 38.7°$ and $\theta_1 = 2\theta_2 = \boxed{77.5°}$.

P35.44 (a) $\theta_1' = \theta_1 = \boxed{30.0°}$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1.00 \sin 30.0° = 1.55 \sin\theta_2$$
$$\theta_2 = \boxed{18.8°}$$

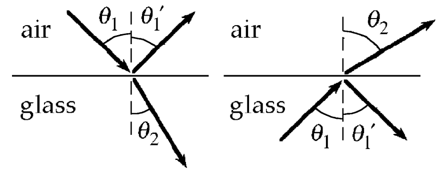


FIG. P35.44

(b) $\theta_1' = \theta_1 = \boxed{30.0°}$

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)$$
$$= \sin^{-1}\left(\frac{1.55 \sin 30.0°}{1}\right) = \boxed{50.8°}$$

(c), (d) The other entries are computed similarly, and are shown in the table below.

(c) air into glass, angles in degrees			(d) glass into air, angles in degrees		
incidence	reflection	refraction	incidence	reflection	refraction
0	0	0	0	0	0
10.0	10.0	6.43	10.0	10.0	15.6
20.0	20.0	12.7	20.0	20.0	32.0
30.0	30.0	18.8	30.0	30.0	50.8
40.0	40.0	24.5	40.0	40.0	85.1
50.0	50.0	29.6	50.0	50.0	none*
60.0	60.0	34.0	60.0	60.0	none*
70.0	70.0	37.3	70.0	70.0	none*
80.0	80.0	39.4	80.0	80.0	none*
90.0	90.0	40.2	90.0	90.0	none*

*total internal reflection

P35.45 For water, $\sin\theta_c = \dfrac{1}{4/3} = \dfrac{3}{4}$

Thus $\theta_c = \sin^{-1}(0.750) = 48.6°$

and $d = 2\left[(1.00 \text{ m})\tan\theta_c\right]$

$$d = (2.00 \text{ m})\tan 48.6° = \boxed{2.27 \text{ m}}$$

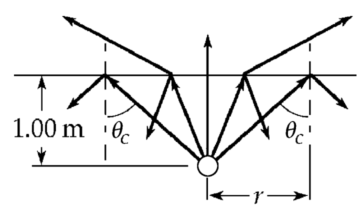


FIG. P35.45

P35.46 (a) We see the Sun moving from east to west across the sky. Its angular speed is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{86\,400 \text{ s}} = 7.27 \times 10^{-5} \text{ rad/s}$$

The direction of sunlight crossing the cell from the window changes at this rate, moving on the opposite wall at speed

$$v = r\omega = (2.37 \text{ m})(7.27 \times 10^{-5} \text{ rad/s}) = 1.72 \times 10^{-4} \text{ m/s} = \boxed{0.172 \text{ mm/s}}$$

(b) The mirror folds into the cell the motion that would occur in a room twice as wide:

$$v = r\omega = 2(0.174 \text{ mm/s}) = \boxed{0.345 \text{ mm/s}}$$

(c), (d) As the Sun moves southward and upward at 50.0°, we may regard the corner of the window as fixed, and both patches of light move $\boxed{\text{northward and downward at } 50.0°}$.

P35.47 Horizontal light rays from the setting Sun pass above the hiker. The light rays are twice refracted and once reflected, as in Figure (b). The most intense light reaching the hiker, that which represents the visible rainbow, is located between angles of 40° and 42° from the hiker's shadow. The hiker sees a greater percentage of the violet inner edge, so we consider the red outer edge. The radius R of the circle of droplets is

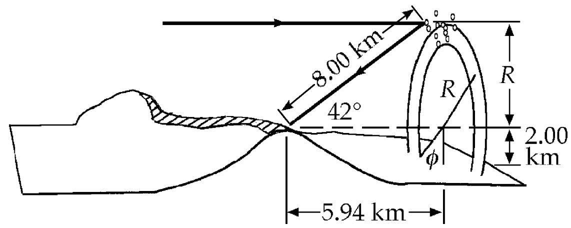


Figure (a)

