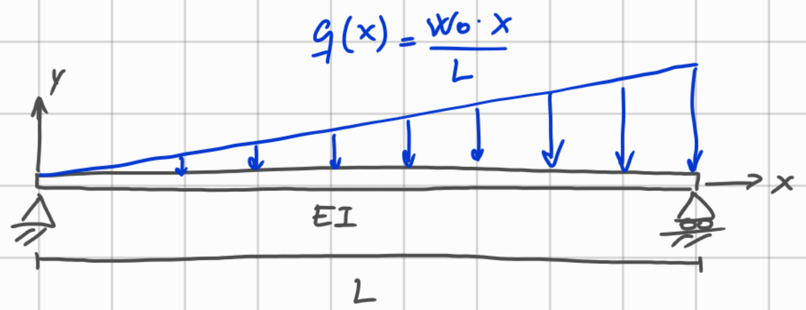
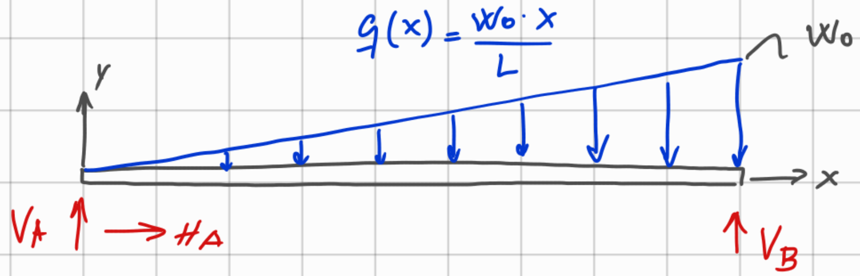
Tarea 8

P1





Calculo de reacciones

$\Sigma F_y = 0$:

$$V_A + V_B - \frac{w_0 \cdot L}{2} = 0$$

$\Sigma F_x = 0$:

$$H_A = 0$$

$\Sigma M_A = 0$:

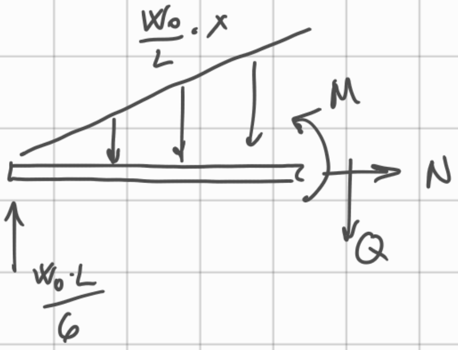
$$-\frac{w_0 \cdot L}{\cancel{2}} \cdot \frac{\cancel{2}}{3} \cancel{L} + V_B \cdot \cancel{L} = 0$$

$$V_B = \frac{w_0 L}{3}$$

$$\Rightarrow \quad V_A = \frac{w_0 \cdot L}{2} - \frac{w_0 L}{3}$$

$$V_A = \frac{w_0 L}{6}$$

Calculo de esfuerzos internos



$$M(x) + \frac{w_0 \cdot x}{L} \cdot \frac{x}{2} \cdot \frac{1}{3} x - \frac{w_0 L}{6} \cdot x = 0$$

$$M(x) = \frac{w_0 L}{6} x - \frac{w_0 x^3}{6L}$$

$$M(x) = \frac{w_0}{6L} \left(L^2 x - x^3 \right)$$

Campo de tensiones por Navier

$$\sigma_{xx} = -\frac{M_z \cdot y}{I_z} = -\frac{y}{I} \cdot \frac{w_0}{6L}\left(L^2 x - x^3\right)$$

$$\sigma_{xx} = -\frac{y\, w_0}{6LI}\left(L^2 x - x^3\right)$$

Campo de deformaciones por ley constitutiva

$$\sigma_{xx} = E\,\varepsilon_{xx} \Rightarrow \varepsilon_{xx} = -\frac{y\, w_0}{6L \cdot EI}\left(L^2 x - x^3\right)$$

