Pushback design optimisation in open-pit mine planning

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How does an open-pit mine look like?

Figure 1: Chuquicamata copper mine (Chile).

Elements of an open-pit

Figure 2: Mining pushback (Radomiro Tomic copper mine, Chile).

Outline

Open-pit long-range mine planning problem (OPLRMPP)

Main questions

- **What** to mine? $\rightarrow$ pit limit.
- **When** to extract? $\rightarrow$ mining sequence / scheduling
- **Where** to process? $\rightarrow$ opportunity cost / cut-off grade

Steps of the current practice in mine planning:

1. Pit limit definition.
2. Mining sequence optimisation.
3. Pushback design.
4. Production scheduling.
5. Operational and capital expenditure calculation.
6. Economic evaluation.

The Ultimate Pit Limit Problem (UPL)

p_b : profit associated to the extraction of block $b \in \mathcal{B}$ ($\mathcal{B}$ set of blocks in the block model).

x_b : equal to one if the block is extracted, zero otherwise.

$\hat{b} \in \hat{\mathcal{B}}_b$: blocks constituting vertical precedences (upwards) for block b.

$$\max z(x) = \sum_{b \in \mathcal{B}} p_b \cdot x_b$$
$$x_b - x_{\hat{b}} \leq 0, \forall b \in \mathcal{B}$$
$$x_b \in [0, 1]$$

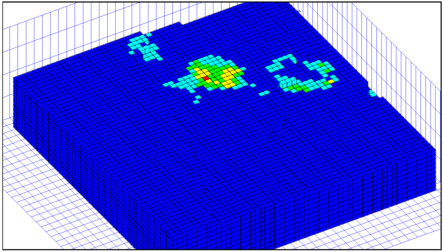


Figure 3: Geological block model.

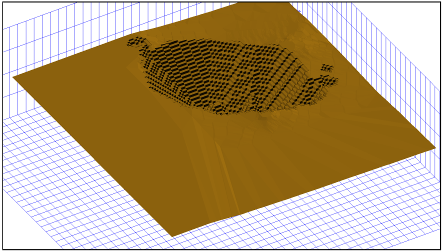


Figure 4: Ultimate pit limit contour.

The Next Best Ore Problem

- *"There are virtually unlimited number of ways of reaching the ultimate pit limit"*[1].
- L&G introduce the parametrisation analysis, which consist on finding **smaller pits** through the relaxation of the volume constrained UPL.

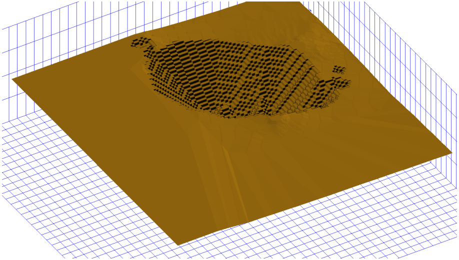


Figure 5: Example of the parametrisation method.

[1] Lerchs, H. and Grossmann, I. F. (1965). *Optimum Design of Open Pit Mines*

Pit limit and the mining sequence (current practice)

- Lerchs and Grossmann (1965) introduced an **efficient algorithm** to calculate the ultimate pit limit (UPL).
- The **parametrisation method** comprises generating a mining sequence by iteratively calculating UPL for different factors (nested pits).
- The parametrisation method is the **prevalent** approach to define the mining sequence.
- However, in most of the cases, nested pits need to be **greatly modified** to be used as pushbacks.

The "art" of the Pushback Design:

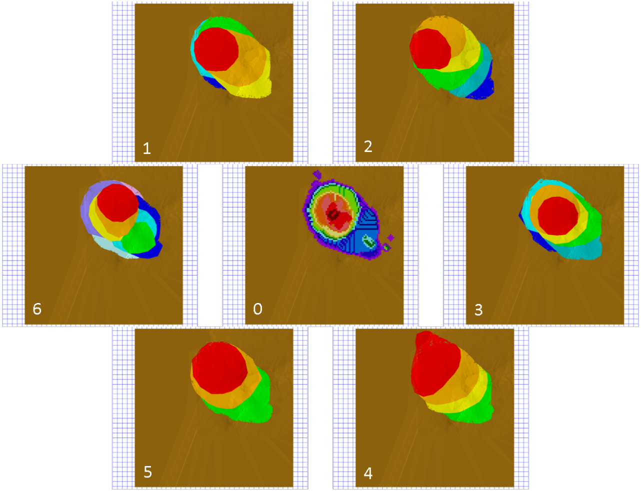


Figure 6: Different practical designs from the same guidance (top view).