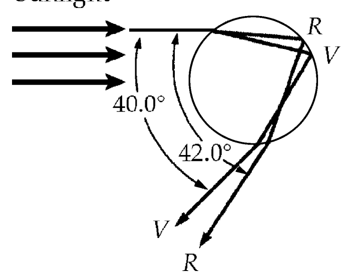


Figure (b)

FIG. P35.47

$$R = (8.00\text{ km})\sin 42.0° = 5.35\text{ km}$$

Then the angle ϕ, between the vertical and the radius where the bow touches the ground, is given by

$$\cos\phi = \frac{2.00\text{ km}}{R} = \frac{2.00\text{ km}}{5.35\text{ km}} = 0.374$$

or $\phi = 68.1°$

The angle filled by the visible bow is $360° - (2\times 68.1°) = 224°$

so the visible bow is $\frac{224°}{360°} = \boxed{62.2\%\text{ of a circle}}$.

P35.48 (a) $\boxed{45.0°}$ as shown in the first figure to the right.

(b) $\boxed{\text{Yes}}$

If grazing angle is halved, the number of reflections from the side faces is doubled.

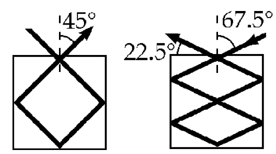


FIG. P35.48

P35.49 As the beam enters the slab,

$$1.00\sin 50.0° = 1.48\sin\theta_2$$

giving $\theta_2 = 31.2°$

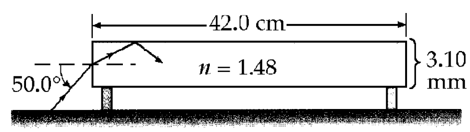


FIG. P35.49

The beam then strikes the top of the slab at $x_1 = \frac{1.55\text{ mm}}{\tan 31.2°}$ from the left end. Thereafter, the beam strikes a face each time it has traveled a distance of $2x_1$ along the length of the slab. Since the slab is 420 mm long, the beam has an additional 420 mm $- x_1$ to travel after the first reflection. The number of additional reflections is

$$\frac{420\text{ mm} - x_1}{2x_1} = \frac{420\text{ mm} - 1.55\text{ mm}/\tan 31.2°}{3.10\text{ mm}/\tan 31.2°} = 81.5 \quad\text{or 81 reflections}$$

since the answer must be an integer. The total number of reflections made in the slab is then $\boxed{82}$.

P35.50 Light passing the top of the pole makes an angle of incidence $\phi_1 = 90.0° - \theta$. It falls on the water surface at distance from the pole

$$s_1 = \frac{L-d}{\tan\theta}$$

and has an angle of refraction ϕ_2 from $1.00\sin\phi_1 = n\sin\phi_2$

Then $s_2 = d\tan\phi_2$

and the whole shadow length is

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\sin\phi_1}{n}\right)\right)$$

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\cos\theta}{n}\right)\right)$$

$$= \frac{2.00\text{ m}}{\tan 40.0°} + (2.00\text{ m})\tan\left(\sin^{-1}\left(\frac{\cos 40.0°}{1.33}\right)\right) = \boxed{3.79\text{ m}}$$

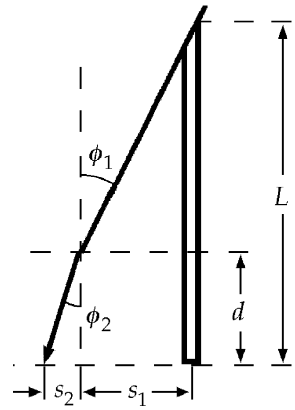


FIG. P35.50

P35.51 Define n_1 to be the index of refraction of the surrounding medium and n_2 to be that for the prism material. We can use the critical angle of 42.0° to find the ratio $\frac{n_2}{n_1}$:

$$n_2 \sin 42.0° = n_1 \sin 90.0°$$

So, $$\frac{n_2}{n_1} = \frac{1}{\sin 42.0°} = 1.49$$

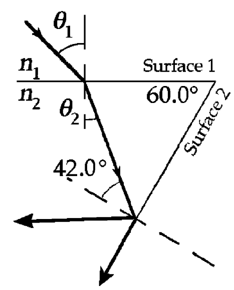


FIG. P35.51

Call the angle of refraction θ_2 at the surface 1. The ray inside the prism forms a triangle with surfaces 1 and 2, so the sum of the interior angles of this triangle must be 180°.

