P2|

$$\underset{\sim}{T} = \begin{pmatrix} 0 & 0 & -Cx_2 \\ 0 & 0 & Cx_1 \\ -Cx_2 & Cx_1 & 0 \end{pmatrix}$$

Punto $(1,2,4)$

$$\Rightarrow \underset{\sim}{T} = \begin{pmatrix} 0 & 0 & -2C \\ 0 & 0 & C \\ -2C & C & 0 \end{pmatrix}$$

a) Esfuerzos principales

$\Rightarrow I_1 = tr(\underset{\sim}{T}) = 0$

$I_2 = \frac{1}{2}\left(I_1^2 - tr(T^2)\right)$; $T^2 = \begin{pmatrix} 0 & 0 & -2C \\ 0 & 0 & C \\ -2C & C & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & -2C \\ 0 & 0 & C \\ -2C & C & 0 \end{pmatrix}$

$$\Rightarrow T^2 = \begin{pmatrix} 4C^2 & -2C^2 & 0 \\ -2C^2 & C^2 & 0 \\ 0 & 0 & 5C^2 \end{pmatrix}$$

$\Rightarrow tr(T^2) = 10C^2$

$\Rightarrow I_2 = -5C^2$

$I_3 = \det \underset{\sim}{T} = 0$

Ec. para valores propios

$\Rightarrow -\lambda^3 + \lambda^2 \cancel{I_1}^{0} - \lambda I_2 + \cancel{I_3}^{0} = 0$

$\Rightarrow -\lambda^3 + 5C^2\lambda = 0$

$\lambda(-\lambda^2 + C^2 5) = 0$

$\Rightarrow$ $\lambda_1 = 0$, $\lambda_2 = \sqrt{5}C$, $\lambda_3 = -\sqrt{5}C$

b) dir principales

$(T - \lambda I)\, n = 0$

$\Rightarrow \lambda_1 = 0$

$$\Rightarrow \begin{pmatrix} 0 & 0 & -2c \\ 0 & 0 & c \\ -2c & c & 0 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow -2c\, n_3 = 0$$
$$c\, n_3 = 0 \quad \Rightarrow \quad n_3 = 0$$
$$-2\cancel{c}\, n_1 + \cancel{c}\, n_2 = 0 \qquad n_2 = 2 n_1$$

normalizando

$\Rightarrow n_1^2 + n_2^2 + n_3^2 = 1$

$\Rightarrow n_1^2 + 4 n_1^2 = 1$

$\Rightarrow n_1 = \frac{1}{\sqrt{5}}$

$\Rightarrow n_2 = \frac{2}{\sqrt{5}}$

$n_3 = 0$

$\lambda_2 = c\sqrt{5}$

$$\Rightarrow \begin{pmatrix} -c\sqrt{5} & 0 & -2c \\ 0 & -c\sqrt{5} & c \\ -2c & c & -c\sqrt{5} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-\cancel{c}\sqrt{5}\, n_1 - 2\cancel{c}\, n_3 = 0$

$-\cancel{c}\sqrt{5}\, n_2 + \cancel{c}\, n_3 = 0$

$\Rightarrow n_3 = -\frac{\sqrt{5}}{2} n_1$

$n_2 = \frac{n_3}{\sqrt{5}} = -\frac{n_1}{2}$

normalizando

$n_1^2 + n_2^2 + n_3^2 = 1$

$\frac{4 n_1^2}{4} + \frac{n_1^2}{4} + \frac{5}{4} n_1^2 = 1 \Rightarrow \frac{\overset{5}{\cancel{10}}\, n_1^2}{\underset{2}{\cancel{4}}} = 1$

$\Rightarrow n_1 = \sqrt{\frac{5}{2}}$

$\Rightarrow n_2 = -\frac{1}{2}\sqrt{\frac{5}{2}}$

$n_3 = -\frac{\sqrt{5}}{2} \cdot \sqrt{\frac{5}{2}}$

$\lambda_3 = -c\sqrt{5}$

$$\Rightarrow \begin{pmatrix} c\sqrt{5} & 0 & -2c \\ 0 & c\sqrt{5} & c \\ -2c & c & c\sqrt{5} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$n_1 \cancel{c}\sqrt{5} - 2\cancel{c}\, n_3 = 0$$

$$c\sqrt{5}\, n_2 + c\, n_3 = 0$$

$$\Rightarrow \quad n_3 = n_1 \frac{\sqrt{5}}{2}$$

$$n_2 = -\frac{n_3}{\sqrt{5}} = -\frac{n_1}{2}$$

Normalizando

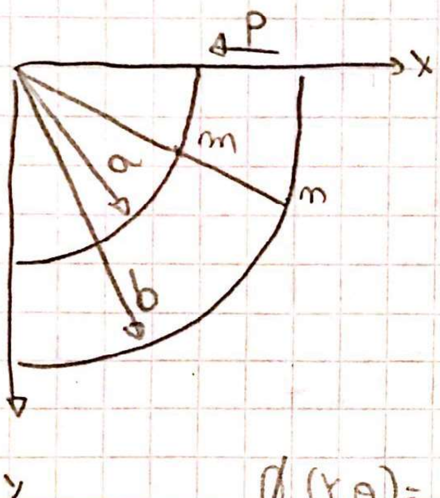
$$n_1^2 + n_2^2 + n_3^2 = 1$$

$$\Rightarrow n_1 = \sqrt{\frac{5}{2}}, \quad n_2 = -\sqrt{\frac{5}{2}} \cdot \frac{1}{2}$$

$$n_3 = \sqrt{\frac{5}{2}} \cdot \frac{\sqrt{5}}{2}$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{c\sqrt{5} + c\sqrt{5}}{2} = c\sqrt{5}$$

