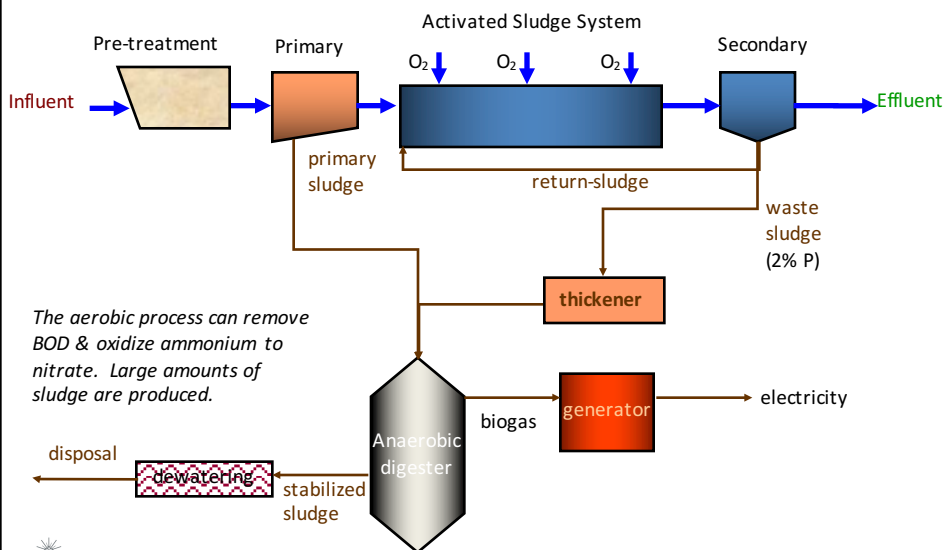


Tema 7 – Lodos Activados Activated Sludge

CI7115 – Biotecnología Ambiental
Prof. Ana Lucía Prieto Santa



Tratamiento Convencional para ARes Municipal



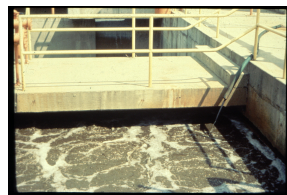
Lodos Activados

Aerobic dispersed growth treatment
Largest application of biotechnology in the world



Empty basin showing aeration manifold

Michigan, USA



Mixed liquor under
aeration



Hyperion WWTP,
Playa Del Rey, CA

For aquaculture



Consideraciones

- Nutrient and oxygen requirements
- Oxygen transfer and mixing
 - Diffused aeration/mixing vs mechanical aeration/mixing
- θ_x - Solids retention time (sludge age)
- Sludge settling and bulking
 - Recycle Ratio ($R = Q^r / Q^o$) (range: 0-3, typically <1)
 - Sludge Volume Index (SVI)
 - Volume occupied by one gram of SS after settling for 30 min (ml/gSS)
 - <50 (very good settling), 100 (typical), >200 (poor settling, bulking)

Consideraciones

- **Volumetric loading**

kg BOD or COD applied per day per volume of aeration tank (i.e., mass loading normalized to reactor volume) (e.g., kg BOD₅/m³-d)

$$\text{Volumetric loading} = Q^\circ S^\circ / V$$

- **Food-to-microorganism ratio (F/M)**

kg BOD or COD applied per day per kg of suspended solids in the aeration tank (e.g., kg BOD₅/kg X_v-d)

$$F/M = Q^\circ S^\circ / VX \text{ (for TSS)}$$

$$F/M_v = Q^\circ S^\circ / VX_v \text{ (for VSS)}$$

See pp 324-326 in text

Typical range: 0.25 to 0.5 kg BOD₅/kg X_v-d

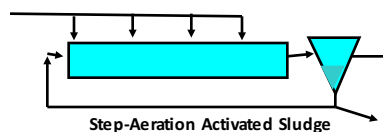


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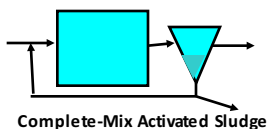
Activated sludge process configurations



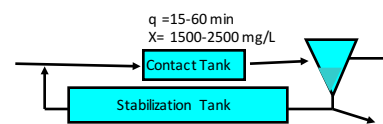
*Advantage: High efficiency,
Disadvantages: uneven O₂ demand, S_b >> S_e,
perhaps causing toxicity*



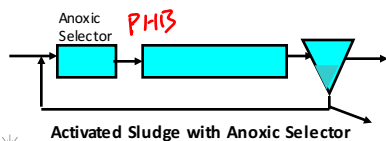
Advantages: reduces max concentration in tank, evens out O₂ demand



*Advantage: S << S^o for toxic wastes
Disadvantage: reduced efficiency, selects for poorly settling biomass*



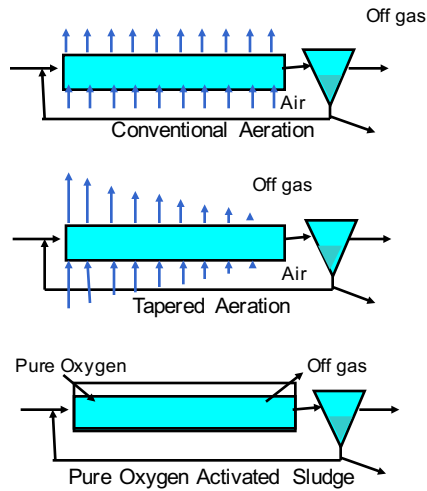
Advantage: Reduced tank volume for a given q_x