Thus, $(90.0° - \theta_2) + 60.0° + (90.0° - 42.0°) = 180°$

Therefore, $\theta_2 = 18.0°$

Applying Snell's law at surface 1, $n_1 \sin\theta_1 = n_2 \sin 18.0°$

$$\sin\theta_1 = \left(\frac{n_2}{n_1}\right)\sin\theta_2 = 1.49\sin 18.0° \qquad \boxed{\theta_1 = 27.5°}$$

***P35.52** (a) As the mirror turns through angle θ, the angle of incidence increases by θ and so does the angle of reflection. The incident ray is stationary, so the reflected ray turns through angle 2θ. The angular speed of the reflected ray is $2\omega_m$. The speed of the dot of light on the circular wall is $2\omega_m R = 2(35\text{ rad/s})(3\text{ m}) = \boxed{210\text{ m/s}}$.

(b) The two angles marked θ in the figure to the right are equal because their sides are perpendicular, right side to right side and left side to left side.

We have $$\cos\theta = \frac{d}{\sqrt{x^2+d^2}} = \frac{ds}{dx}$$

and $$\frac{ds}{dt} = 2\omega_m\sqrt{x^2+d^2}.$$ So

$$\frac{dx}{dt} = \frac{ds}{dt}\frac{\sqrt{x^2+d^2}}{d} = 2\omega_m\frac{x^2+d^2}{d} = 2(35\text{ rad/s})\frac{x^2+(3\text{ m})^2}{(3\text{ m})} = \boxed{23.3\frac{x^2+9\text{ m}^2}{\text{m}\cdot\text{s}}}$$

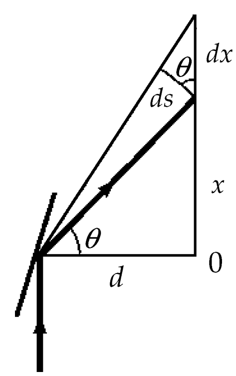


FIG. P35.52

continued on next page

(c) The minimum value is $\boxed{210\text{ m/s}}$ for $x = 0$, just as for the circular wall.

(d) The maximum speed goes to infinity as x goes to $\boxed{\text{infinity}}$, which happens when the mirror turns by 45°.

(e) To turn by 45° takes the time interval $(\pi/4\text{ rad})/(35\text{ rad/s}) = \boxed{22.4\text{ ms}}$.

***P35.53** (a) For polystyrene *surrounded by air,* internal reflection requires

$$\theta_3 = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2°$$

Then from geometry, $\theta_2 = 90.0° - \theta_3 = 47.8°$

From Snell's law, $\sin\theta_1 = 1.49\sin 47.8° = 1.10$

This has no solution.

Therefore, total internal reflection $\boxed{\text{always happens}}$ or the greatest angle is 90°.

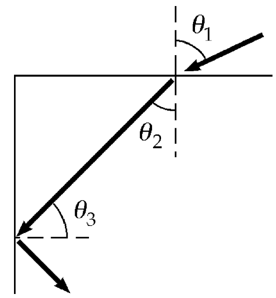


FIG. P35.53

(b) For polystyrene *surrounded by water,* $\theta_3 = \sin^{-1}\left(\frac{1.33}{1.49}\right) = 63.2°$

and $\theta_2 = 26.8°$

From Snell's law, $\theta_1 = \boxed{30.3°}$

(c) $\boxed{\text{No internal refraction is possible}}$ since the beam is initially traveling in a medium of lower index of refraction. No angle exists.

***P35.54** (a) The optical day is longer. Incoming sunlight is refracted downward at the top of the atmosphere, so an observer can see the rising Sun when it is still geometrically below the horizon. Light from the setting Sun reaches her after the Sun is below the horizon geometrically.