Pushbacks

There are **several** (sometimes contradictory) definitions of pushbacks in the literature.

Definitions[1]

- **Pushbacks:** Pushbacks are a set of disjoint and mineable volumes aimed to maximise the financial return from a mine. The union of the pushbacks form the UP.
  Each pushback is a connected volume (aggregation of blocks) with sufficient operational width, and as such the slope conditions are honoured. Each pushback is designed with a haul-road that connects all their benches from top to bottom.
- **Semi-practical pushback:** a pushback without a haul-road.

[1] Yarmuch, Juan L. (2020). *Optimisation in open-pit mine planning. PhD Thesis. University of Melbourne.*

Haul roads

semi-practical pushbacks → pushbacks

- At the time of this work, **there was no** mathematical model for open-pit haul roads available in the literature.
- Most of the found haul road models are developed for the **forest industry** and **civil engineering**.
- Those models assume **low stripping** (no-valuable material required to be removed), which does not apply to in-pit ramps.
- **Two models** that include **gradient** and **curvature** constraints are developed: ex-pit roads (low stripping) and in-pit ramps (high stripping).
- For the in-pit ramps an **integer programming** model is developed, and for the ex-pit roads a **shortest path** approach is undertaken.
- In both cases, the models can be **solved exactly**.

- The idea is to use a regular **2D lattice of cells**, where each cell, $c \in \mathcal{C}$, represents a rectangular area of the terrain.
- At each cell there is a **node** representing a **direction** in which the cell will be accessed.

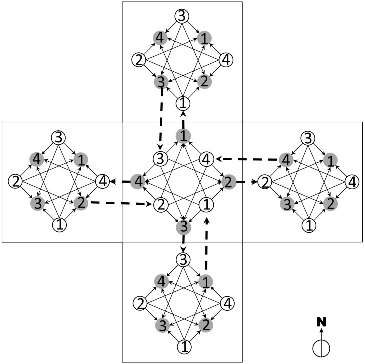


Figure 7: A representation of a simple 4-choice of directions graph $\mathcal{G}$.

Ex-pit haul road

- The **weights** associated with the arcs of the graph $\mathcal{G}$ are calculated as the **costs** of changing the direction and the cost of building a section of the road.
- The minimum cost haul road is solved using **Dijkstra's algorithm** for the weighted graph $\mathcal{G}$.

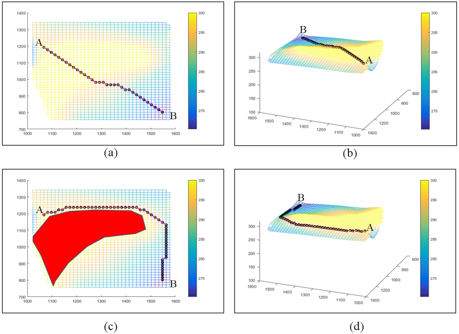


Figure 8: Ex-pit road design.

In-pit ramps

- A **binary linear model** to find the minimum cost ramp is formulated.
- A set of blocks above the ramp need to be extracted to consider the **stripping** associated with the ramp.
- The ramp is represented as a **connected path** of adjacent blocks from S to T (two artificial blocks).

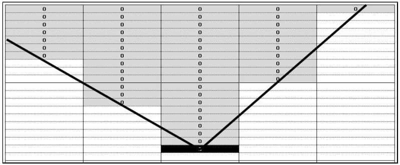


Figure 9: Stripping.

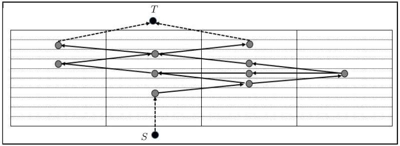


Figure 10: Arcs constituting possible elements of the ramp.

In-pit ramps: Formulation and solution method

- Min: $\sum_{i \in \mathcal{V}_1} C_i \cdot x_i + \sum_{(i,j) \in \mathcal{A}_3''} H_{i,j} \cdot a_{i,j} + \sum_{i \in \mathcal{V}_3} \sum_{d \in \mathcal{D}} P_i^d \cdot v_i^d$
- S.T: 1) Wall slope constraints. 2) Ramp connectivity. 3) Ramp cannot be built over excavated blocks. 4) Account for change of directions.