Advantage: High efficiency, selects flocculent biomass that settles well



Activated sludge modifications to aeration



- Pure Oxygen Activated Sludge**
- Results in biomass that settles well.
 - Economics have recently become more favorable.
 - pH control becomes more important because P_{CO_2} is high in the headspace

Ex: Howard F. Curren Advanced Wastewater Treatment Plant
<https://www.tampagov.net/wastewater/info/advanced-wastewater-treatment-plant>

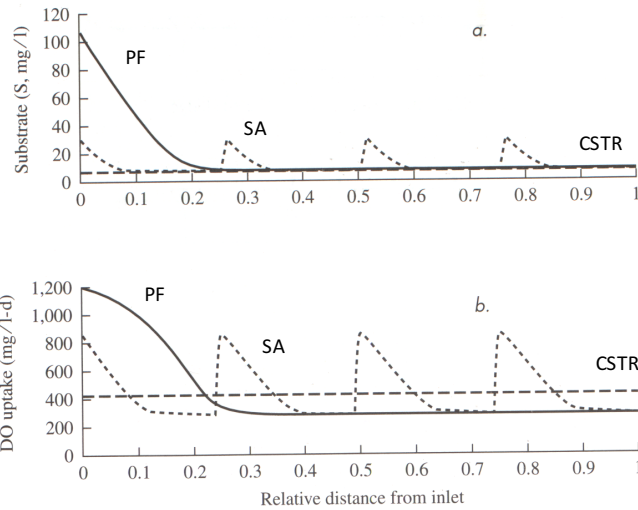


Figure 6.2 Changes in contaminant (substrate) concentration and oxygen (DO) uptake rate along the reactor length for plug flow (PF, solid lines), step aeration (SA, small-dash lines), and continuous-stirred tank (CSTR, large-dash lines) reactors for a typical loading with a dilute wastewater.



Mean Cell residence time (sludge age) and loading rates for different activated sludge systems

Process	Typical θ_x^d days	Safety factor	Volumetric loading kg BOD ₅ /m ³ -d	F/M kg BOD ₅ /kg X _v -d	Typical BOD ₅ removal efficiency
Extended aeration	>14	>70	0.3	0.05-0.2	85-95
Conventional	4-14	20-70	0.6	0.2-0.5	95
Tapered Aeration	4-14	20-70	0.6	0.2-0.5	95
Step Aeration	4-14	20-70	0.8	0.2-0.5	95
Contact Stabilization	4-14	20-70	1.0	0.2-0.5	90
Modified Aeration	0.8-4	4-20	1.5-6	0.5-3.5	60-85

Check table 6.2 of your book for more details



Soluble Microbial Products (SMP)

Definition: dissolved substances resulting from

- cell lysis
- leaking cell membranes
- excretion

Molecular weight range: 100s to 1000s

Significance

- metal complexation (ex: siderophores)
- cell-cell signaling (ex: homoserine lactones)
- ecological warfare (ex: antibiotics, toxins)
- fouling of membranes
- color
- foaming (ex: biosurfactants)



$$SMP = UAP + BAP$$

Utilization Associated Products (UAP) = released during growth

k_1 in units of (COD_p/COD_s), is a fraction of substrate that becomes UAP

Specific rate of formation = k_1q

$$\text{Specific rate of degradation} = q_{UAP} = \frac{\hat{q}_{UAP}UAP}{K_{UAP}+UAP}$$

Biomass Associated Products (BAP) = released during decay

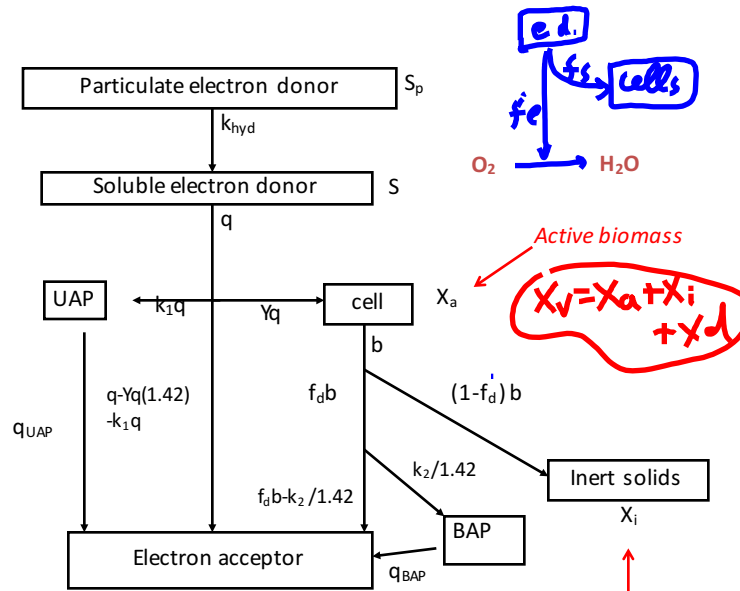
k_2 has units of (gCOD_p/gVSS -d), need to divide by 1.42 gCOD/gVSS to result in same units as b (1/d)

Specific rate of formation = k_2

$$\text{Specific rate of degradation} = q_{BAP} = \frac{\hat{q}_{BAP}BAP}{K_{BAP}+BAP}$$