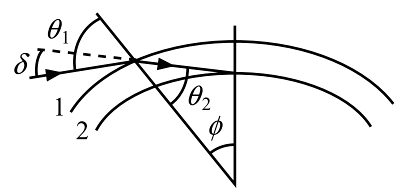


FIG. P35.54

(b) The picture illustrates optical sunrise. At the center of the Earth,

$$\cos\phi = \frac{6.37\times10^6\text{ m}}{6.37\times10^6\text{ m} + 8\,614}$$

$$\phi = 2.98°$$

$$\theta_2 = 90 - 2.98° = 87.0°$$

At the top of the atmosphere

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$1\sin\theta_1 = 1.000\,293\sin 87.0°$$

$$\theta_1 = 87.4°$$

Deviation upon entry is

$$\delta = |\theta_1 - \theta_2|$$

$$\delta = 87.364° - 87.022° = 0.342°$$

Sunrise of the optical day is before geometric sunrise by $0.342°\left(\frac{86\,400\text{ s}}{360°}\right) = 82.2\text{ s}$.

Optical sunset occurs later too, so the optical day is longer by $\boxed{164\text{ s}}$.

P35.55 $\tan\theta_1 = \dfrac{4.00\text{ cm}}{h}$

and $\qquad \tan\theta_2 = \dfrac{2.00\text{ cm}}{h}$

$$\tan^2\theta_1 = \left(2.00\tan\theta_2\right)^2 = 4.00\tan^2\theta_2$$

$$\frac{\sin^2\theta_1}{1-\sin^2\theta_1} = 4.00\left(\frac{\sin^2\theta_2}{1-\sin^2\theta_2}\right) \quad (1)$$

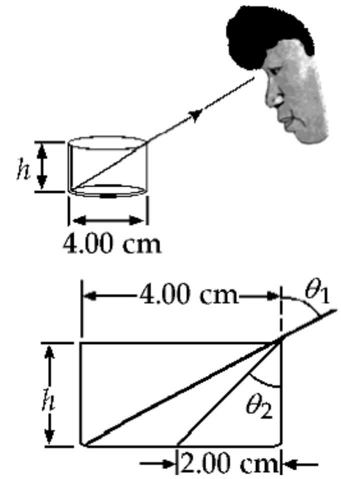


FIG. P35.55

Snell's law in this case is: $\quad n_1\sin\theta_1 = n_2\sin\theta_2$

$$\sin\theta_1 = 1.333\sin\theta_2$$

Squaring both sides, $\quad \sin^2\theta_1 = 1.777\sin^2\theta_2 \quad (2)$

Substituting (2) into (1), $\quad \dfrac{1.777\sin^2\theta_2}{1-1.777\sin^2\theta_2} = 4.00\left(\dfrac{\sin^2\theta_2}{1-\sin^2\theta_2}\right)$

Defining $x = \sin^2\theta$, $\quad \dfrac{0.444}{1-1.777x} = \dfrac{1}{1-x}$

Solving for x, $\quad 0.444 - 0.444x = 1 - 1.777x$ and $x = 0.417$

From x we can solve for θ_2: $\quad \theta_2 = \sin^{-1}\sqrt{0.417} = 40.2°$

Thus, the height is $\quad h = \dfrac{2.00\text{ cm}}{\tan\theta_2} = \dfrac{2.00\text{ cm}}{\tan 40.2°} = \boxed{2.36\text{ cm}}$

P35.56 $\delta = \theta_1 - \theta_2 = 10.0°$ $\qquad$ and $\quad n_1\sin\theta_1 = n_2\sin\theta_2$

with $\quad n_1 = 1,\ n_2 = \dfrac{4}{3}$

Thus, $\quad \theta_1 = \sin^{-1}\left(n_2\sin\theta_2\right) = \sin^{-1}\left[n_2\sin\left(\theta_1 - 10.0°\right)\right]$

(You can use a calculator to home in on an approximate solution to this equation, testing different values of θ_1 until you find that $\theta_1 = \boxed{36.5°}$. Alternatively, you can solve for θ_1 exactly, as shown below.)

