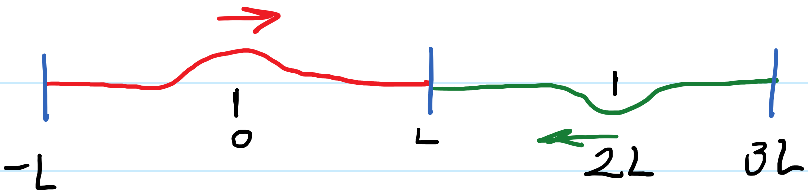
Auxiliar 4



$$u(x,0)=\frac{1}{x^2+1}=f(x)$$

$$u(x,t)=f(x-ct)+\ ?$$

Borde fijo: $u(L,t)=0 \wedge u(-L,t)=0$

$$\boxed{u(x,t)=f(x-ct)-f((x-2L)+ct)}$$

$$u(L,t)=f(L-ct)-f(-L+ct)$$

$$=f(L-ct)-f(L-ct)=0\ \checkmark$$

$$u(x,t)=f(x-ct)-f((x-2L)+ct)+f((x+4L)-ct)$$

$$u(-L,t)=-f(-3L+ct)+f(3L-ct)$$

$$=-f(-3L+ct)+f(-3L+ct)=0$$

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$u(x,t) = A\,\text{sen}(\kappa x)\cdot\cos(\omega t)$: onda estacionaria

$$u(x,t) = A\,\text{sen}(\kappa x)\underbrace{\left[\cos(\omega_1\cdot t) + \cos(\omega_2\cdot t)\right]}_{g(t)}$$

$$e^{i\phi} = \cos\phi + i\,\text{sen}\,\phi$$

$$g(t) = \cos(\omega_1\cdot t) + \cos(-\omega_2 t) \qquad \cos(-\phi) = \cos(\phi)$$

$$e^{\alpha}\cdot e^{\beta} = e^{\alpha+\beta}$$

$$\tilde{g}(t) = e^{i\omega_1 t} + e^{-i\omega_2 t}$$

$$\tilde{g}(t) = e^{i\left(\frac{\omega_1-\omega_2}{2}\right)t}\left[e^{i\left(\frac{\omega_1+\omega_2}{2}\right)t} + e^{i\left(\frac{\omega_1+\omega_2}{2}\right)t}\right]$$

$$g(t) = \cos\left(\frac{\omega_1-\omega_2}{2}\cdot t\right)\left[\cos\left(\frac{\omega_1+\omega_2}{2}t\right) + \cos\left(\frac{\omega_1+\omega_2}{2}\,t\right)\right]$$

$$= 2\cos\left(\left(\frac{\omega_1-\omega_2}{2}\right)t\right)\cos\left(\left(\frac{\omega_1+\omega_2}{2}\right)t\right)$$

$$\Omega = \frac{\omega_1+\omega_2}{2} \qquad \omega = \frac{\omega_1-\omega_2}{2}$$

$$g(t) = 2\cos(\omega t)\cos(\Omega t)$$

$$\lambda f = c$$

$$f = \frac{c}{\lambda}$$

$$\Omega = \frac{1}{2}(\omega_1 + \omega_2)$$

$$\omega = 2\pi f = \frac{2\pi c}{\lambda}$$

$$= \frac{1}{2}\left(\frac{2\pi c}{\lambda_1} + \frac{2\pi c}{\lambda_2}\right) = \frac{1}{2}\left(\frac{2\pi\cdot 340}{0{,}06} + \frac{2\pi\cdot 340}{0{,}065}\right)$$

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$$p(x,t) = -B \frac{\partial u}{\partial x}(x,t)$$

$$u(x,t) = A\cos(Kx - \omega t)$$

$$\frac{\partial u}{\partial x} = -AK\,\text{sen}(Kx - \omega t)$$

$$p(x,t) = +\underbrace{BAK}_{}\,\text{sen}(Kx - \omega t)$$

$$c = \lambda f \Rightarrow \lambda = \frac{c}{f}$$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$$

$$A \cdot B \cdot K < P_0 = 10\ Pa$$

$$A = 1\ \mu m$$

$$A = 1 \cdot 10^{-6} m$$

$$10^{-6} \cdot 1{,}42 \cdot 10^{5} \cdot \frac{2\pi \cdot f}{340} < 10$$

$$f < \frac{10 \cdot 340}{2\pi \cdot 10^{5} \cdot 1{,}42 \cdot 10^{-6}}$$

$$\boxed{f_{max} = 3810{,}75\ hz}$$

$$20hz < f_{max} < 20khz$$

$\rightarrow$ sí es audible

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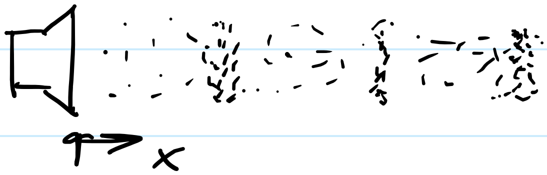

$$p(x,t) = p_0 \cdot \cos(kx) \cdot \cos(\omega t)$$

$$p(x=\tilde{x}, t) = 0 = p_0 \cos(k\tilde{x}) \cos(\omega t)$$

$$\rightarrow \cos(k\tilde{x}) = 0$$

$$\rightarrow k\tilde{x} = \frac{\pi}{2} + n\pi \quad , \quad n \in \mathbb{Z}$$

$$k = \frac{2\pi}{\lambda} \quad \rightarrow \quad \frac{2\cancel{\pi}}{\lambda}\tilde{x} = \frac{\cancel{\pi}}{2} + n\cancel{\pi}$$

$$\tilde{x} = \frac{\lambda}{2}\left(\frac{2n+1}{2}\right)$$

$$= \left(\frac{2n+1}{4}\right)\lambda$$

$$\tilde{x} = \left\{ \frac{\lambda}{4}, \frac{3}{4}\lambda, \ldots \right\}$$