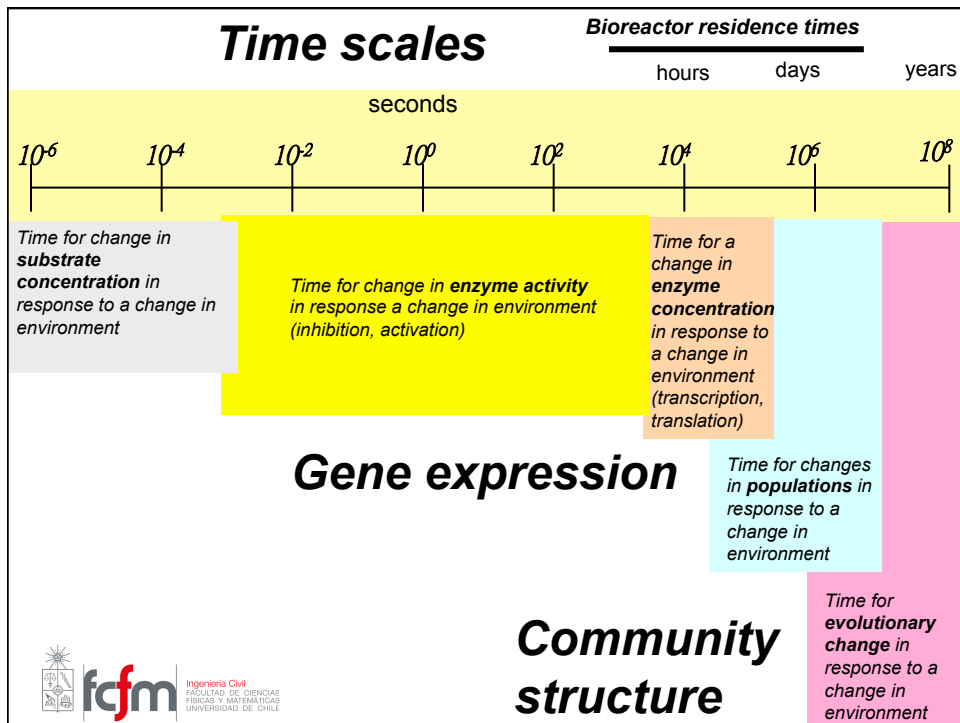


# Tema 5 – Cinética Microbiana

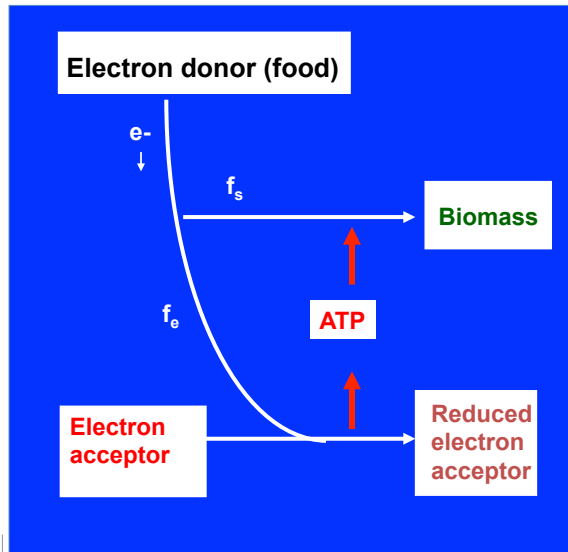
## Microbial Kinetics

CI7115 – Biotecnología Ambiental  
 Prof. Ana Lucía Prieto Santa



# Metabolismo

What microorganisms do



# Metabolismo

Includes many hazardous chemicals, such as lightly chlorinated molecules, aromatics