We are given that $\quad \sin\theta_1 = \dfrac{4}{3}\sin\left(\theta_1 - 10.0°\right)$

This is the sine of a difference, so $\quad \dfrac{3}{4}\sin\theta_1 = \sin\theta_1\cos 10.0° - \cos\theta_1\sin 10.0°$

Rearranging, $\quad \sin 10.0°\cos\theta_1 = \left(\cos 10.0° - \dfrac{3}{4}\right)\sin\theta_1$

$\dfrac{\sin 10.0°}{\cos 10.0° - 0.750} = \tan\theta_1$ $\quad$ and $\quad \theta_1 = \tan^{-1}\left(0.740\right) = \boxed{36.5°}$

P35.57 Observe in the sketch that the angle of incidence at point P is γ, and using triangle OPQ:

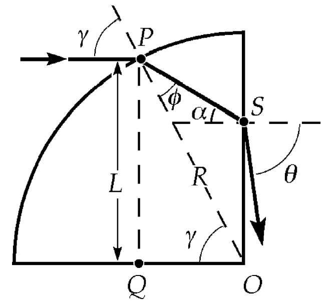


FIG. P35.57

$$\sin\gamma = \frac{L}{R}$$

Also, $$\cos\gamma = \sqrt{1-\sin^2\gamma} = \frac{\sqrt{R^2-L^2}}{R}$$

Applying Snell's law at point P, $1.00\sin\gamma = n\sin\phi$.

Thus, $$\sin\phi = \frac{\sin\gamma}{n} = \frac{L}{nR}$$

and $$\cos\phi = \sqrt{1-\sin^2\phi} = \frac{\sqrt{n^2R^2-L^2}}{nR}$$

From triangle OPS, $\phi+(\alpha+90.0°)+(90.0°-\gamma)=180°$ or the angle of incidence at point S is $\alpha=\gamma-\phi$. Then, applying Snell's law at point S

gives $$1.00\sin\theta = n\sin\alpha = n\sin(\gamma-\phi)$$

or $$\sin\theta = n[\sin\gamma\cos\phi-\cos\gamma\sin\phi] = n\left[\left(\frac{L}{R}\right)\frac{\sqrt{n^2R^2-L^2}}{nR}-\frac{\sqrt{R^2-L^2}}{R}\left(\frac{L}{nR}\right)\right]$$

$$\sin\theta = \frac{L}{R^2}\left(\sqrt{n^2R^2-L^2}-\sqrt{R^2-L^2}\right)$$

and $$\theta = \boxed{\sin^{-1}\left[\frac{L}{R^2}\left(\sqrt{n^2R^2-L^2}-\sqrt{R^2-L^2}\right)\right]}$$

***P35.58** (a) In the textbook figure we have $r_1=\sqrt{a^2+x^2}$ and $r_2=\sqrt{b^2+(d-x)^2}$. The speeds in the two media are $v_1=c/n_1$ and $v_2=c/n_2$ so the travel time for the light from P to Q is indeed

$$t=\frac{r_1}{v_1}+\frac{r_2}{v_2}=\frac{n_1\sqrt{a^2+x^2}}{c}+\frac{n_2\sqrt{b^2+(d-x)^2}}{c}$$

(b) Now $\frac{dt}{dx}=\frac{n_1}{2c}(a^2+x^2)^{-1/2}2x+\frac{n_2}{2c}(b^2+(d-x)^2)^{-1/2}2(d-x)(-1)=0$ is the requirement

for minimal travel time, which simplifies to $\frac{n_1x}{\sqrt{a^2+x^2}}=\frac{n_2(d-x)}{\sqrt{b^2+(d-x)^2}}$.

(c) Now $\sin\theta_1=\frac{x}{\sqrt{a^2+x^2}}$ and $\sin\theta_2=\frac{d-x}{\sqrt{b^2+(d-x)^2}}$ so we have directly $n_1\sin\theta_1=n_2\sin\theta_2$.

P35.59 To derive the law of *reflection,* locate point O so that the time of travel from point A to point B will be minimum.