Algorithm 1: MEGAP, Mutually Exclusive Greedy Adaptive Path

```
Data: G_3 with edges sorted by stripping in increasing order, G_tc,
      starting node S, final node T
Result: A path p that is a feasible solution for In-pit ramp problem
begin
    p ← ∅;
    cN ← ∅;
    foreach u ∈ V_3 do
        u.visited ← False;
        u.predecessor ← null;
    MEGAPvisit(S)
    p ← p ∪{T}
    pred ← T.predecessor
    while pred ≠ null do
        p ← p ∪{pred}
        pred ← pred.precedessor
    Output p
    Function MEGAPvisit (u):
        foreach v such that (u, v) ∈ A_tc do
            cN ← cN ∪ {v}
        u.visited ← True;
        foreach v such that (u, v) ∈ A_3 do
            if (v.visited=False) and (v ∉ cN) and (T.visited=False)
             then
                v.visited ← True;
                v.predecessor ← u;
                MEGAPvisit(v)
```

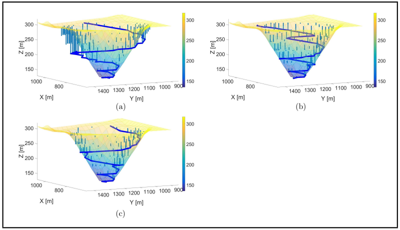


Figure 11: Example of an optimised in-pit ramp vs a manual design.

Details in: *Yarmuch, J. L., Brazil, M., Rubinstein, H., & Thomas, D. A. (2020). Optimum ramp design in open pit mines. Computers & Operations Research.*

Pushbacks (first approach)

Ramps and Pushbacks

- The pushback problem is formulated as a **binary linear programming** model.
- The model maximises an **approximate discounted cash-flow** (no production schedule) to keep the problem tractable.
- A key idea in this formulation is to use the haulage ramp as a **relative coordinate system** to control the shape of the pushback.
- The operational width and connectivity are modelled by **biasing** the objective function.
- A constraint over the **tonnage** of the pushback is required as we are not considering the production schedule.

Formulation.

Obj: Max $\sum_{p \in \mathcal{P}} \delta_p (\sum_{b \in \mathcal{B}} W_b \cdot V_b \cdot x_{b,p} + \lambda_p \sum_{b \in \mathcal{B}} \sum_{\breve{b} \in \breve{\mathcal{B}}_b} \frac{u_{b,\breve{b},p}}{D_{b,\breve{b}}} \cdot)$

Some important constraints

- Material content at each pushback.
- Geotechnical constraints (slope stability).
- Ramps cannot be built in the air (previously mined blocks).
- Every mined block must have access to their respective ramp.
  - $x_{\vec{b},p} - \sum_{\vec{b} \in \vec{\mathcal{B}}} \sum_{s' \in S'_{\vec{b}}} z_{s',p} \leq 0, \qquad l \in \mathcal{L} - \{1\}, \quad \vec{b} \in \vec{\mathcal{B}}_l, \quad p \in \mathcal{P}$
- Ramp continuity constraints.
  - $\sum_{s'' \in \mathcal{S}''_b} z_{s'',p} - \sum_{s' \in \mathcal{S}'_b} z_{s',p} = 0, \qquad b \in \mathcal{B}, \quad p \in \mathcal{P}$
- Ramp accessibility constraints.
  - $\sum_{\rho \leq p} r_{b,\rho} - \sum_{\rho \leq p} x_{b^+,\rho} \leq 0, \qquad b \in \mathcal{B}, \quad p \in \mathcal{P}$

Details in: *Yarmuch, J. L., Brazil, M., Rubinstein, H., & Thomas, D. A. (2021). A mathematical model for mineable pushback designs. International Journal of Mining, Reclamation and Environment, 35(7), 523–539.*

Case study.

- Test instance based on a copper mine in Chile.
- Number of blocks: 3916 blocks (re-blocked).
- Optimisation parameters:
  - Mining width 2 blocks.
  - Discount factor of 20% per pushback.
- Solution method: Greedy approach (pushbacks one-by-one).
- Solution time: 3 hours approx.

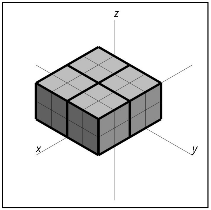


Figure 12: Example of a 2 by 2 reblocking.

A comparison against the traditional methodology

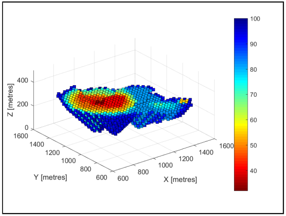


Figure 13: Nested pits.