**Electron donor (organics,  $H_2$ ,  $NH_3$ )**

**Biomass**

**Electron acceptor**

**Reduced electron acceptor**

$O_2$  **Aerobes**  $H_2O$

$NO_3^-$  **Denitrifiers**  $N_2$

$SO_4^{2-}$  **Sulfate-reducers**  $H_2S$

$CO_2$  **Methanogens**  $CH_4$

**Highly chlorinated molecules**

**Dehalorespirers** **Lightly chlorinated molecules**



## Reacciones cinéticas simples

### Growth:

X = cells

$$r = dX/dt = k \quad X(t) = X_0 + kt \text{ (zero order)}$$

$$r = dX/dt = kX \quad X(t) = X_0 e^{kt} \text{ (first order)}$$

### Degradation:

S = substrate

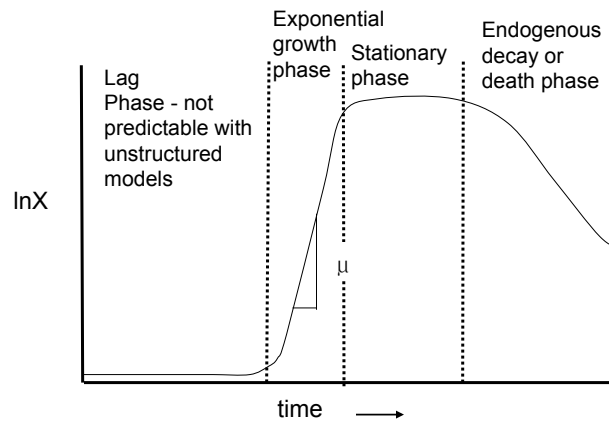
$$r = dS/dt = -k \quad S(t) = S_0 - kt \text{ (zero order)}$$

$$r = dS/dt = -kS \quad S(t) = S_0 e^{-kt} \text{ (first order)}$$



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## Crecimiento microbiano



X = cell mass

$\mu$  = specific growth rate

How to model?



## Crecimiento exponencial

X	$\Delta X/\Delta t$
1	$1/t_d$
2	$2/t_d$
4	$4/t_d$
8	$8/t_d$

$\frac{\Delta X}{\Delta t} \propto X$        $\frac{dX}{dt} = \mu X$

$$\mu = \frac{dX/dt}{X}$$

Specific growth rate

If  $\mu = \text{constant}$ ,

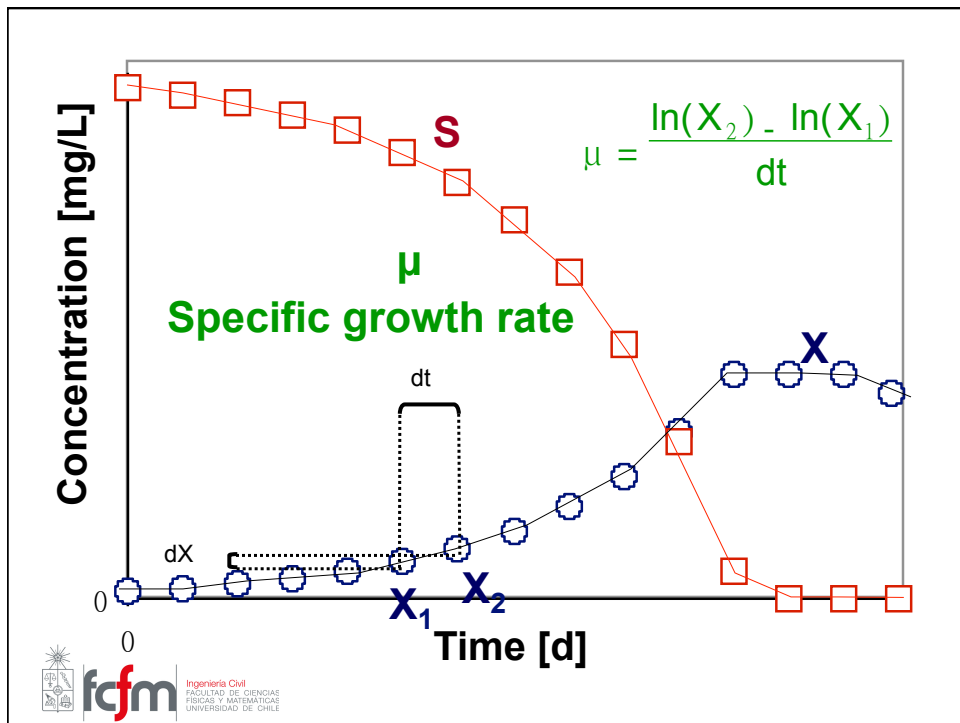
$$\int_{X_0}^X \frac{dX}{X} = \mu \int_0^t dt$$