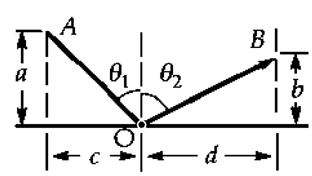


FIG. P35.59

The *total* light path is $L=a\sec\theta_1+b\sec\theta_2$.

The time of travel is $t=\left(\frac{1}{v}\right)(a\sec\theta_1+b\sec\theta_2)$.

If point O is displaced by dx, then

$$dt=\left(\frac{1}{v}\right)(a\sec\theta_1\tan\theta_1 d\theta_1+b\sec\theta_2\tan\theta_2 d\theta_2)=0 \qquad (1)$$

(since for minimum time $dt=0$).

continued on next page

Also, $c + d = a\tan\theta_1 + b\tan\theta_2 = \text{constant}$

so, $a\sec^2\theta_1 d\theta_1 + b\sec^2\theta_2 d\theta_2 = 0$ (2)

Divide equations (1) and (2) to find $\boxed{\theta_1 = \theta_2}$.

P35.60 As shown in the sketch, the angle of incidence at point A is:

$$\theta = \sin^{-1}\left(\frac{d/2}{R}\right) = \sin^{-1}\left(\frac{1.00\text{ m}}{2.00\text{ m}}\right) = 30.0°$$

If the emerging ray is to be parallel to the incident ray, the path must be symmetric about the centerline CB of the cylinder. In the isosceles triangle ABC,

$$\gamma = \alpha \qquad \text{and} \qquad \beta = 180° - \theta$$

Therefore, $\alpha + \beta + \gamma = 180°$

becomes $2\alpha + 180° - \theta = 180°$

or $\alpha = \dfrac{\theta}{2} = 15.0°$

Then, applying Snell's law at point A,

$$n\sin\alpha = 1.00\sin\theta$$

or $n = \dfrac{\sin\theta}{\sin\alpha} = \dfrac{\sin 30.0°}{\sin 15.0°} = \boxed{1.93}$

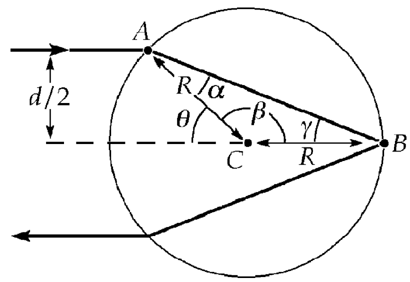


FIG. P35.60

P35.61 (a) The apparent radius of the glowing sphere is R_3 as shown. For it

$$\sin\theta_1 = \frac{R_1}{R_2}$$

$$\sin\theta_2 = \frac{R_3}{R_2}$$

$$n\sin\theta_1 = 1\sin\theta_2$$

$$n\frac{R_1}{R_2} = \frac{R_3}{R_2} \qquad \boxed{R_3 = nR_1}$$

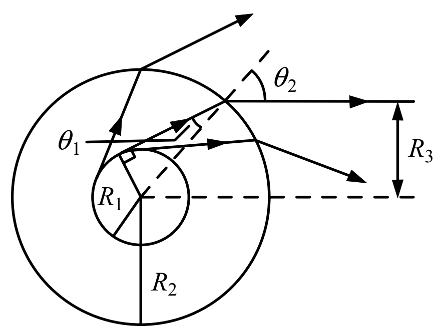


FIG. P35.61 (a)

(b) If $nR_1 > R_2$, then $\sin\theta_2$ cannot be equal to $\dfrac{nR_1}{R_2}$.

The ray considered in part (a) undergoes total internal reflection. In this case a ray escaping the atmosphere as shown here is responsible for the apparent radius of the glowing sphere and $\boxed{R_3 = R_2}$.

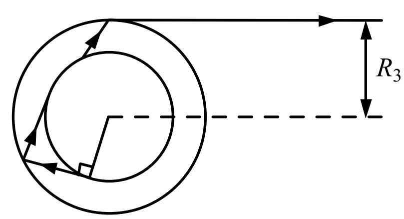


FIG. P35.61 (b)

P35.62 (a) At the boundary of the air and glass, the critical angle is given by $\sin\theta_c = \dfrac{1}{n}$

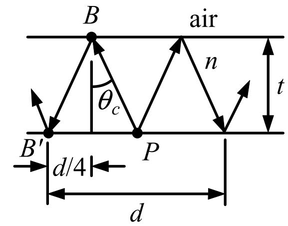


FIG. P35.62

Consider the critical ray PBB': $\tan\theta_c = \dfrac{d/4}{t}$ or $\dfrac{\sin\theta_c}{\cos\theta_c} = \dfrac{d}{4t}$.