Adding structure: accounting for more of what we see (p. 176-178 in text)



$$SMP = UAP + BAP$$



Parameter estimates - see p. 178 in text

UAP product (UAP)
 Aerobic
 $k_1 = 0.12 \text{ g COD}_p/\text{gCOD}_s$
 $q_{UAP} = 1.8 \text{ g COD}_p/\text{g vss-d}$
 $K_{UAP} = 100 \text{ mg/L}$
 substrate

$$\frac{\hat{q}_{UAP}}{K_{UAP}} = 18 \text{ L/g vss-d}$$

BAP product (BAP)
 Aerobic
 $k_2 = 0.09 \text{ gCOD}_p/\text{g vss-d}$
 $\hat{q}_{BAP} = 0.1 \text{ g COD}_p/\text{g vss-d}$
 $K_{BAP} = 85 \text{ mg/L}$
 biomass

$$\frac{q_{BAP}}{K_{BAP}} = 1.2 \text{ L/g vss-d}$$

Methanogenic
 $k_1 = 0.21 \text{ g COD}_p/\text{gCOD}_s$

$$\frac{\hat{q}_{UAP}}{K_{UAP}} = 2.4 \text{ L/g vss-d}$$

Methanogenic
 $k_2 = 0.035 \text{ g COD}_p/\text{g vss-d}$

$$\frac{\hat{q}_{BAP}}{K_{BAP}} = 0.31 \text{ L/g vss-d}$$



Key points

- Significant fraction of COD becomes released as UAP (12% for aerobes and 21% for methanogen) rather than EA or biomass
- Formation of BAP is a significant fraction of biomass loss (b)
- UAP is degraded faster than BAP (18:1.2 for aerobe and 2.4:0.31 for methanogen), resulting in accumulation of BAP.



$$\text{SMP} = \text{UAP} + \text{BAP}$$

Utilization Associated Products (UAP) = released during growth

k_1 in units of $(\text{COD}_p/\text{COD}_s)$, is a fraction of substrate that becomes UAP

Specific rate of formation = $k_1 q$

$$\text{Specific rate of degradation} = q_{\text{UAP}} = \frac{\hat{q}_{\text{UAP}} \text{UAP}}{K_{\text{UAP}} + \text{UAP}}$$

Biomass Associated Products (BAP) = released during decay

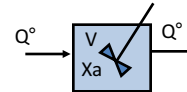
k_2 has units of $(\text{gCOD}_p/\text{gVSS} \cdot \text{d})$, need to divide by 1.42 gCOD/gVSS to result in same units as b (1/d)

Specific rate of formation = k_2

$$\text{Specific rate of degradation} = q_{\text{BAP}} = \frac{\hat{q}_{\text{BAP}} \text{BAP}}{K_{\text{BAP}} + \text{BAP}}$$



Steady state CSTR mass balances on UAP and BAP



	source	sink	out
<u>UAP</u>	$0 = k_1 q X_a V$	$- \frac{\hat{q}_{\text{UAP}} (\text{UAP}) X_a V}{K_{\text{UAP}} + \text{UAP}}$	$- Q^\circ (\text{UAP})$

<u>BAP</u>	$0 = k_2 X_a V$	$- \frac{\hat{q}_{\text{BAP}} (\text{BAP}) X_a V}{K_{\text{BAP}} + \text{BAP}}$	$- Q^\circ (\text{BAP})$
------------	-----------------	---------------------------------------------------------------------------------	--------------------------

The above equations can be solved for UAP and BAP.
Solutions are available in the text (ch. 8)



Ecuaciones de SMP para el cálculo de UAP and BAP

$$\text{SMP} = \text{UAP} + \text{BAP}$$

$$\text{UAP} = \frac{-(\hat{q}_{\text{UAP}}X_a\theta + K_{\text{UAP}} - K_1qX_a\theta)}{2} + \sqrt{\frac{(\hat{q}_{\text{UAP}}X_a\theta + K_{\text{UAP}} - K_1qX_a\theta)^2 + 4K_{\text{UAP}}K_1qX_a\theta}{2}}$$

$$\text{BAP} = \frac{-[K_{\text{BAP}} + (\hat{q}_{\text{BAP}}X_a - k_{\text{hyd}}(\text{EPS})\theta)]}{2} + \sqrt{\frac{-[K_{\text{BAP}} + (\hat{q}_{\text{BAP}}X_a - k_{\text{hyd}}(\text{EPS})\theta)]^2 + 4K_{\text{BAP}}k_{\text{hyd}}(\text{EPS})\theta}{2}}$$



Parámetros SMP

Information to define the SMP kinetic parameters is sparse. Noguera (1991) analyzed aerobic data and obtained the following best-fit values:

$$\begin{aligned} k_1 &= 0.12 \text{ g COD}_p/\text{g COD}_s \\ k_2 &= 0.09 \text{ g COD}_p/\text{g VSS}_a\text{-d} \\ \hat{q}_{\text{UAP}} &= 1.8 \text{ g COD}_p/\text{g VSS}_a\text{-d} \\ K_{\text{UAP}} &= 100 \text{ mg COD}_p/\text{l} \\ \hat{q}_{\text{UAP}}/K_{\text{UAP}} &= 18 \text{ l/g VSS}_a\text{-d} \\ \hat{q}_{\text{BAP}} &= 0.1 \text{ g COD}_p/\text{g VSS}_a\text{-d} \\ K_{\text{BAP}} &= 85 \text{ mg COD}_p/\text{l} \\ \hat{q}_{\text{BAP}}/K_{\text{BAP}} &= 1.2 \text{ l/g VSS}_a\text{-d} \end{aligned}$$

While these values can be considered only provisional, they do point out several key features of SMP. First, a significant fraction of substrate COD is shunted to UAP formation; $k_1 = 0.12 \text{ g COD}_p/\text{mg COD}_s$ means that 12 percent of the substrate "utilized" is neither sent to the electron acceptor nor converted to biomass, but is released as UAP. Second, the formation of BAP constitutes a significant fraction of biomass loss; $k_2 = 0.09 \text{ g COD}_p/\text{g VSS}_a\text{-d}$ converts to a first-order "decay" rate of approximately 0.06/d. Third, UAP is biodegraded considerably faster than is BAP; thus, we should expect to see preferential buildup of BAP in most situations.