$$\ln \left[ \frac{X}{X_0} \right] = \mu t = \ln X - \ln X_0$$

$$X = X_0 e^{\mu t}$$

Time to double,  $t_d = \frac{\ln 2}{\mu} = \frac{0.693}{\mu}$

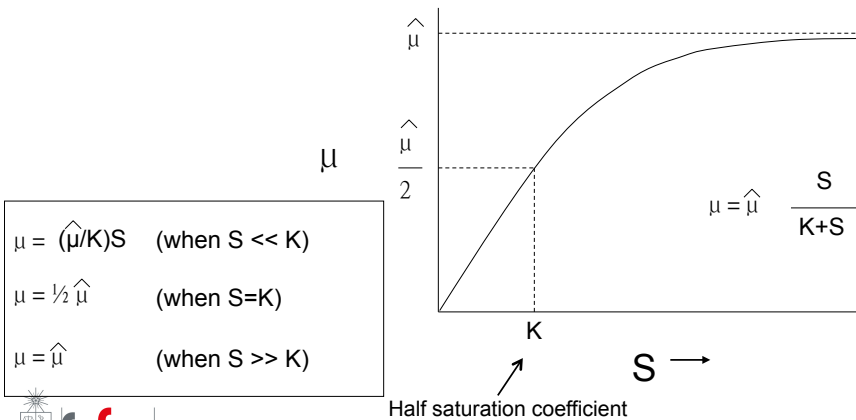
No. generations =  $t/t_d$



## Cinética del metabolismo

### Monod kinetics

$\mu$  = specific growth rate [new biomass/biomass-time]



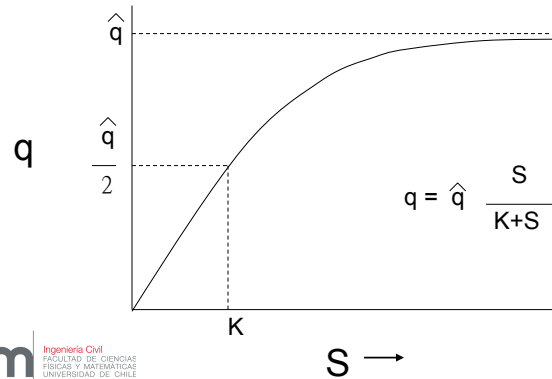
## Cinética del metabolismo

$$\mu = Yq$$

$\mu$  = specific growth rate [new biomass/biomass-time]

$Y$  = yield = new biomass/substrate consumed

$q$  = specific rate of substrate utilization =  $\mu/Y$   
 = [substrate consumed/biomass-time]



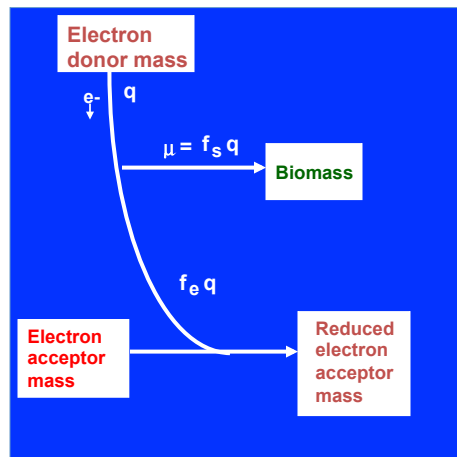
## Yield (Y)

$$Y = \frac{\text{Biomass produced}}{\text{Substrate consumed}} \quad \text{e.g., (g VSS/ g COD)}$$

- Y can be measured and is directly related to  $f_s$ .
- Y can be changed it into  $f_s$  by converting both the numerator and the denominator into electron equivalents (or into  $O_2$  equivalents, whichever is easier).

$$Y \text{ (g VSS/g COD)} = f_s \times (5.65 \text{ g VSS/eq}) / (8 \text{ g COD/eq}) \\ = f_s / 1.42 \text{ (gCOD/gVSS)}$$

$$Y = f_s (5.65/8) = f_s / 1.42$$



When substrate and biomass are expressed as either electron equivalents or oxygen equivalents,  $\mu = f_s q$  and the specific rate of use of the electron acceptor is  $f_e q$ .