Squaring the last equation gives: $\dfrac{\sin^2\theta_c}{\cos^2\theta_c} = \dfrac{\sin^2\theta_c}{1-\sin^2\theta_c} = \left(\dfrac{d}{4t}\right)^2$.

Since $\sin\theta_c = \dfrac{1}{n}$, this becomes $\dfrac{1}{n^2-1} = \left(\dfrac{d}{4t}\right)^2$ or $\boxed{n = \sqrt{1+\left(\dfrac{4t}{d}\right)^2}}$

(b) Solving for d, $d = \dfrac{4t}{\sqrt{n^2-1}}$.

Thus, if $n = 1.52$ and $t = 0.600$ cm, $d = \dfrac{4(0.600\text{ cm})}{\sqrt{(1.52)^2-1}} = \boxed{2.10\text{ cm}}$

(c) Since violet light has a larger index of refraction, it will lead to a smaller critical angle and the inner edge of the white halo will be tinged with $\boxed{\text{violet}}$ light.

P35.63 (a) Given that $\theta_1 = 45.0°$ and $\theta_2 = 76.0°$

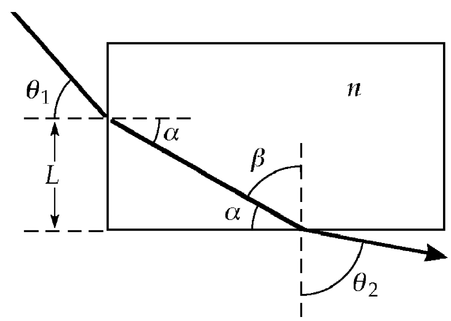


FIG. P35.63

Snell's law at the first surface gives

$$n\sin\alpha = 1.00\sin 45.0° \qquad (1)$$

Observe that the angle of incidence at the second surface is

$$\beta = 90.0° - \alpha.$$

Thus, Snell's law at the second surface yields

$$n\sin\beta = n\sin(90.0° - \alpha) = 1.00\sin 76.0°$$

or $\quad n\cos\alpha = \sin 76.0°. \qquad (2)$

Dividing Equation (1) by Equation (2), $\tan\alpha = \dfrac{\sin 45.0°}{\sin 76.0°} = 0.729$

or $\alpha = 36.1°$

Then, from Equation (1), $n = \dfrac{\sin 45.0°}{\sin\alpha} = \dfrac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$

(b) From the sketch, observe that the distance the light travels in the plastic is $d = \dfrac{L}{\sin\alpha}$. Also, the speed of light in the plastic is $v = \dfrac{c}{n}$, so the time required to travel through the plastic is

$$\Delta t = \frac{d}{v} = \frac{nL}{c\sin\alpha} = \frac{1.20(0.500\text{ m})}{(3.00\times10^8\text{ m/s})\sin 36.1°} = 3.40\times10^{-9}\text{ s} = \boxed{3.40\text{ ns}}$$

***P35.64**

$\sin\theta_1$	$\sin\theta_2$	$\dfrac{\sin\theta_1}{\sin\theta_2}$
0.174	0.131	1.330 4
0.342	0.261	1.312 9
0.500	0.379	1.317 7
0.643	0.480	1.338 5
0.766	0.576	1.328 9
0.866	0.647	1.339 0
0.940	0.711	1.322 0
0.985	0.740	1.331 5

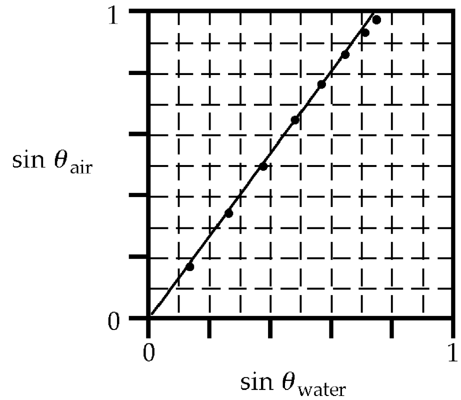


FIG. P35.64

The straightness of the graph line demonstrates Snell's proportionality of the sine of the angle of refraction to the sine of the angle of incidence.