Campo de desplazamientos por ecuación de la elástica se tiene

con

$$M(x) = EI\,\frac{d^2v}{dx^2} \qquad \frac{d^2v}{dx^2} = \phi(x)$$

$$\frac{dv}{dx} = \theta(x)$$

$v(x)$: flecha

$$\phi(x) = \frac{M(x)}{EI} = \frac{w_0}{6L \cdot EI}\left(L^2 x - x^3\right) \quad / \int$$

$$\frac{dv}{dx} = \frac{w_0}{6L \cdot EI}\left(L^2 \int x\,dx - \int x^3\,dx\right) + C_1$$

$$\frac{dv}{dx} = \frac{w_0}{6L \cdot EI}\left(\frac{L^2 x^2}{2} - \frac{x^4}{4}\right) + C_1 \quad / \int$$

$$v(x) = \frac{w_0}{6L\,EI}\left(\frac{L^2}{2}\int x^2\,dx - \frac{1}{4}\int x^4\,dx\right) + C_1 x + C_2$$

$$v(x) = \frac{w_0}{6LEI}\left(\frac{L^2 x^3}{6} - \frac{x^5}{20}\right) + C_1 x + C_2$$

Por condición de borde

$v(x=0) = 0$ (1) $\wedge$ $v(x=L) = 0$ (2)

apoyo simple — apoyo deslizante

De (1)

$$v(x=0) = C_2 = 0$$

De (2)

$$v(x=L) = \frac{w_0}{6LEI}\left(\frac{L^5}{6} - \frac{L^5}{20}\right) + C_1 \cdot L = 0$$

$$\frac{w_0}{6LEI}\left(\frac{7L^5}{60}\right) + C_1 \cdot L = 0$$

$$C_1 = -\frac{w_0}{6EI}\left(\frac{7L^3}{60}\right)$$

$$C_1 = \frac{-7 \cdot w_0 \cdot L^3}{360\ EI}$$

$$\Rightarrow \quad v(x) = \frac{w_0}{6L \cdot EI}\left(\frac{L^2x^3}{6} - \frac{x^5}{20}\right) - \frac{7w_0L^3}{360\ EI} \cdot x$$

$$v(x) = \frac{-w_0 \cdot x}{360L \cdot EI}\left(3x^4 - 10L^2x^2 + 7L^4\right)$$

Calculando la flecha máxima

$$\frac{dv}{dx} = 0 = \frac{w_0}{6L \cdot EI}\left(\frac{L^2x^2}{2} - \frac{x^4}{4}\right) - \frac{7\ w_0 \cdot L^3}{360\ EI} = 0$$

resolviendo

$$x^* = \frac{L \cdot \sqrt{-15(2\sqrt{30} - 15)}}{15} = 0.5193\ L$$

$$v(x=x^*) = -\frac{w_0(0.5193\cancel{L})}{360\cancel{L} \cdot EI}\left(3 \cdot (0.5193 \cdot L)^4 - 10 \cdot L^2(0.5193L)^2 + 7 \cdot L^4\right)$$

$$v_{max} = \frac{-0{,}00652\ w_0 L^4}{EI}$$

Curvatura

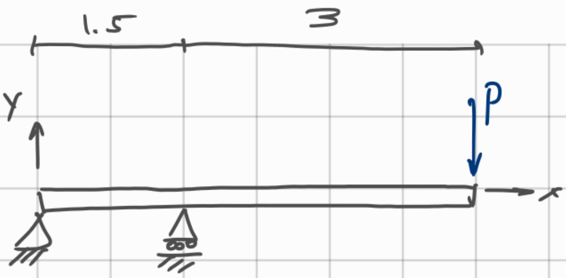
$$\phi(x) = \frac{w_o}{6LEI}\left(L^2x - x^3\right)$$

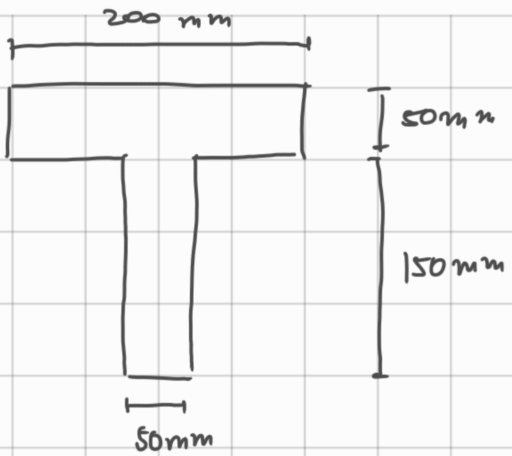
Giro

$$\theta(x) = \frac{w_o}{6L\,EI}\left(\frac{L^2x^2}{2} - \frac{x^4}{4}\right) - \frac{7\,w_o L^3}{360\,EI}$$