McCarty found that  $f_e \hat{q}$  is  $\sim 1$  mole  $e^-$ /g vss-d (8 g COD/g VSS-d) at 25°C for a wide range of microorganisms.

**McCarty found that the maximum specific rate of electron transfer for energy was one mole of electrons per gram VSS per day at 25°C.**

**Given: Aerobic heterotrophs have a yield of 0.42 g vss/g COD**

What is  $\hat{q}$ , the maximum specific rate of substrate consumption ?

$$f_s = 0.42 \text{ g vss/g COD} \times 8 \text{ g COD/eq} \times \text{eq}/5.65 \text{ g vss} = 0.60$$

$$f_e = 1 - f_s = 1 - 0.60 = 0.40$$

$$f_e \hat{q} = 8 \text{ g COD/g VSS-d}$$

$$\hat{q} = f_e \hat{q} / f_e = 8 / 0.4 = 20 \text{ g COD/g vss-d}$$

$$0.693 / 8.4 \times 24 = 1.98 \text{ hr}$$

Doubles approx every 2 hr

What is  $\hat{\mu}$ ?

$$\hat{\mu} = Y \hat{q} = 0.42 \times 20 = 8.4 / \text{d}$$



**McCarty found that the maximum specific rate of electron transfer for energy was one mole of electrons per gram VSS per day at 25°C.**

**Given: Methanogens have a maximum specific rate of substrate consumption  $q$  of 8.4 g COD/g vss-d.**

What is  $\hat{Y}$ ?

$$f_e = f_e \hat{q} / \hat{q} = 8 \text{ COD/g vss-d} \div 8.4 \text{ g COD/g vss-d} = 0.95$$

$$f_s = 1 - f_e = 0.05 \quad Y = 0.05 \times 5.65 / 8 = 0.03 \text{ g vss/g COD}$$

What is  $\hat{\mu}$ ?

$$\hat{\mu} = Y \hat{q} = (0.03)(8.4) = 0.28 / \text{d}$$



McCarty found that the maximum specific rate of electron transfer for energy was one mole of electrons per gram vss per day at 25°C.

**Given:** Sulfate-reducing bacteria have a maximum specific growth rate  $\hat{\mu}$  of 0.5/d.

What is  $\hat{q}$ ?

(Convert  $\hat{\mu}$  from VSS to COD units)

$$\hat{\mu} = 0.5 \text{ g vss/g vss-d} \times 1.42 \text{ g COD/g vss} = \mathbf{0.71 \text{ g COD /g vss-d}}$$

$$\hat{q} = \underset{\text{Energy}}{8 \text{ g COD/g vss-d}} + \underset{\text{growth}}{0.71 \text{ g COD/g vss-d}} = \mathbf{8.71 \text{ g COD/g vss-d}}$$

What is Y?

$$f_s = \hat{\mu} / \hat{q} = 0.71 / 8.71 = 0.08$$

$$Y = 0.08 \times 5.65 / 8 = \mathbf{0.06 \text{ g vss/g COD}}$$



## Expresiones cinéticas

$$\mu = \hat{\mu} \frac{S}{K+S}$$

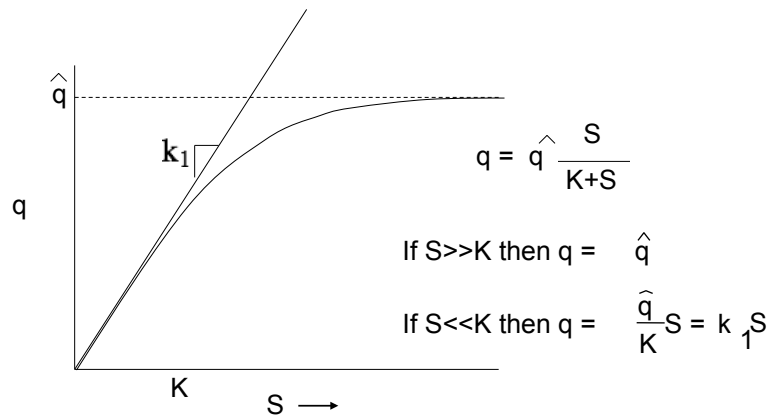
$$q = \hat{q} \frac{S}{K+S}$$

How can we apply these expressions?

