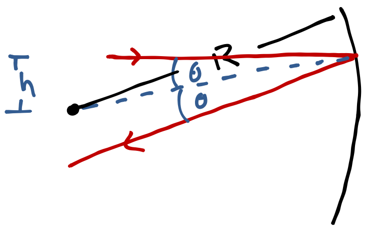
P1

(a)



$\sin\theta = \dfrac{h}{R}$

$\Rightarrow\ y = tg(2\theta)\cdot(x - x_o)$

$\Rightarrow\ h = tg(2\theta)\cdot(R\cos\theta - x_o)$

$\Rightarrow\ h = \dfrac{2\sin\theta\sqrt{1-\sin^2\theta}}{1-2\sin^2\theta}\cdot\left(R\sqrt{1-\sin^2\theta} - x_o\right)$

$\Rightarrow\ \cancel{h} = \dfrac{2\cancel{h}\sqrt{R^2-h^2}}{R^2-2h^2}\left(\sqrt{R^2-h^2} - x_o\right) \Rightarrow \boxed{x_o = \sqrt{R^2-h^2} - \dfrac{R^2-2h^2}{\sqrt{R^2-h^2}}}$

(*) Si hubieramos hecho la paraxial $\theta \ll 1$

$\Rightarrow\ \sin\theta \approx tg\theta \approx \theta$; $\cos\theta \approx 1$

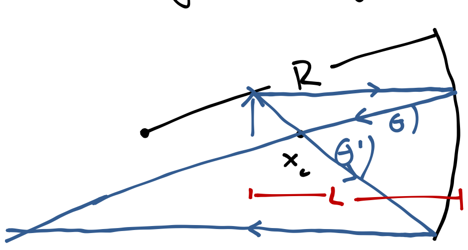
$\Rightarrow\ \cancel{h} \approx \dfrac{2\cancel{h}}{R}\cdot(R - x_o) \Rightarrow \boxed{x_o \approx R/2}$

Otra forma de verlo es que $h \ll R \Rightarrow z \equiv h/R \ll 1$

$\Rightarrow\ x_o = R\sqrt{1-z^2} - \dfrac{R(1-2z^2)}{2\sqrt{1-z^2}}$

$\Rightarrow\ \boxed{x_o \simeq R/2}$

- La imagen reflejada entonces debe ser:



- Los haces deben cumplir con

$$h = \frac{-2h'\sqrt{R^2-h'^2}}{R^2-2h'^2}\left((R-L) - \sqrt{R^2-h'^2} + \frac{R^2-2h^2}{\sqrt{R^2-h^2}}\right)$$

$$h' = \frac{2h\sqrt{R^2-h^2}}{R^2-2h^2}\left((R-L') - \sqrt{R^2-h^2} + \frac{R^2-2h^2}{\sqrt{R^2-h^2}}\right)$$

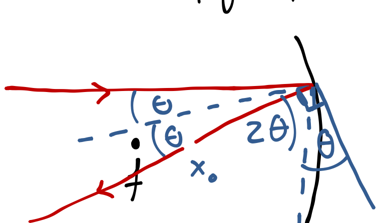
- Notamos que es muy complicado como para hacer analíticamente. Por lo que probamos con la aproximación paraxial, donde:

$$h \simeq \frac{2h'}{R}\left(R - L - \frac{R}{2}\right) \ ; \ h' \simeq \frac{2h}{R}\left(R - L' - \frac{R}{2}\right)$$

$$\Rightarrow h' = \frac{hR}{R-2L} \ ; \ L' = \frac{R}{2} - \frac{R^2}{2(R-2L)} \Rightarrow L' = \frac{-\cancel{2}LR}{\cancel{2}(R-2L)} \Rightarrow \boxed{\frac{1}{L'} = \underbrace{\frac{2}{R}}_{x_o} - \frac{1}{L}}$$

(b)

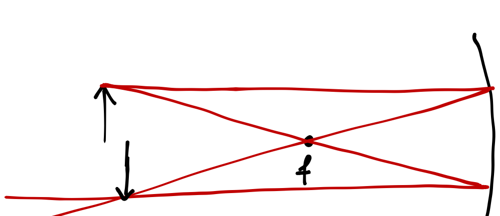
- Para un espejo parabólico:



- Acá tenemos: $x = -\dfrac{y^2}{4f} \Rightarrow tg\theta = -\dfrac{dx}{dy} = \dfrac{y}{2f}$

$y = tg(2\theta)(x - x_o) \Rightarrow y = \dfrac{2tg\theta}{1-tg^2\theta}(x - x_o)$

$\Rightarrow\ \cancel{h} = \dfrac{-4f\cancel{h}}{4f^2-h^2}\left(\dfrac{+h^2}{4f} + x_o\right) \Rightarrow \boxed{x_o = -f}$ → Sin aproximaciones!

