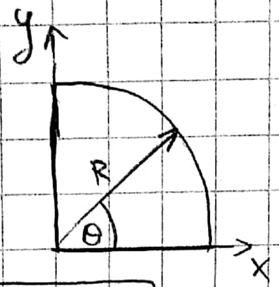


$$A = \frac{\pi R^2}{4}$$

$$\bar{y} = \frac{1}{A}\int_A y \, dA$$

polares

$$y = r \operatorname{sen} \theta$$

$$dA = r \, dr \, d\theta$$

$$\int_A y \, dA = \int_0^{\pi/2}\int_0^{R} r^2 \operatorname{sen}\theta \, dr \, d\theta$$

$$= \int_0^{\pi/2} \operatorname{sen}\theta \, d\theta \cdot \int_0^{R} r^2 \, dr$$

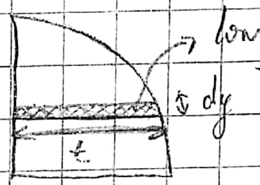
$$= -\cos(\theta)\Big|_0^{\pi/2} \cdot \frac{r^3}{3}\Big|_0^{R}$$

$$= (0 - {}^-1)\cdot \frac{R^3}{3} = \frac{R^3}{3}$$

$$\Rightarrow \bar{y} = \frac{4}{\pi R^2}\cdot\frac{R^3}{3}$$

$$\Rightarrow \boxed{\bar{y} = \frac{4}{3}\frac{R}{\pi}}$$

cartesianos



$$x^2 + y^2 = R^2$$

$$\Rightarrow x = \sqrt{R^2 - y^2} = t$$

$$\Rightarrow dA = t \cdot dy = \sqrt{R^2 - y^2}\, dy$$

$$\int_A y \, dA = \int_0^R y\sqrt{R^2 - y^2}\, dy$$

usando $u = R^2 - y^2$; $du = -2y\,dy$

$$\Rightarrow \int_A y \, dA = -\int_{R^2}^{0} \frac{\sqrt{u}}{2}\, du = \frac{1}{2}\int_0^{R^2} u^{\frac{1}{2}}\, du$$

$$= \frac{1}{\cancel{2}}\frac{\cancel{2}}{3} u^{3/2}\Big|_0^{R^2} = \frac{R^3}{3}$$

$$\Rightarrow \bar{y} = \frac{4}{\pi \cancel{R^2}}\cdot\frac{R^3}{3}$$

$$\Rightarrow \boxed{\bar{y} = \frac{4}{3}\frac{R}{\pi}}$$