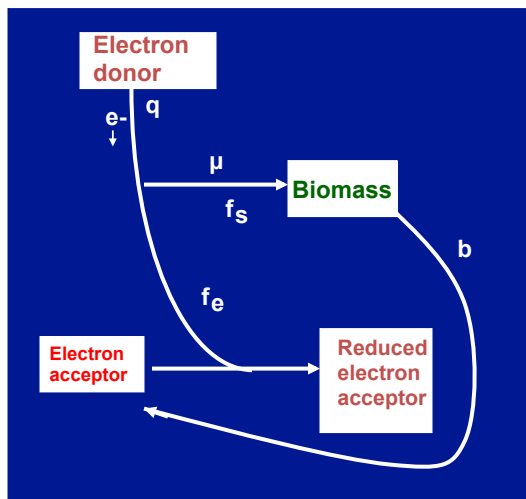




## Simplificaciones



## Modelo de Herbert



$Y_{eff} \ll Y$   
because of decay

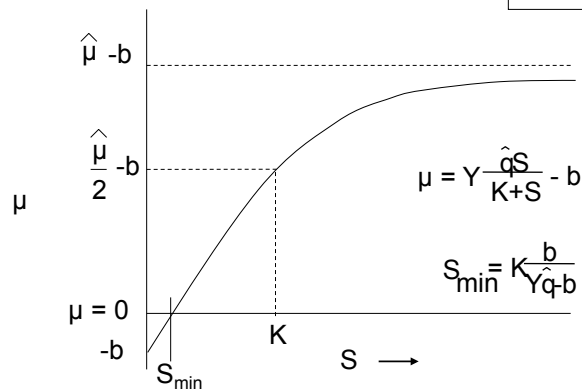
$f_s \ll f_s^\circ$   
because of decay

$b = \text{endogenous decay } (T^{-1})$

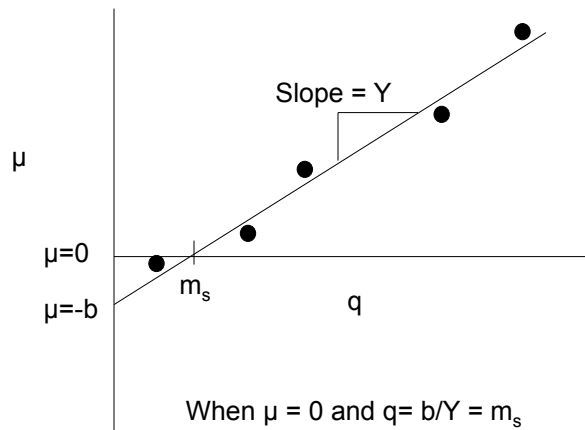
## Qué pasa con el decaimiento?

Herbert:  $\mu = Yq - b$

$$q = \hat{q} \frac{S}{K+S}$$



$\mu = Yq - b$



$m_s$  is the maintenance coefficient [substrate mass/biomass-time]



