Aux #14.

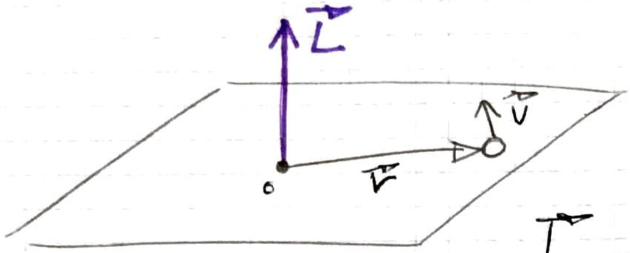
5/Junio/2020

Fzas. Centrales: $\vec{F} = f(r)\hat{r}$ $\qquad \vec{F} = -\nabla U$

⊛ gravedad

$$\vec{\tau} = \vec{r} \times \vec{F} = r\hat{r} \times f(r)\hat{r} = 0 \,.\, ⊛$$

$\Rightarrow$ El movimiento es plano.



$$\vec{\tau} = \frac{d\vec{L}}{dt} = 0$$

$\Rightarrow \vec{L} = L_0$ (cte).

$$\vec{L} = m(\vec{r} \times \vec{v}) = m\rho^2\dot{\phi} = L_0.$$

$$h = \frac{L_0}{m} = \rho^2\dot{\phi}$$

- Ec de mov: $m(\ddot{\rho} - \rho\dot{\phi}^2) = f(\rho)$

$$\Rightarrow \boxed{m\ddot{\rho} = f(\rho) + \frac{L_0^2}{m\rho^3}}$$

Energía: $\frac{1}{2}m\dot{\rho}^2 + \underbrace{\left[\frac{mh^2}{2\rho^2} + U(\rho)\right]}_{U \text{ efectivo.}} = cte$

$$\frac{1}{2}m(\dot{\rho}^2 + (\rho\dot{\phi})^2) + U(\rho) = cte.$$

P1 | $\vec{F} = -c\hat{r}$

ⓐ U_{eff}. $E = \frac{1}{2}m\dot{r}^2 + \underbrace{\frac{1}{2}m\left(\frac{h}{r}\right)^2 + U(r)}$

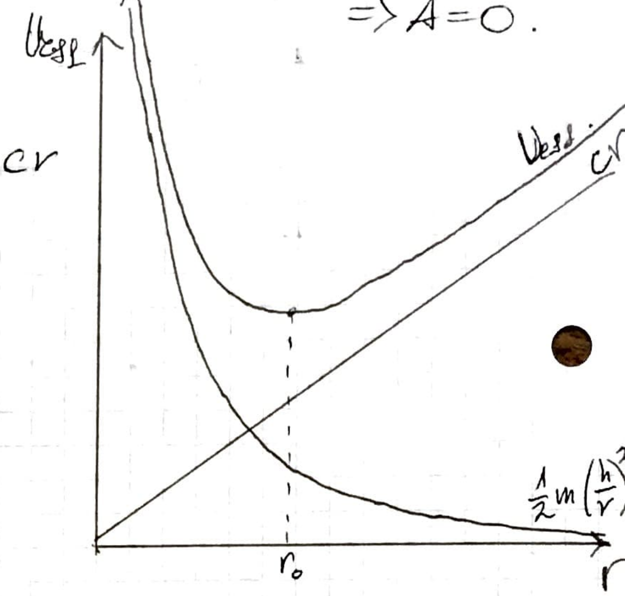
$\vec{F} = -\nabla U$.

y $(-c) = -\frac{\partial U}{\partial r}\cdot\hat{r}$ $\iff$ $c = \frac{\partial U}{\partial r}$ $/ \int dr$

$\Rightarrow U(r) = A + c\cdot r$. pero quiero que $U(0)=0$ $\Rightarrow A = 0$.

$U(r) = c\cdot r$.

$\Rightarrow U_{eff} = \frac{1}{2}m\left(\frac{h}{r}\right)^2 + cr$



ⓑ ¿escapa?

K tendría que superar a U.

pero $U \to \infty$.

$\Rightarrow$ NO puede escapar.

ⓒ $r = 2r_0$. ΔE? $h = \frac{L_0}{m}$

$E_1 = E(r_0) = \frac{1}{2}m\cancel{\dot{r}_0^2} + \frac{1}{2}m\cdot\left(\frac{L_0}{mr_0}\right)^2 + cr_0$

Despejamos L_0 de la ec de movimiento:

porque es circunferencial. $\cancel{m\ddot{r}} = f(r) + \frac{L_0^2}{mr^3}$

$0 = -c + \frac{L_0^2}{mr_0^3} \Rightarrow L_0^2 = cmr_0^3$.

