Aux #18

Centro de masa: $\vec{R}_{cm} = \frac{1}{M} \cdot \sum_{i=1}^{n} m_i \cdot \vec{r}_i$

Fuerzas Internas: centrales.

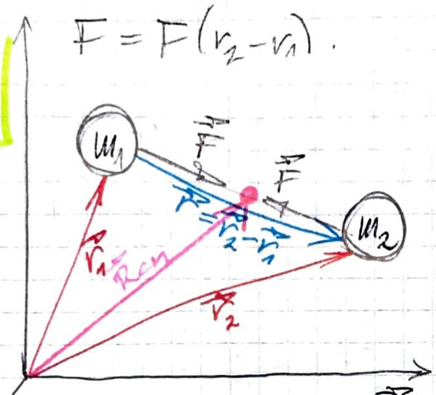
$\sum \vec{F}_{internas} = 0 \rightarrow$ porque se cancelan entre sí.

$\sum \vec{F}_{externas} = \dot{\vec{p}}$ Si no hay $\vec{F}$ externas, $\vec{p}$ se CONSERVA.

Momento Angular: $\vec{l}_i = \vec{r}_i \times \vec{p}_i$ $\quad \vec{L} = \sum_{i=1}^{n} l_i$.

$F = F(r_2 - r_1)$.

P1



Ecuaciones + reducir a 1 part.

$m_1 \ddot{r}_1 = F(r_2 - r_1)\hat{r}_{12}$

$m_2 \ddot{r}_2 = F(r_2 + r_1) \cdot (+\hat{r}_{12})$

Para reducir: $\vec{R}_{cm} = \frac{1}{M} \sum_{i=1}^{2} m_i \vec{r}_i = \frac{1}{(m_1+m_2)} \cdot (m_1 \vec{r}_1 + m_2 \cdot \vec{r}_2)$

Reducir:

Cambio de Variable: $\vec{r} = \vec{r}_2 - \vec{r}_1$.

$\Rightarrow \vec{r}_1 = \vec{r}_2 - \vec{r} \quad ; \quad \vec{r}_2 = \vec{r} + \vec{r}_1$.

Para despejar $\vec{r}_2$.

En el $\vec{R}_{cm}$: $R_{cm} = \frac{m_1(\vec{r}_2 - \vec{r}) + \vec{r}_2 m_2}{m_1 + m_2} = \frac{\vec{r}_2 (m_1 + m_2) + - m_1 \vec{r}}{m_1 + m_2}$

$\vec{R}_{cm} = \vec{r}_2 - \frac{m_1 \vec{r}}{m_1 + m_2} \Rightarrow \boxed{\vec{r}_2 = \vec{R}_{cm} + \frac{m_1 \cdot \vec{r}}{m_1 + m_2}}$.

Para despejar $\vec{r}_1$: $\vec{R}_{cm} = \frac{m_1 \vec{r}_1 + m_2(\vec{r} + r_1)}{m_1 + m_2} = \frac{\vec{r}_1(m_1+m_2) + \vec{r} \cdot m_2}{m_1 + m_2}$

$\vec{R}_{cm} = \vec{r}_1 + \frac{\vec{r} \cdot m_2}{m_1 + m_2} \Rightarrow \boxed{\vec{r}_1 = \vec{R}_{cm} - \frac{\vec{r} \cdot m_2}{m_1 + m_2}}$.

~~Restamos~~ Restamos las ecs de movimiento.

$m_1 \ddot{r}_1 - m_2 \ddot{r}_2 = 2F(r_2 - r_1) \cdot \hat{r}_{12}$

~~...~~

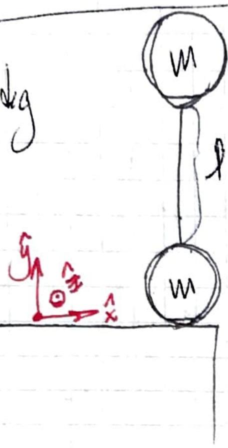
$$m_1\ddot{\vec R}_{cm}-\frac{m_1 m_2\ddot{\vec r}}{m_1+m_2}-m_2\ddot R_{cm}-\frac{m_2\cdot m_1\ddot{\vec r}}{m_1+m_2}=2F\cdot\hat r.$$

No hay fzas externas $\Rightarrow$ $\dot{\vec p}$ se conserva $\iff$ $\vec V_{cm}=cte.$ $\Rightarrow$ $\ddot{\vec R}_{cm}=0$

$$\Rightarrow -\frac{2m_1 m_2\ddot{\vec r}}{m_1+m_2}=2F\hat r.$$

$$\left.\frac{m_1 m_2\ddot{\vec r}}{m_1+m_2}=-F\hat r\right\}$$ sólo 1 partícula.

P2 $\downarrow g$



$\to \vec V_0$

@ $\vec P_{cm}$ y $\vec L_{cm}$.

$$\vec R_{cm}=\frac{m\vec r_1+m\vec r_2}{2m}=\frac{\vec r_1+\vec r_2}{2}.$$

$\vec F_{ext}=0\hat x-2mg\hat y$ → gravedad.

$\dot{\vec P}_{cm}=\vec F_{ext}=-2mg\hat y+0\hat x$

$\Rightarrow$ ~~...~~

$\cancel{2m}\ddot X_{cm}=0$

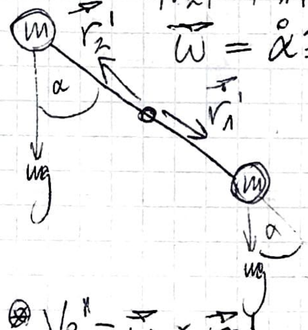
$\dot x_{cm}=0$ $\Rightarrow$ $\boxed{\dot x_{cm}=0}$

$\cancel{2m}\ddot y_{cm}=-\cancel{2m}g \iff \ddot y_{cm}=-g \Rightarrow \dot y_{cm}=-gt.$

$$\Rightarrow \boxed{\vec P_{cm}=cte\,\hat x+-2mgt\,\hat y}$$

$|\vec r_2'|=|\vec r_1'|=\frac{l}{2}.$

$\vec\omega=\dot\alpha\hat z$



$$\vec L_{cm}=\sum\vec r_i'\times\vec p_i'=\sum m_i\cdot\vec r_i'\times\vec V_i'$$

$$=m_1\cdot\vec r_1'\times(\dot\alpha\hat z\times\vec r_1')+m_2\cdot\vec r_2'\times(\dot\alpha\hat z\times\vec r_2')$$

$$=2m\left(\frac{l}{2}\right)^2\cdot\dot\alpha=\vec L_0$$

⊗ $V_i'=\vec\omega\times\vec r_i'$

Conservación de Momentum:

$$\overset{t=0}{m\cdot\frac{l}{2}\cdot V_0}=\overset{t>0}{2m\left(\frac{l}{2}\right)^2\cdot\dot\alpha}\Rightarrow\boxed{\dot\alpha=\frac{V_0}{l}}$$

$$\Rightarrow\vec L_{cm}=\frac{\cancel{2}ml^{\cancel{2}}\cdot V_0}{2\cancel{2}\cancel{l}}\qquad\boxed{\vec L_{cm}=\frac{mlV_0}{2}}$$