**TYPICAL VALUES**  
for  $Y$ ,  $\hat{q}$ ,  $\mu_{max}$ , and  $b$

Process	Y (g cell per g of limiting substrate)	$\hat{q}$ (g substrate per g cell per day)	b (d <sup>-1</sup> )	$\mu_{max}$ (d <sup>-1</sup> )	$t_d$ (d)
<b>Aerobic:</b>					
organic removal	0.45 g vss/g COD	22 g COD/g vss S-d	0.2	9.7	0.1
nitrification	0.2 g vss /g NH <sub>3</sub> -N	2 g NH <sub>3</sub> -N/g vss -d	0.05	0.35	2.0
S <sup>2-</sup> oxidation	0.59 g vss /g S	2.5 g S/g vss -d	0.1	1.4	0.5
Fe <sup>2+</sup> oxidation	0.0075 g vss /g Fe <sup>2+</sup>	60 g Fe <sup>2+</sup> /g vss -d	0.1	0.3	2.0
H <sub>2</sub> oxidation	1.38 g vss /g H <sub>2</sub>	1.3 g H <sub>2</sub> /g vss -d	0.1	1.7	0.4
<b>Anaerobic:</b>					
denitrification	0.35 g vss /g COD 0.8 g vss /g NO <sub>3</sub> -N	14 g COD/g vss -d 4 g NO <sub>3</sub> -N/g vss -d	0.1 0.1	4.8 3.1	0.14 0.22
sulfate reduction	0.1 g vss /g COD	9.3 g COD/g vss -d	0.05	0.9	0.8
methane fermentation:					
fats	0.031 g vss /g COD	8.4 g COD/g vss -d	0.02	0.2	2.9
proteins	0.081 g vss /g COD	8.4 g COD/g vss -d	0.02	0.7	1.1
carbohydrates	0.23 g vss /g COD	8.4 g COD/g vss-d	0.05	1.9	0.4
sewage sludge	0.081 g vss/gCOD	8.4 g COD/g vss -d	0.05	0.6	1.1

## Valores típicos de K

Process	K (mg substrate/L)
<b>Aerobic:</b>	
organic mixtures	50-150 mg COD/L
single organics	1-10 mg COD/L
nitrification	0.4 - 2 mg NH <sub>3</sub> -N/L
<b>Anaerobic:</b>	
denitrification	0.06-0.20 mg NO <sub>3</sub> -N/L
methane fermentation:	
acetate, propionate	600-900 mg COD/L
sewage sludge	2,000-3,000 mg COD/L

## Sustancias tóxicas

### Naturally Occuring Biodegradable Substances

- Benzene
- Toluene
- Polycyclic aromatic hydrocarbons

### Microorganisms can fortuitously transform new chemicals

- Structure is similar to enzyme's natural substrate
- Enzymes are not very specific

**Metabolism** - the organism is able to obtain energy from the reaction or make use the products of the reaction

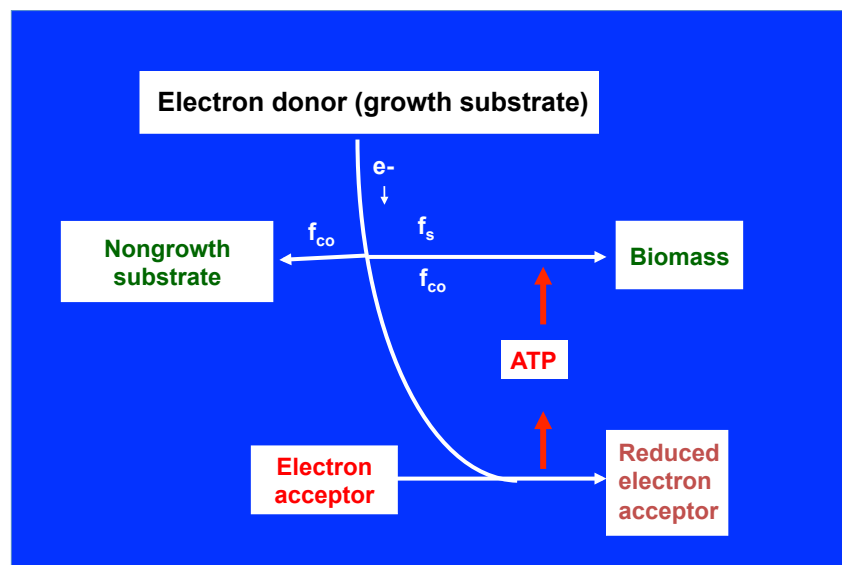
**Cometabolism** - the organism cannot derive energy from the reaction or make use of the products