$$\theta(x) = \frac{-w_o}{360LEI}\left(15x^4 - 30L^2x^2 + 7L^4\right)$$

P2

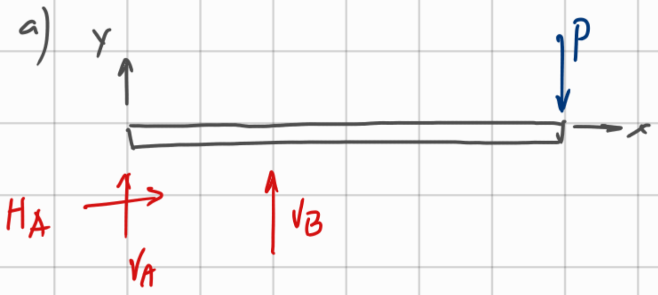




$P = 4500\ N$

$E_m = 10\ GMPa$

a)



$H_A = 0$

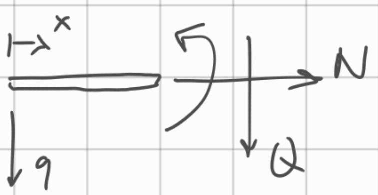
$V_A + V_B - P = 0$

$-3 \cdot P - 1{,}5 \cdot V_A = 0$

$V_A = -2P = -9\ kN$

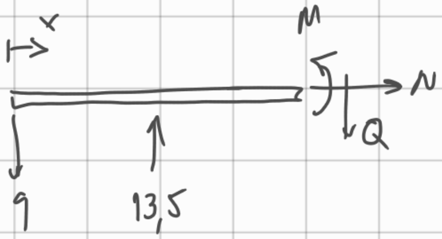
$V_B = 3P = 13{,}5\ KN$

TRAMO AB



$M_{AB}(x) = -9 \cdot x$

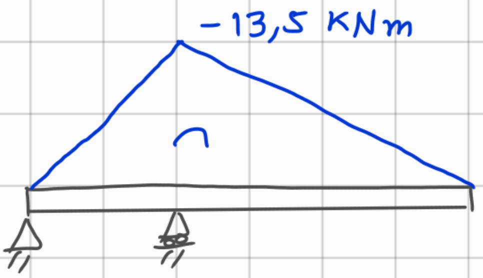
TRAMO BC



$M_{BC}(x) = 13{,}5 \cdot (x - 1{,}5) - 9 \cdot x$

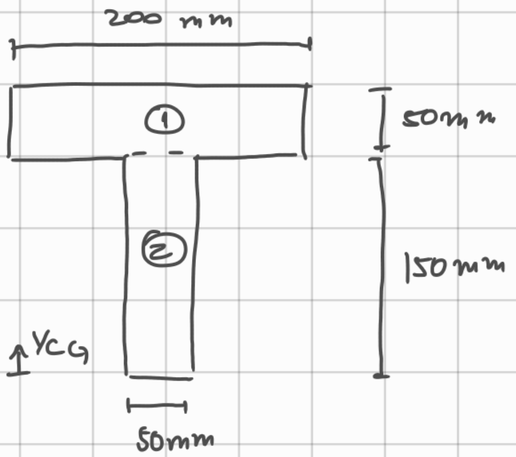
$= 13{,}5\,x - 20{,}25 - 9\,x$

$M_{BC}(x) = 4{,}5\,x - 20{,}25$



$M_{max}(x = 1{,}5m) = -13{,}5\ KNm$

Propiedades geométricas de la sección



$$A_T = A_1 + A_2$$

$$A_T = 10000\ mm^2 + 7500\ mm^2$$

$$A_T = 17500\ mm^2$$

$$y_{CG} = \frac{\sum A_i \cdot y_{CGi}}{\sum A_i} = \frac{A_1 \cdot (150+25) + A_2(75)}{A_T} = 132{,}143\ mm$$