The slope of the line is $\bar{n} = 1.327\,6 \pm 0.01$

The equation $\sin\theta_1 = n\sin\theta_2$ shows that this slope is the index of refraction,

$$n = \boxed{1.328 \pm 0.8\%}$$

***P35.65** Consider an insulated box with the imagined one-way mirror forming one face, installed so that 90% of the electromagnetic radiation incident from the outside is transmitted to the inside and only a lower percentage of the electromagnetic waves from the inside make it through to the outside. Suppose the interior and exterior of the box are originally at the same temperature. Objects within and without are radiating and absorbing electromagnetic waves. They would all maintain constant temperature if the box had an open window. With the glass letting more energy in than out, the interior of the box will rise in temperature. But this is impossible, according to Clausius's statement of the second law. This reduction to a contradiction proves that it is impossible for the one-way mirror to exist.

ANSWERS TO EVEN PROBLEMS

P35.2 227 Mm/s

P35.4 (a) and (b) See the solution. (c) 0.055 7°

P35.6 See the solution.

P35.8 27.1 ns

P35.10 15.4°, 2.56 m. The light wave slows down as it moves from air to water but the sound wave speeds up by a large factor. The light wave bends toward the normal and its wavelength shortens, but the sound wave bends away from the normal and its wavelength increases.

P35.12 (a) 474 THz (b) 422 nm (c) 200 Mm/s

P35.14 (a) 2.0×10^8 m/s (b) 474 THz (c) 4.2×10^{-7} m

P35.16 6.30 cm

P35.18 3.39 m

P35.20 106 ps

P35.22 23.1°

P35.24 ~10^{-11} s; between 10^3 and 10^4 wavelengths

P35.26 (a) 41.5° (b) 18.5° (c) 27.6° (d) 42.6°

P35.28 $\sin^{-1}\left(\sqrt{n^2-1}\sin\Phi-\cos\Phi\right)$

P35.30 See the solution.

P35.32 0.171°

P35.34 62.5°

P35.36 67.2°

P35.38 (a) 10.7° (b) air (c) Sound falling on the wall from most directions is 100% reflected.

P35.40 25.7°

P35.42 (a) $\frac{h}{c}\left(\frac{n+1}{2}\right)$ (b) larger by $\frac{n+1}{2}$ times

P35.44 (a) See the solution; the angles are $\theta_1'=30.0°$ and $\theta_2=18.8°$ (b) See the solution; the angles are $\theta_1'=30.0°$ and $\theta_2=50.8°$. (c) and (d) See the solution.

P35.46 (a) 0.172 mm/s (b) 0.345 mm/s (c) Northward at 50.0° below the horizontal (d) Northward at 50.0° below the horizontal

P35.48 (a) 45.0° (b) Yes; see the solution.

P35.50 3.79 m

P35.52 (a) 210 m/s (b) 23.3 $(x^2+9\text{ m}^2)$/m·s (c) 210 m/s for $x=0$; this is the same as the speed on the circular wall. (d) The speed goes to infinity as x goes to infinity, when the mirror turns through 45° from the $x=0$ point. (e) 22.4 ms

P35.54 (a) The optical day is longer. Incoming sunlight is refracted downward at the top of the atmosphere, so an observer can see the rising Sun when it is still geometrically below the horizon. Light from the setting Sun reaches her after the Sun is already below the horizon geometrically.
(b) 164 s

P35.56 36.5°

P35.58 See the solution.

P35.60 1.93

P35.62 (a) $n=[1+(4t/d)^2]^{1/2}$ (b) 2.10 cm (c) violet

P35.64 See the solution; $n=1.328\pm0.8\%$.