When a bioreactor is designed for cometabolism, growth substrate must be added



Opportunities for cometabolism because many waste sites have **mixed wastes**


## Cometabolismo



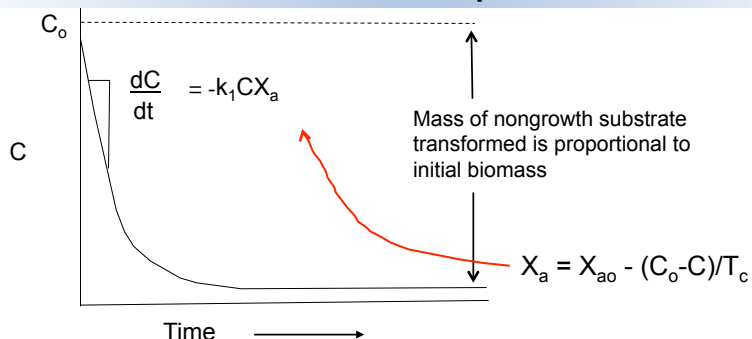
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## Ejemplos de cometabolismo

Growth substrates	Nongrowth substrates
Methane Toluene Phenol Propane Ammonia Cresol	} <u>Many</u> halogenated aliphatics, alkanes, and aromatics
Napththalene	PAHs
Aliphatic hydrocarbons	Alkyl-substituted cyclic hydrocarbons
Chlorobenzoate	Monofluorobenzoates
Biphenyl benzoate	PCBs



## Cinéticas típicas de cometabolismo en ausencia de sustrato para crecimiento



$\frac{dC}{dt} = -k_1 C X_a$


Mass of nongrowth substrate transformed is proportional to initial biomass

$X_a = X_{a0} - (C_0 - C)/T_c$

$T_c = \text{transformation capacity} = \frac{dC}{dX_a}$

Integrated result:

$$C = C_0 \frac{F e^{-k_1 F t}}{X_{a0} - \frac{C_0}{T_c} e^{-k_1 F t}} \quad \text{where } F = X_{a0} - \frac{C_0}{T_c}$$



## Elección de sustrato para crecimiento

Many growth substrates induce enzyme activity that can destroy TCE aerobically:

- ammonia (ammonia monooxygenase) ammonia oxidizers
- phenol (phenol hydroxylase)
- methane (methane monooxygenase) methanotrophs
- toluene (toluene dioxygenase)

**Let's compare TCE cometabolism kinetics for the phenol degraders and the methanotrophs:**

TCE kinetics for phenol-degraders:

$$T_c = 0.35 \text{ mg TCE/mg vs}$$

$$k_1 = 0.18 \text{ L/mg vs-d}$$

TCE kinetics for methane (methanotrophs):

$$T_c = 0.047 \text{ mg TCE/mg vs}$$

$$k_1 = 1.4 \text{ L/mg vs-d}$$

Which is more advantageous for sustained activity?

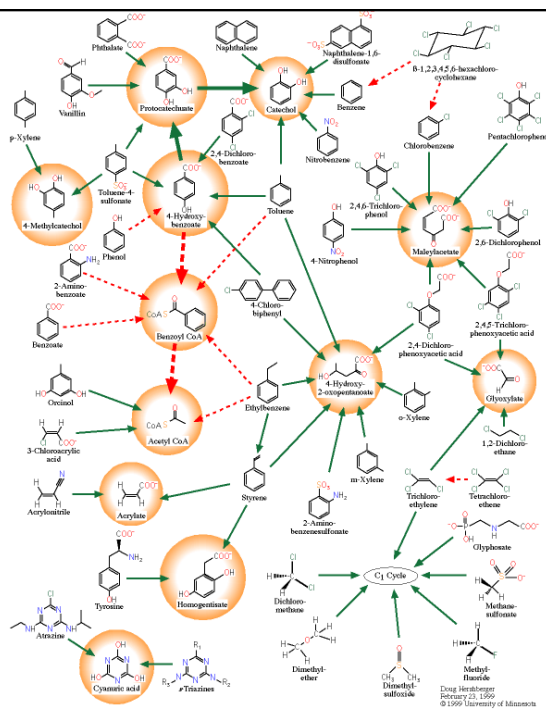


## Vías para biodegradación

On the web at: <http://umbdd.ahc.umn.edu/>

- Degraded from the indicated **starting** compound to an intermediate compound, **aerobically**
- Degraded from the indicated **intermediate** compound to another intermediate compound, **aerobically**
- - - → Degraded from the indicated **starting** compound to an intermediate compound, **anaerobically**
- - - → Degraded from the indicated **intermediate** compound to another intermediate compound, **anaerobically**

The circled compounds are common intermediates for trunk pathways. Trunk pathways feed into common pathways of metabolism, such as the tricarboxylic acid cycle.