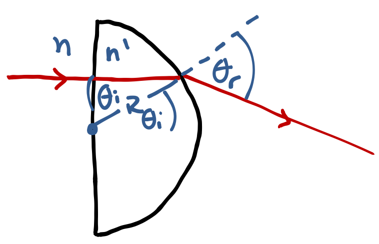


$$h' = \frac{4f}{4f^2-h^2}(L' + f) \ ; \ h = \frac{-4fh'}{4f^2-h'^2}(L+f)$$

$$\Rightarrow 4hf^2 - hh'^2 = -4fh'(L+f) \Rightarrow h'^2 - \frac{4f(L+f)}{h}h' - 4f^2 = 0$$

P2

(a)



- Si el haz viene a una altura $h \Rightarrow \sin\theta_i = h/R$

↳ Snell: $n'\sin\theta_i = n\sin\theta_r \Rightarrow \sin\theta_r = \dfrac{hn'}{Rn}$

$$\Rightarrow y = -tg(\theta_r-\theta_i)\cdot(x-x_o) = -\frac{tg\,\theta_r + tg\,\theta_i}{1 + tg\theta_r\, tg\theta_i}(x-x_o)$$

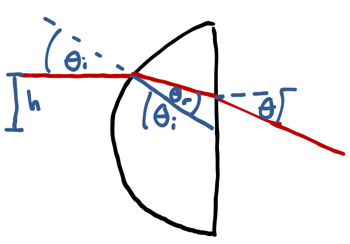
$$\Rightarrow h = -\frac{tg\theta_r + tg\theta_i}{1+tg\theta_r\, tg\theta_i}(R\cos\theta_i - x_o)$$

$$\Rightarrow x_o = -R\,tg\theta_r\cos\theta_i + R\sin\theta_i - h - h\,tg\,\theta_r\,tg\,\theta_i$$

- Notamos que conocemos todas las variables, por lo que conocemos x_o. Analíticamente en la aproximación paraxial:

$$h = -(\theta_r-\theta_i)(R-x_o) \Rightarrow R = \left(-\frac{n'}{n}+1\right)(R-x_o) \Rightarrow x_o = R + \frac{Rn}{n'-n} = \frac{R\,n'}{n'-n}$$

(b)



$$\sin\theta_i = h/R \Rightarrow \sin\theta_r = \frac{hn}{Rn'} \Rightarrow \sin\theta = \frac{n'}{n}\sin(\theta_i-\theta_r)$$

$$y' = -tg(\theta_i-\theta_r)(x-x_o')$$
$$y = -tg\theta\,(x-x_o)$$
$$\Rightarrow x_o = \frac{tg(\theta_i-\theta_r)}{tg\theta}\,x_o'$$

- Entonces, tenemos que encontrar solo x_o'. Para eso solo reemplazamos y' en $(R\cos\theta_i, h)$

$$\Rightarrow h = -tg(\theta_i-\theta_r)(R\cos\theta_i - x_o') \Rightarrow x_o' = R\cos\theta_i + \frac{h}{tg(\theta_i-\theta_r)}$$

- Ahora podemos usar una calculadora para resolver. Si usamos la aproximación paraxial:

$$\theta_i \approx \frac{h}{R} \Rightarrow \theta_r \approx \frac{hn}{Rn'} \Rightarrow \theta \approx \frac{hn}{Rn'}\left(1-\frac{n}{n'}\right) \;;\; x_o' = R + \frac{R}{\left(1-n/n'\right)} = \frac{Rn'}{n'-n}$$

$$\Rightarrow x_o = \frac{\cancel{h}}{\cancel{R}}\left(\frac{\cancel{n'-n}}{\cancel{n'}}\right)\cdot\frac{\cancel{R}\,n'^2}{\cancel{h}\,n\,(\cancel{n'-n})}\cdot\frac{R\cancel{n'}}{n'-n}$$

$$\Rightarrow \boxed{x_o = \frac{Rn'^2}{n(n'-n)}}$$