Noguera et al. (1994) estimated SMP parameters for a methanogenic system:

$$\begin{aligned} k_1 &= 0.21 \text{ g COD}_p/\text{g COD}_s \\ k_2 &= 0.035 \text{ g COD}_p/\text{g VSS}_a\text{-d} \\ \hat{q}_{\text{UAP}}/K_{\text{UAP}} &= 2.4 \text{ l/g VSS}_a\text{-d} \\ \hat{q}_{\text{BAP}}/K_{\text{BAP}} &= 0.31 \text{ l/g VSS}_a\text{-d} \end{aligned}$$

Most notable is the relatively slower degradation kinetics, compared to the aerobic system.

Since k_2 can be a significant fraction of b , we shall assume that b implicitly includes BAP generation, as well as biomass oxidation and conversion to inert biomass. By differences, the portion of endogenous decay that results in direct biomass oxidation is $b - f_d b - k_2 = (1 - f_d)b - k_2$.



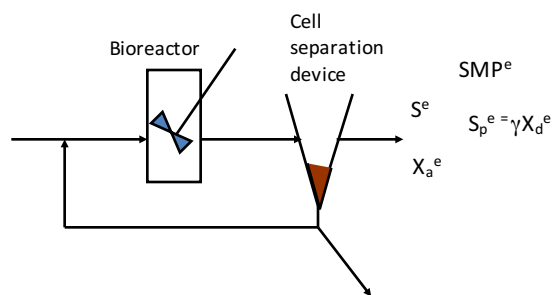
Table 8.1 Estimates for EPS- and SMP-Related Production and Consumption Parameters

Parameter Name	Parameter Symbol	Units	Typical Value	All Microbial Types or Only Heterotrophs
EPS-formation fraction	k_{EPS}	g CODE ^{EPS} /g COD ^S	0.18	All
UAP-formation fraction	k^1	g COD ^{UAP} /g COD ^S	0.05	All
Net biomass-formation fraction	$1 - k^{EPS} - k^1$	g COD ^X /g COD ^S	0.77	All
EPS-hydrolysis rate	k_{hyd}	1/d	0.17	Heterotrophs
UAP maximum specific utilization rate	q_{UAP}	g COD ^{UAP} /g COD ^X -d	1.3	Heterotrophs
BAP maximum specific utilization rate	q_{BAP}	g COD ^{BAP} /g COD ^X -d	0.35	Heterotrophs
UAP half-maximum-rate concentration	K_{UAP}	g COD ^{UAP} /l	0.1	Heterotrophs
BAP half-maximum-rate concentration	K_{BAP}	g COD ^{BAP} /l	0.085	Heterotrophs

Table 8.2 Estimates for SMP- and EPS-Affected Parameter Values for Key Types of Microorganisms

Parameters and Units	Aerobic Heterotrophs	Denitrifying Heterotrophs	Denitrifying H ₂ -Oxidizing Autotrophs	Ammonium-Oxidizing Autotrophs	Acetate-Cleaving Methanogens
Y_r , g COD ^{Xa} /g COD ^S	0.64	0.50	0.20	0.14	0.04
Y_r' , g COD ^{Xa} /g COD ^S	0.49	0.39	0.15	0.11	0.031
$\hat{q}q^A$, g COD ^S /g COD ^X -d	7	7	5	5.6	8
$\hat{\mu}$, 1/d	4.5	3.5	1.0	0.78	0.32
$\hat{\mu}'$, 1/d	3.4	2.7	0.77	0.68	0.25
b , 1/d	0.3	0.15	0.05	0.05	0.03
K , mg COD ^S /l	10	10	0.6	0.5	30
$[\theta_{minx}]lim[\theta_{xmi}n]lim'$, d	0.32	0.39	1.4	1.8	4.5
S'_{min} , mg COD ^S /l	0.96	0.59	0.04	0.045	4.4

Cálculo de la BOD_L efluente



Soluble BOD_L = S^e + SMP^e (assumes all biodegradable SMP)

$$\text{Total BOD}_L = \text{Soluble BOD}_L + 1.42 f_d X_a^e + S_p^e$$

BOD_L from particulate fraction
Note: 1.42 g COD/g VSS

Typically, $S < \text{SMP}^e < 1.42 f_d X_a^e$

Rough estimate of effluent BOD₅^e

$$\text{BOD}_t^e = S^e(1 - e^{-kt}) + \text{SMP}^e(1 - e^{-k_{\text{SMP}}t}) + 1.42f_d X_a^e(1 - e^{-bt}) + S_p^e(1 - e^{-k_{\text{hyd}}t})$$

when $t = 5$ days, $\text{BOD}_t^e = \text{BOD}_5^e$

Refer to Example 3.3 in the text

O₂ UTILIZATION RATE ADJUSTED FOR SMP

$$\text{O}_2 \text{ utilization rate} = Q^{\circ}(S^{\circ}_{\text{eff}} - S) - Q^{\circ}(\text{SMP}) - 1.42(\text{cell production rate})$$

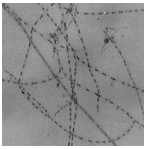
$$= Q^{\circ}(S^{\circ}_{\text{eff}} - S) f_e - Q^{\circ}(\text{SMP})$$

$$= \Sigma \text{O}_2 \text{ equivalent input} - \Sigma \text{O}_2 \text{ equivalent output (biomass+SMP)}$$


Note: the value of S_p^e depends upon the removal efficiency of the separation device. $S_p^e = \gamma (X_d/X_v) X_v^e$, where X_v^e is obtained from a knowledge of clarifier performance and $X_d = (\theta_x^d/\theta) [X_d^{\circ}/(1 + k_{\text{hyd}}\theta_x^d)]$. SMP values are calculated using equations 3.38-3.40 in the text.