$E_1 = E(r_0) = \frac{1}{2}\cancel{m}\cdot\frac{c\cancel{m}r_0^{\cancel{3}}}{\cancel{m^2}\cdot\cancel{r_0^2}} + cr_0 = \frac{3}{2}cr_0$.

$E_2 = E(2r_0) = \frac{1}{2}m\cdot\frac{L_0^2}{m^2(2r_0)^2} + c(2r_0)$ | $L_0^2 = cmr_0^3$

* El momento angular NO cambia.

$E(2r_0) = \frac{1}{2}\cancel{m}\frac{c\cancel{m}r_0^{\cancel{3}}}{\cancel{m^2}(2\cancel{r_0})^2} + 2cr_0$

$E(2r_0) = \frac{cr_0}{8} + 2cr_0 = \frac{17}{8}cr_0$.

$$\boxed{\Delta E = \left(\frac{17}{8} - \frac{3}{2}\right)c\cdot r_0}$$

P2 | ⓐ V_B antes de frenar.

$$\vec{F}_G = \frac{GMm}{r^2}$$

$$U_G = -\frac{GMm}{r}$$

$$\vec{F}_G = -\nabla U_G$$

Energía: $E = \frac{1}{2} m V_B^2 + U_{gravitacional}$.

$$E_p = \frac{1}{2} m V_B^2 - \frac{GMm}{r_B} = 0$$

¿ $E = 0$?

~~excentricidad~~: cómo la órbita se desvía de una circunferencia perfecta

~~$e<1$ ELIPSE~~
~~$e=1$ PARÁBOLA~~
~~$e>1$ HIPÉRBOLA~~

	circunferencia	elipse	parábola	hipérbola
excentricidad e	$e=0$	$e<1$	$e=1$	$e>1$
E mecánica	$E<0$	$E<0$	$E=0$	$E>0$.

$$E_{\cancel{p}} = \frac{1}{2}\cancel{m} V_B^2 - \frac{GM\cancel{m}}{r_B} = 0 \Rightarrow \boxed{V_B^2 = \frac{2GM}{r_B}}$$

E entre B y C

ⓑ El impulso es tangencial $\Rightarrow$ ~~[illegible]~~ L cambia.

$$E_e = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r}$$

cambió de órbita $\Rightarrow$ E cambia.

En B y C: $\dot{r} = 0. \Rightarrow E_e = E_{B_e} = E_c$

$$E_{Be} = E_c \Leftrightarrow$$

$$\frac{L^2}{2mr_B^2} - \frac{GMm}{r_B} = \frac{L^2}{2mR_m^2} - \frac{GMm}{R_m}$$

$$\frac{L^2}{2m}\left(\frac{1}{r_B^2} - \frac{1}{R_m^2}\right) = GMm\left(\frac{1}{r_B} - \frac{1}{R_m}\right)$$

$$L^2\left(\frac{1}{r_B} + \frac{1}{R_m}\right)\cancel{\left(\frac{1}{r_B} - \frac{1}{R_m}\right)} = 2GMm^2\cancel{\left(\frac{1}{r_B} - \frac{1}{R_m}\right)}$$

$$\Rightarrow L^2 = \frac{2GMm^2 \cdot R_m \cdot r_B}{R_M \cdot r_B}$$

$E_n \quad E_e(R_M) = \cancel{\frac{1}{2}mv^2} + \frac{1}{\cancel{2}\cancel{m}R_M^{\cancel{2}}}\left(\frac{\cancel{2}GMm^{\cancel{2}}\cancel{R_M}r_B}{R_M + r_B}\right) - \frac{GMm}{R_M}$

$= E_e(r_B)$

$$E_e = \frac{GMm r_B}{R_M(R_M + r_B)} - \frac{GMm}{R_M}.$$

c) V_c $\quad E_e = \frac{1}{2}mV_c^2 - \frac{GMm}{R_M} \longrightarrow V_c^2.$

$$L^2 = \cancel{m}^2 R_M^{\cancel{2}} V_c^2 = \frac{2GM\cancel{m^2}\cancel{R_M}r_B}{R_M + r_B}.$$

$$\boxed{V_c^2 = \frac{2GMr_B}{R_M(R_M + r_B)}}$$