

MA1002-8: Cálculo Diferencial e Integral

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## Guia primitivas

### Algebra de primitivas

- $\int (f \pm g) = \int f \pm \int g$
- $\int (\lambda f) = \lambda \int f$ , con  $\lambda$  constante
- $\int f'g = fg - \int fg'$
- $\int \frac{f'g - fg'}{g^2} = \left(\frac{f}{g}\right)$

**Primitivas esenciales** (Todos los resultados llevan un  $+C$  que obviaremos para simplificar la notacion)

- $\int 0dx = C$
- $\int 1dx = x$
- $\int x^n dx = \frac{x^{n+1}}{n+1}$
- $\int e^x dx = e^x$
- $\int \frac{1}{x} dx = \ln|x|$
- $\int n^x dx = \frac{n^x}{\ln(n)}$
- $\int \cos(x) dx = \sin(x)$
- $\int \sin(x) dx = -\cos(x)$
- $\int \sec^2(x) dx = \tan(x)$
- $\int \tan(x) \sec(x) dx = \sec(x)$
- $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x)$
- $\int -\frac{1}{\sqrt{1-x^2}} dx = \arccos(x)$
- $\int \frac{1}{1+x^2} dx = \arctan(x)$
- $\int -\frac{1}{1+x^2} dx = \text{arccotan}(x)$

### Anexo: Funciones hiperbólicas

- $\sinh(x) = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x}$
- $\cosh(x) = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x}$
- $\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
- $\cosh^2(x) - \sinh^2(x) = 1$

### Anexo: Primitiva de funciones hiperbólicas

- $\int \sinh(x) dx = \cosh(x)$
- $\int \cosh(x) dx = \sinh(x)$
- $\int \tanh(x) dx = \ln(\cosh(x))$
- $\int \frac{1}{\sqrt{x^2+1}} dx = \text{arcsinh}(x)$
- $\int \frac{1}{\sqrt{x^2-1}} dx = \text{arccosh}(x)$
- $\int \frac{1}{1-x^2} dx = \text{arctanh}(x)$ , para  $|x| < 1$

**Ejercicios:** Calcule las siguientes primitivas:

Polinomios

- $\int x^2 + 3x + 2dx$
- $\int x^2 + 2x + 1dx$
- $\int \frac{x^3}{3} + 3x^2 + 10dx$
- $\int \frac{2}{3}x^9 + \frac{3}{2}x^3dx$
- $\int x^3 + 3x^2 + 3x + 1dx$
- $\int 10x^9 + 5x^3 - x^2dx$

Producto de funciones

- $\int x \arctan(x)dx$
- $\int \sin(ax) \sin(bx)dx$
- $\int \cos(ax) \cos(bx)dx$
- $\int x \tan^2(x)dx$
- $\int x \sqrt{1+x}dx$
- $\int \sin(ax) \cos(bx)dx$

Division de funciones

- $\int \frac{x}{1+x}dx$
- $\int \frac{\ln(x)}{\sqrt{x}}dx$
- $\int \frac{\tan(x)}{\arctan(x)}dx$
- $\int \frac{x-1}{x+1}dx$
- $\int \frac{\cos(x)}{\arccos(x)}dx$
- $\int \frac{e^x}{\sqrt{1+e^x}}dx$

Primitivas, nivel 1

- $\int \frac{\ln(x)}{x}dx$
- $\int e^{2x}dx$
- $\int \sqrt{2x+4}dx$
- $\int \ln(x^2)dx$
- $\int \tan(x+1)dx$
- $\int \operatorname{cosec}(x)dx$

Primitivas, nivel 2

- $\int \frac{x^2}{\sqrt{x^2+4}}dx$
- $\int \frac{\ln^2(x)}{x^2}dx$
- $\int \sec^2(x) \operatorname{cosec}^2(x)dx$
- $\int \sqrt{1 + \frac{1}{3x}}dx$
- $\int \frac{\sqrt{x}}{\sqrt{1+\sqrt{x}}}dx$
- $\int \frac{\sin(x) + \cos(x)}{\cos(x)}dx$
- $\int \arctan(x)dx$
- $\int x^3 \sqrt{1-x^2}dx$
- $\int x^2 \cotan(x) + \frac{1}{x}dx$
- $\int x \tan^2(x)dx$
- $\int \frac{\sqrt{a^2+x}}{x}dx$
- $\int \frac{1}{e^x - e^{-x}}dx$

Primitivas, nivel 3

- $\int \frac{1 + \cos^2(x)}{1 + \cos(2x)}dx$
- $\int \frac{x^2 - 1}{x^2 + 1} \frac{1}{\sqrt{1+x^4}}dx$
- $\int \frac{x}{\sqrt{x^4 + 10x^2 - 96x - 71}}dx$
- $\int \frac{1}{\sqrt{25x^2 + 2}}dx$
- $\int \frac{3}{x^3 - 1}dx$
- $\int \frac{\arctan(x)}{1 + \frac{1}{x^2}}dx$
- $\int \frac{\tan^3(\ln(x))}{x}dx$
- $\int \cos(x) - \int \cos(x)dx$
- $\int \sqrt{\tan(x)}dx$