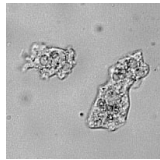
Ecología de los lodos activados



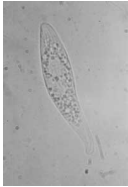
bacterial filaments



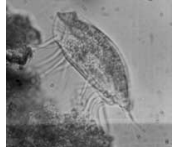
flagellates




amoeba




free-swimming ciliates




crawling ciliates




attached ciliates



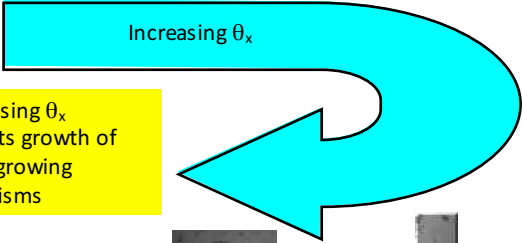
aquatic worms



nematodes




rotifers




Increasing θ_x

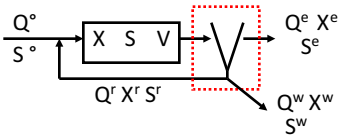
Increasing θ_x permits growth of slow-growing organisms





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Tanques de Sedimentación



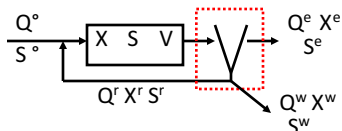






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Tanques de Sedimentación



Solids balance around clarifier.

$$(Q^{\circ} + Q^r)X = Q^r X^r + Q^e X^e$$

$$X = \frac{Q^r X^r}{Q^{\circ} + Q^r} = \frac{X^r}{\frac{Q^{\circ}}{Q^r} + 1}$$

$$R = \frac{Q^r}{Q^{\circ}} \text{ so that } X = \frac{X^r}{\frac{1}{R} + 1}$$

		X
R	X/X ^r	Normal compaction (X ^r = 10,000 mg/L)
0.1	0.09	900
0.25	0.2	2,000
0.5	0.33	3,300
1	0.5	5,000
2	0.67	6,700

Settling tank criteria – X^r_{max}

Normal – 10,000 mg/L (1%)

Good – 20,000 mg/L (2%)

Poor < 5,000 mg/L (0.5%)

As R increases X approaches X^r, R must increase if compaction is bad to maintain adequate solids in the reactor.



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Problemas con la separación de biosólidos lodos

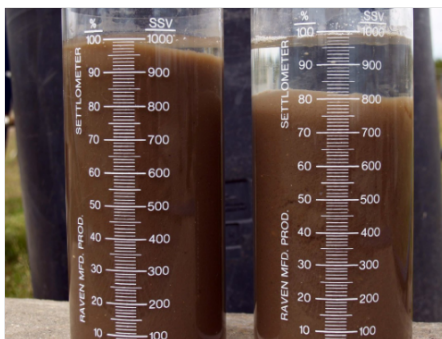
Biosolids Separation Problem	Cause of Problem	Effect of Problem
Bulking	Filamentous organisms extend from flocs into the bulk solution and interfere with compaction and settling	High SVI with clear supernatant. Overflow of sludge blanket can occur. Solids handling processes become hydraulically overloaded
Viscous bulking or nonfilamentous bulking	Microorganisms present in large amounts of exocellular slime. In severe cases, the slime imparts a jelly-like consistency	Reduced settling and compaction rates. Can result in overflow of sludge blanket from secondary clarifier or formation of a viscous foam
Dispersed growth	Microorganisms do not form flocs, but are dispersed, forming only small clumps or single cells	Turbid effluent. No zone settling of sludge
Pin floc or pinpoint floc	Small, compact, weak, roughly spherical flocs. Larger aggregates settle rapidly, smaller ones slowly	Low SVI and cloudy turbid effluent
Foaming/Scum formation	Caused by (i) nondegradable surfactants or (ii) the presence of <i>Nocardia</i> sp. and/or <i>Microthrix parvicella</i>	Foams float large amounts of biosolids to surface of treatment units. Causes solids overflow into secondary effluent and onto walkways. Anaerobic digester foaming also can result
Rising sludge	Denitrification in the settler releases poorly soluble N ₂ gas which attaches to activated sludge flocs and floats them to the clarifier surface	"Chunks" of activated sludge collect on the surface of the settler and may result in turbid effluent

Source: Jenkins (1992), Jenkins et al. (2004), Wanner and Grau (1989).

