P3. Calcule mediante coeficientes indeterminados:

$$(D+1)^2 y = 3xe^{-x} + 2e^{-x}[cos(2x) + sen(2x)]$$

$y_H = Ae^{-x} + Bxe^{-x}$

Sol de $(D+1)^2$

Notando que podemos separar el L.D en 2

$y_{p_1} = (A_1 + A_2 x)e^{-x} x^2$, x^2 (2 grados de resonancia)

$y_{p_2} = (B_1 \cos(2x) + B_2 \sin(2x))e^{-x}$

Ojo no es resonante, porque tiene que ser igual lambida no sólo parte real

$(D+1)^2 y_{p_1} = D^2 y_{p_1} + 2D y_{p_1} + y_{p_1}$

$D y_{p_1} = D\left((A_1 x^2 + A_2 x^3)e^{-x}\right) = \left((A_1 x^2 + A_2 x^3)' - (A_1 x^2 + A_2 x^3)\right)e^{-x}$

$= \left(2A_1 x + (3A_2 - A_1)x^2 - A_2 x^3\right)e^{-x}$

De manera similar

$D^2 y_{p_1} = \left(2A_1 + 2(3A_2 - A_1)x - 3A_2 x^2 - (2A_1 x + (3A_2 - A_1)x^2 - A_2 x^3)\right)e^{-x}$

$= \left(2A_1 + (6A_2 - 5A_1)x + (A_1 - 6A_2)x^2 + A_2 x^3\right)e^{-x}$

Haciendo las 4 igualdades con $(0 + 3x + 0x^2 + 0x^3)e^{-x}$

$\Rightarrow 2A_1 = 0 \Rightarrow A_1 = 0$

$6A_2 - 5A_1 + 4A_1 = 3 \Rightarrow A_2 = \frac{1}{2} \Rightarrow \boxed{y_{p_1} = \frac{x^3 e^{-x}}{2}}$

$(A_1 - 6A_2 + 6A_2 - 2A_1 + A_1) = 0$

$A_2 - 2A_2 + A_2 = 0$

$$y_{p_2} = \left(B_1 \cos(2x) + B_2 \sin(2x)\right) e^{-x}$$

$$D y_{p_2} = \left((2B_2 - B_1)\cos(2x) + (-2B_1 - B_2)\sin(2x)\right) e^{-x}$$

$$D^2 y_{p_2} = \Big((B_1 - 2B_2 - 4B_1 - 2B_2)\cos(2x) +$$
$$(2B_1 - 4B_2 + 2B_1 + B_2)\sin(2x)\Big) e^{-x}$$

$$= \left((-3B_1 - 4B_2)\cos(2x) + (4B_1 - 3B_2)\sin(2x)\right) e^{-x}$$

$$\boxed{\Rightarrow} \; (-3B_1 - \cancel{4B_2} + \cancel{4B_2} - 2B_1 + B_1 = 2$$
$$-4B_1 = 2 \Rightarrow \boxed{B_1 = -\tfrac{1}{2}}$$

$$\cancel{4B_1} - 3B_2 - \cancel{4B_1} - 2B_2 + B_2 = 2$$
$$-4B_2 = 2 \Rightarrow \boxed{B_2 = -\tfrac{1}{2}}$$

$$\Rightarrow y_{p_2} = \frac{-(\cos(2x) + \sin(2x))e^{-x}}{2}$$

$$y_p = \frac{1}{2} e^{-x}\left(x^3 - (\cos(2x) + \sin(2x))\right)$$