P2.1



$$\phi(r,\theta) = f(r)\sin\theta$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

$$\Rightarrow \nabla^2\nabla^2\phi = 0$$

$$\Rightarrow \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial \theta^2}\right) = 0$$

$$= \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(f'' + \frac{1}{r}f' - \frac{1}{r^2}f\right)\sin\theta = 0$$

$$= \left(\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \cancelto{0}{\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}}\right)\left(f'' + \frac{1}{r}f' - \frac{1}{r^2}f\right)\right)\sin\theta$$

$$+ \left(\left(\cancel{\frac{\partial^2}{\partial r^2}} + \cancel{\frac{1}{r}\frac{\partial}{\partial r}} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\sin\theta\right)\left(f'' + \frac{1}{r}f' - \frac{1}{r^2}f\right)$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\left(f'' + \frac{1}{r}f' - \frac{1}{r^2}f\right)\sin\theta - \frac{1}{r^2}\left(f'' + \frac{1}{r}f' - \frac{1}{r^2}f\right)\sin\theta = 0$$

$\forall \theta$

$$\Rightarrow \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\right)\left(f'' + \frac{1}{r}f' - \frac{1}{r^2}f\right) = 0$$

2:) $f(r) = Ar^3 + B\frac{1}{r} + Cr + Dr\ln(r)$

$$\Rightarrow \sigma_\theta = \frac{\partial^2\phi}{\partial r^2} = \left(6Ar + 2\frac{B}{r^3} + \frac{D}{r}\right)\operatorname{sen}\theta$$

$$\sigma_r = \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = \left(2Ar - \frac{2B}{r^3} + \frac{D}{r}\right)\operatorname{sen}\theta$$

$$\tau_{r\theta} = \left(-2Ar + \frac{2B}{r^3} - \frac{D}{r}\right)\cos\theta$$

Cond de borde

en a y B

$$\Rightarrow T\, n \underset{\text{en a y b}}{=} 0$$

$$\begin{pmatrix} \sigma_r & \tau_{r\theta} \\ \tau_{r\theta} & \sigma_\theta \end{pmatrix}\begin{pmatrix} \mp 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} \sigma_r(a,b) = 0 \\ \tau_{r\theta}(a,b) = 0 \end{matrix}$$

$\theta = 0$
ticket b

$\Rightarrow T n = t_p$

$$\begin{pmatrix} \sigma_r & \tau_{r\theta} \\ \tau_{r\theta} & \sigma_\theta \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -t_p \\ 0 \end{pmatrix}$$

$$-\tau_{r\theta} = -t_p \qquad \int_0^1 \int_a^b dA$$

$$\Rightarrow A(b^2 - a^2) - B\left(\frac{1}{b^2} - \frac{1}{a^2}\right) - D \ln\left(\frac{b}{a}\right) = P$$

$$\Rightarrow \mathcal{N} = (a^2 - b^2) + (a^2 + b^2) \ln(b/a)$$

$$\Rightarrow \sigma_r = \left(\frac{P}{\mathcal{N}} r + \frac{P a^2 b^2}{\mathcal{N}} \frac{1}{r^3} - \frac{P}{\mathcal{N}} (a^2 + b^2) \frac{1}{r}\right) \sin\theta$$

$$\sigma_\theta = \left(\frac{3P}{\mathcal{N}} r - \frac{P a^2 b^2}{\mathcal{N}} \frac{1}{r^3} - \frac{P}{\mathcal{N}} (a^2 + b^2) \frac{1}{r}\right) \sin\theta$$

$$\tau_{r\theta} = \left(-\frac{P}{\mathcal{N}} r - \frac{P a^2 b^2}{\mathcal{N}} \frac{1}{r^3} + \frac{P}{\mathcal{N}} (a^2 + b^2) \frac{1}{r}\right) \cos\theta$$

Topo $\Rightarrow \theta = 0$

$$\Rightarrow \sigma_r = \sigma_\theta = 0, \quad \tau_{r\theta} = -\frac{P}{\mathcal{N}}\left(r + \frac{a^2 b^2}{r^3} - \frac{1}{r}(a^2 + b^2)\right)$$

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en la parte de abajo $\theta = \frac{\pi}{2}$

$$\tau_{r\theta} = 0 \quad ; \quad \sigma_r = \frac{P}{\gamma}\left(r + \frac{a^2 b^2}{r^3} - \frac{1}{r}\left(a^2 + b^2\right)\right)$$

$$\sigma_\theta = \frac{P}{\gamma}\left(3r - \frac{a^2 b^2}{r^3} - \frac{1}{r}\left(a^2 + b^2\right)\right)$$