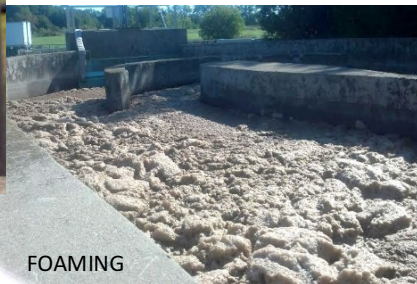


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Problemas con la separación de biosólidos lodos



SVI TEST



FOAMING

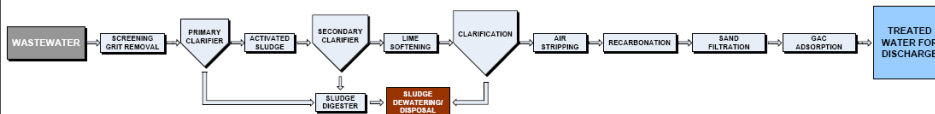


BULKING

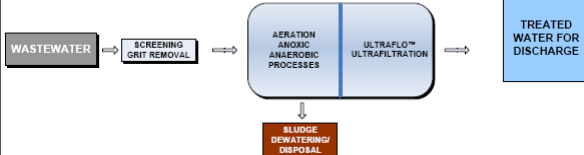
27

Reactores de Membrana o Membrane Bioreactors (MBRs)

CONVENTIONAL ACTIVATED SLUDGE PROCESS



MEMBRANE BIOREACTOR



- Particle-free effluent
- Absolute barrier for retention of biomass
- Decoupling of HRT and SRT
- Remote-monitoring and control appropriate

But...
Energy + maintenance

Comparison between CAS and MBR space and process requirement (adapted from Ultra-Flo,2007)

Ventajas de los MBR

- Particle free effluent
 - Removal of pathogenic protozoa, bacteria, viruses

- Higher biomass concentration (5 to 10 x)
 - More compact
 - May permit direct anaerobic treatment of dilute waste (e.g., sewage) at cooler temperatures

- Solids separation
 - Independent from sludge settling
 - Reduced operator attention: remote monitoring and control may be possible (distributed systems?)



fcfm

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Tecnología compacta de lodos activados

- Largely developed in Japan in the 1990's
 - Used all over the world in municipal and industrial applications
- <https://www.thembrsite.com/largest-membrane-bioreactor-plants-worldwide/>

Two major types:

- submerged
- crossflow

Membrane flux ranges from 5-300 L/m²-h, depending on:

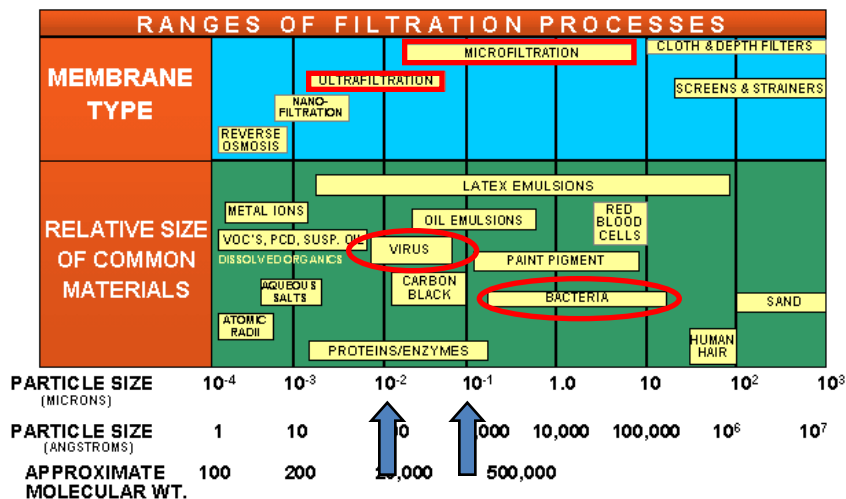
- membrane material (surface characteristics, pore size)
- extent of fouling
- transmembrane pressure
- mode of operation
- module arrangement



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Membrane separation



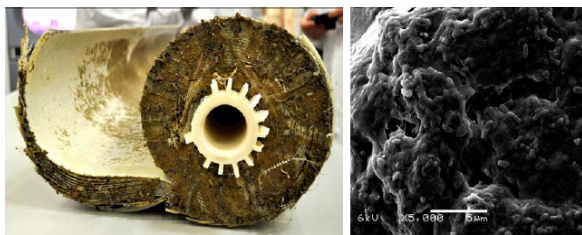
CI7115 - Biotecnología Ambiental

MBR concerns

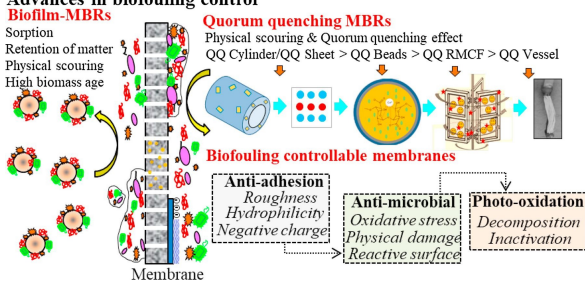
The major concern is biofouling

Strategies to combat biofouling:

- periodic cleaning of membranes
- backwash
- highly turbulent aeration
- high energy needed to shear biomass (2-10 kWh·m⁻³) for crossflow applications, but is reportedly 10 times lower for submerged membranes.