## Whole cell inhibition kinetics

**Competitive inhibition** - apparent  $K$  increases.  
Observed with oxygenases used to degrade gasoline components, chlorinated solvents, ammonium.

$$q = \frac{\hat{q} S}{K \left(1 + \frac{I}{K_i}\right) + S} \quad \text{so that } K_{\text{apparent}} = K \left(1 + \frac{I}{K_i}\right)$$

$I$  is concentration of inhibitory compound

$K_{\text{apparent}}$  can be obtained from a Lineweaver-Burke plot or other non-linear curve fitting procedure.



## Whole cell inhibition kinetics

**Noncompetitive inhibition** -  $\hat{q}_{\text{apparent}}$  decreases.  
Often reported for metals.

$$q = \frac{\hat{q} S}{\left(1 + \frac{I}{K_i}\right)(K + S)} \quad \text{so that } \hat{q}_{\text{apparent}} = \frac{\hat{q}}{1 + \frac{I}{K_i}}$$

$\hat{q}$

$\hat{q}_{\text{apparent}}$  can be obtained from a Lineweaver-Burke plot or other non-linear curve fitting procedure.





## Whole cell inhibition kinetics

### Self inhibition (Andrews equation)

$$q = \frac{\hat{q}S}{K + S + \frac{S^2}{K_i}}$$

High levels of acetic acid show substrate inhibition in methanogenesis.

### General expression for inhibition (Han and Levenspiel, 1988)

$$q = \frac{\hat{q}S}{K \left(1 - \frac{I}{I^*}\right)^m + S} \left(1 - \frac{I}{I^*}\right)^n$$



where  $I^*$  is the concentration that stops growth

## Tipos de inhibición

$$q = \frac{\hat{q}S}{K \left(1 - \frac{I}{I^*}\right)^m + S} \left(1 - \frac{I}{I^*}\right)^n$$

General expression of inhibition

where  $I^*$  is the concentration that stops growth

Inhibition type	Effect on $\hat{q}$	Value of n	Effect on K	Value of m
competitive	none	0	increase	<0
noncompetitive	decrease	>0	none	0
uncompetitive	decrease	>0	decrease	>0
mixed	decrease	>0	increase	<0

Source: Grady et al., 2002. Biological Wastewater Treatment)

Volskay et al.(1990) found that chlorinated organic compounds were mostly mixed inhibitors of aerobic heterotrophs. The lowest value for  $I^*$  was 126 mg/L for tetrachloroethylene.



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**For a batch system, a mass balance on substrate gives**

In: 0  
 Out: 0  
 Source: 0  
 Sink:  $qX_a V_L \Delta t$

$$\Delta M = -qX_a V_L \Delta t \quad (\text{constant liquid volume})$$

For a nonvolatile substrate:  $\Delta M/V_L = \Delta S$

$$\frac{-dS}{dt} = qX_a \quad \frac{-dS}{dt} = \frac{\hat{q}SX_a}{K+S}$$

A mass balance on biomass gives

$$\frac{dX_a}{dt} = Y \left( \frac{-dS}{dt} \right) \quad X_a = X_{a0} + Y(S_0 - S)$$

$$t = -\frac{1}{q} \left( \frac{-K}{X_{a0} + YS_0} - \frac{1}{Y} \right) \ln[X_{a0} + YS_0 - YS] + \left[ \frac{K}{X_{a0} + YS_0} \right] \ln \left[ \frac{SX_{ad}}{S_0} \right] + \frac{1}{Y} \ln X_{ad}$$

- Solids retention time (SRT) = mean cell residence time (MCRT) = sludge age
- $q_x$  = active biomass in the system/  
 production rate of new biomass =  $m^{-1}$