$$I_i = \frac{b_i h_i^3}{12} + A_i (y_{CG} - y_{CGi})^2$$

$$I_1 = \frac{200 \cdot 50^3}{12} + 10000\left(132{,}143 - (150+25)\right)^2$$

$$I_1 = 2{,}045 \times 10^7\ mm^4$$

$$I_2 = \frac{50 \cdot 150^3}{12} + 7500\left(132{,}143 - \left(\frac{150}{2}\right)\right)^2$$

$$I_2 = 3{,}855 \times 10^7\ mm^4$$

$$\Rightarrow I = I_1 + I_2 = 5{,}9 \times 10^7\ mm^4$$

Tensión máxima por Navier

$$\sigma_{xx} = \frac{-M_z \cdot y}{I_{zz}} = 0$$

con $y = 67{,}857\ mm$ (superior)(1)

$y = -132{,}143\ mm$ (inferior)(2)

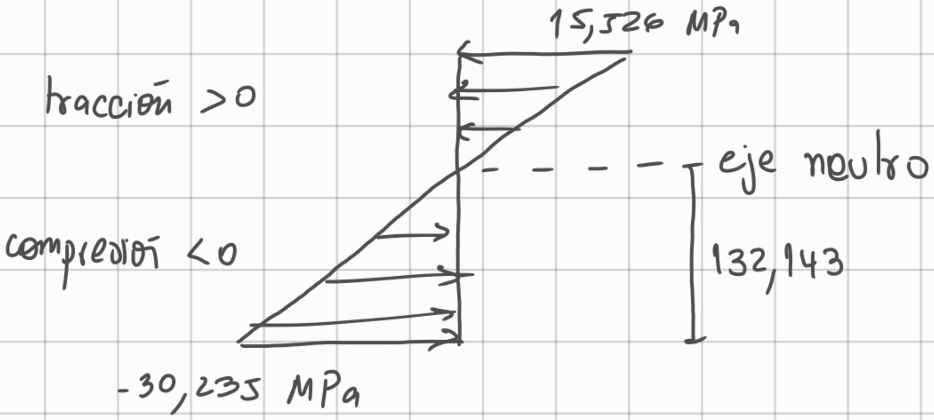
(1) $$\sigma_{x_1} = -\frac{(-13{,}5\ kNm)(67{,}857\ mm)}{5{,}9 \times 10^7\ mm^4}$$

$$\sigma_{x_1} = 15{,}526\ MPa \quad (\text{tracción})$$

para calcularlo se usa $13{,}5 \times 10^6\ N\cdot mm$

(2) $$\sigma_{x_2} = \frac{-(-13{,}5\ kNm)(-132{,}143\ mm)}{5{,}9 \times 10^7\ mm^4}$$

$$\sigma_{x_2} = -30{,}235\ MPa \quad (\text{compresión})$$



b) $\sigma_{adm\ max} = 6{,}2\ MPa$ para tracción y compresión, estime P

Como la tensión mayor se genera en compresión, se usará ese caso para encontrar la carga P.

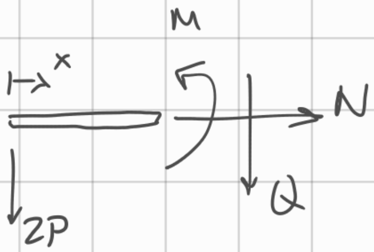
$$\sigma_x = -6{,}2\ MPa = -\frac{(M)(-132{,}143\ mm)}{5{,}9 \times 10^7\ mm^4}$$

$$M = \frac{(6{,}2\ MPa)(5{,}9 \times 10^7\ mm^4)}{(-132{,}143\ mm)}$$

$$M_{adm} = -2{,}768\ KNm$$

Sabiendo que el momento máximo ocurrirá en $x = 1,5\,m$, usando la ecuación del tramo AB

TRAMO AB



$$M_{AB} = -2P \cdot x$$

En $x = 1,5\ m$

$$\Rightarrow \quad M_{adm} = -2,768\ kNm = -2P \cdot 1,5\ m$$

$$P = 0,922\ kN$$

NOTA

$$1\ MPa = 1\ \frac{N}{mm^2} = 1 \times 10^6\ \frac{N}{m^2}$$