Advances in biofouling control



Aslam et al., 2018, Recent developments in biofouling control in membrane bioreactors for domestic wastewater treatment, Separation and Purification Technology, 206, 297-315,

U.S. Filter membranes



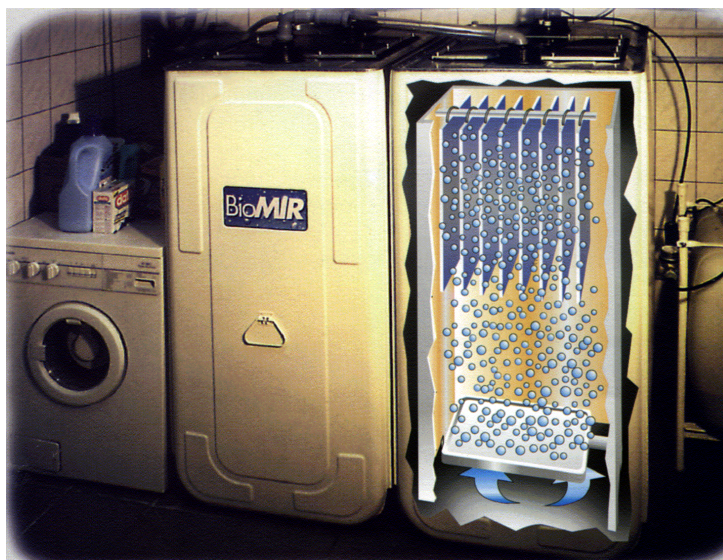
Source: <http://www.usfilter.com>

U.S. Hollow Fiber membranes



Source: <http://www.zenon.com/>

Submerged plate and frame membrane modules
(KUBOTA, license VA Tech WABAG, Germany)



**Design Example 1: Activated Sludge
(Example 6.2 in the text, pages 347-352):**

1. Determine waste characteristics and effluent requirements:

$Q^0 = 1000 \text{ m}^3/\text{d}$
 $S^0 = 500 \text{ mg/L}$
 $S_p^0 = 0$ (not typical of sewage)
 $X_i^0 = 50 \text{ mg/L}$
 $X_{in}^0 = 20 \text{ mg/L}$
 $S_{e_{max}} = 20 \text{ mg/L}$
 Nonbiodegradable COD = 0

2. Select coefficients and design factors.

$Y = 0.4 \text{ mg vss/mg BOD}_L$
 $\hat{q} = 10 \text{ mg BOD}_L/\text{mg vs-d}$
 $b = 0.1 \text{ d}^{-1}$
 $K = 10 \text{ mg BOD}_L/\text{d}$
 $f_d = 0.8$
 $S.F. = 20$
 X_v (if recycle) = 2500 mg/L
 $X_c^e = 15 \text{ mg/L}$ (expected performance of clarifier)

3. Calculate $[\theta_x^{min}]_{m} = 1/[(0.4)(10)-0.1] = 0.26 \text{ d}$

4. Select $\theta_x^d = S.F. \cdot [\theta_x^m]_{m} = (20)(0.26) = 5.2 \text{ days}$

5. Solve $S^e = \frac{K(1+b\theta_x^d)}{\theta_x^d(Y\hat{q}-b)-1} = \frac{10[1+(0.1)(5.2)]}{5[(0.4)(10)-0.1]-1} = 0.81 \text{ mg/L}$

$S_{e_{max}} = 20 \text{ mg/L}$ which is $\gg 0.81$, so we are OK.

6. Use the expression:

$$X_v = \frac{\theta_x}{\theta} \left[X_i^o + \frac{X_d^o}{1 + k_{mg}\theta_x} + \frac{Y(S_{eff}^o - S^e)(1 + 0.2b\theta_x)}{1 + b\theta_x} \right] =$$

$$\frac{\theta_x}{\theta} [X_i^o + X_a + X_{i, syn}] = \frac{\theta_x}{\theta} [X_v]$$

Activated sludge systems are designed for recycle, so we rearrange the above equation to solve for θ using a design value for X_v of 2,500 mg/L. This value is chosen based on the expected settleability of the activated sludge.

In this example, $X_d^o = 0$, so the second term inside the parenthesis drops out, and $S_{eff}^o = S^o$:

$$\begin{aligned} \theta &= \frac{\theta_x}{X_v} \left[X_i^o + \frac{Y(S_{eff}^o - S^e)(1 + 0.2b\theta_x)}{1 + b\theta_x} \right] = \frac{5}{2500} \left[50 + \frac{0.4(500 - 0.8)(1 + 0.2(0.1)(5))}{1 + (0.1)(5)} \right] \\ &= \frac{5}{2500} [50 + 133 + 13] = \frac{5}{2500} [196] = 0.39 \text{ d} = 9.4 \text{ h} \end{aligned}$$

7. Solve $V=Q^o\theta$

$$V = (1000 \text{ m}^3/\text{d})(0.39 \text{ d}) = 390 \text{ m}^3$$

8. Solve for waste sludge production and biomass production rate:

$$X_v \text{ production rate} = \frac{X_v V}{\theta_x^d} = \left(\frac{(2500 \text{ g/m}^3)(390 \text{ m}^3)}{5 \text{ d}} \right) \left(\frac{\text{kg}}{1000 \text{ g}} \right) = 196 \text{ kg/d}$$

$$\text{Alternatively, } X_v \text{ production rate} = Q \bar{X}_v = (1000 \text{ m}^3/\text{d})(196 \text{ g/m}^3)(\text{kg}/1000 \text{ g}) = 196 \text{ kg/d}$$

The wasting rate of volatile suspended solids is obtained by subtracting out solids that are accidentally lost from the system in the clarifier effluent ($=Q^o X_e^o$)

$$\text{Wasting rate} = \text{production rate} - \text{accidental loss rate} = 196 - (1000)(15)/1000 = 181 \text{ kg vss/d}$$

8. Continued

X_{in} removal rate = rate at which X_{in} enters system = $Q^\circ X_{in} = (1000)(20)/1000 = 20$ kg ss/d

Some X_{in} is also generated as X_v is produced because MLVSS is 90% organic and 10% inorganic (ash). For that ratio, the additional inorganic solids produced = $(196)(10/90) = 22$ kg/d.

Suspended solids (organic + inorganic) production rate = $196 + 20 + 22 = 238$ kg/d

What is the biological suspended solids production rate?

$$\overline{X}_{syn} = \overline{X}_a + \overline{X}_{i,syn} = 133 + 13 = 146 \text{ g/m}^3$$

Biological solids production rate = $Q \overline{X}_{syn} = (1000 \text{ m}^3/\text{d})(146 \text{ g/m}^3)(\text{kg}/1000 \text{ g}) = 146$ kg/d



9. Stoichiometry and materials balance:

Substrate removal rate = $Q^\circ(S^\circ - S) = 1000(500 - 0.81)/1000 = 499$ kg BOD_L/d

Volumetric BOD_L removal rate = $499/390 = 1.28$ kg BOD_L/m³-d

N requirement = $0.124 \times$ Cell production rate = $(0.124)(146) = 18$ kg N/d

P requirement = $0.025 \times$ Cell production rate = $(0.025)(146) = 3.6$ kg P/d

10. Computation of SMP - use equation 3.38 and 3.39 and coefficients in the book

UAP = $f(\text{volumetric BOD}_L \text{ removal rate}, X_a, \theta) = 4.8$ mg/L

BAP = $f(X_a, \theta) = 39$ mg/L

SMP = $4.8 + 39 = 43.8$ mg COD/L



11. Estimate effluent quality:How much COD is in the effluent?

$$\text{Effluent COD} = S_e + (1.42)(X_v^e) + \text{SMP} = 0.8 + 21.3 + 44 = 66 \text{ mg COD/L}$$

How much BOD_L is in the effluent?

$$\text{Assume } X_a^e = X_v^e (X_a / X_v) = 15(133/196) = 10.2 \text{ mg vss/L}$$

Since $f_d = 0.8$,

$$\text{Effluent BOD}_L = S^e + (1.42)(0.8)(X_a^e) + \text{SMP} = 0.8 + 11.6 + 44 = 56 \text{ mg BOD}_L/\text{L}$$

How much BOD₅ is in the effluent?

For this we need to know the first order decay rates for the BOD test, the decay rates of the cells, and the decay rate of SMP.

$$\text{Assume } k = 0.23 \text{ d}^{-1}$$

$$b = 0.1 \text{ d}^{-1}$$

$$b_{\text{smp}} = 0.03 \text{ d}^{-1}$$

$$\text{BOD}_5^e = S^e(1 - e^{-5k}) + f_d(1.42)X_a^e(1 - e^{-5b}) + \text{SMP}(1 - e^{-5k_{\text{smp}}}) =$$

$$S^e(0.68) + 0.8(1.42)X_a^e(0.4) + \text{SMP}(0.14) = 0.5 + 4.6 + 6.1 = 11.2 \text{ mg/L}$$

12. Estimate O₂ use rate:Oxygen equivalents entering system:

$$\text{Substrate: } Q^{\circ}S^{\circ} = (1000)(500)/1000 = 500 \text{ kg/d}$$

$$\text{VSS: } 1.42 Q^{\circ}X_i^{\circ} = (1.42)(1000)(50)/1000 = 71 \text{ kg/d}$$

(Note: Since X_i does not exert an O₂ demand, it could be omitted from the oxygen equivalents balance, but if X_i is not separated out in the computation of oxygen equivalents leaving the system, then it must be included here and multiplied by 1.42, so that it cancels out X_i leaving the system that is of nonbiological origin).

$$\text{Sum: } 500 + 71 = 571 \text{ kg/d}$$

Oxygen equivalents leaving system:

$$\text{Substrate: } Q^{\circ}S^e = (1000)(0.81)/1000 = 0.8 \text{ kg/d}$$

$$\text{SMP: } Q^{\circ}\text{SMP}^{\circ} = (1000)(43.8)/1000 = 43.8 \text{ kg/d}$$

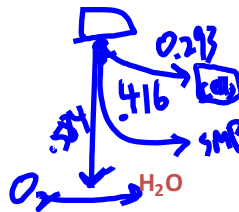
VSS discharged in effluent or wasted = $(1.42)(196) = 278 \text{ kg/d}$ (Note that this value includes X_i° that has passed through the system undegraded)

$$\text{Sum: } 0.8 + 43.8 + 278 = 323 \text{ kg/d}$$

O₂ uptake rate = Oxygen equivalents entering system - Oxygen equivalents leaving system

$$\text{O}_2 \text{ uptake rate} = 571 - 323 = 248 \text{ kg O}_2/\text{d}$$

Alternative method:



$$Y_{\text{eff}} = \frac{Y(1 + 0.2b\theta_x)}{1 + b\theta_x} = \frac{(0.4)(1 + 0.2(0.1)(15))}{1 + (0.1)(15)} = 0.293 \text{ g vss/g BOD}_L$$

$$f_s = 1.42Y = (0.293)(1.42) = 0.416$$

$$f_e = 1 - f_s = 1 - 0.41 = 0.584$$

$$\text{O}_2 \text{ use if there were no SMP in the effluent} = (0.584)(499) = 291 \text{ kg/d}$$

$$\text{O}_2 \text{ use corrected for SMP in the effluent} = 291 - 43.8 = 248 \text{ kg/d}$$