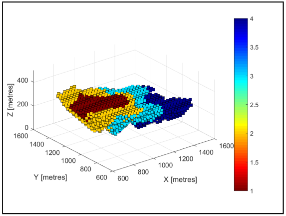


Figure 14: Engineer design.

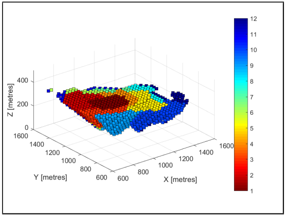


Figure 15: Output of the new model.

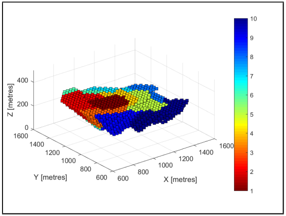
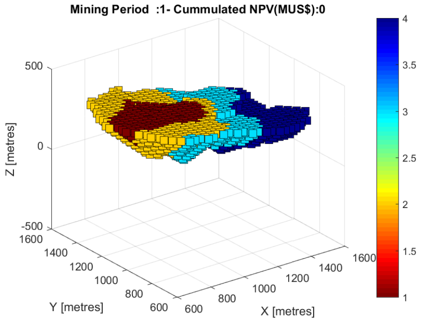
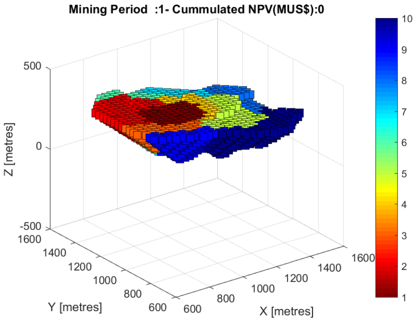


Figure 16: Smoothed output.

Production schedule





Summary

Pushback optimisation

- Pros
  - The model gives *more operational* guidelines compared to the nested pits (traditional methodology).
  - The model improved the NPV of the mine by 5.4%.
  - The idea of using the ramp to control the shape of the pushbacks is worthwhile to explore.
- Cons
  - The model optimise an approximate discounted cash-flow biased by a factor λ to force connectivity and mining width.
  - The value of λ is determined by exploration.
  - The number of variables related to the ramp segments (z_{sp}) grows exponentially with the length of the ramp.
  - The model requires bounds in the material content per pushback.
  - The model does not account for the production schedule.

Semi-practical pushbacks optimisation

Mining width and connectivity

- **A mathematical formulations** is developed to incorporate the mining width and connectivity conditions without requiring external factors (such as λ).
- The formulations is a **binary linear programming** model.
- The model maximises an **approximate discounted cash-flow**.
- A constraint over the **tonnage** of the pushback is required.

Mining width and connectivity approaches

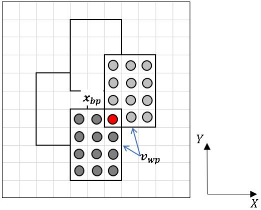


Figure 17: Rectangular template.

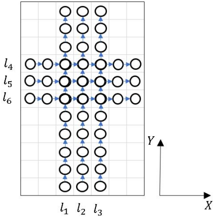


Figure 18: Directional *lines*.

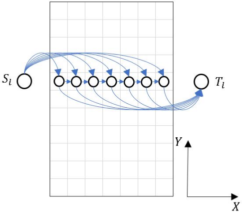


Figure 19: Example of a *linear* graph.

Formulations.

Obj: Max $\sum_{p\in\mathcal{P}} \delta_p \sum_{b\in\mathcal{B}} W_b \cdot V_p \cdot x_{bp}$

Some important constraints

- Material content at each pushback.
- Geotechnical constraints (slope stability).
- Mining width constraint (rectangular template approach).
- Connectivity constraints:
  - $\sum_{(i,j)\in\mathcal{A}_l} a_{ijp} - \sum_{(j,k)\in\mathcal{A}_l} a_{jkp} \leq \begin{cases} -1, & \text{if } j = S_l \\ 1, & \text{if } j = T_l \end{cases} \quad l \in \mathcal{L}, \quad p \in \mathcal{P}, j \in \mathcal{B} \cup \{S_l, T_l\}$
  - $\sum_{(i,j)\in\mathcal{A}_l | j\neq S_l, j\neq T_l} a_{ijp} - \sum_{(j,k)\in\mathcal{A}_l | j\neq S_l, j\neq T_l} a_{jkp} = 0, \quad l \in \mathcal{L}, \quad p \in \mathcal{P}$
  - $\sum_{(i,j)\in\mathcal{A}_l, j\neq T_l} a_{ijp} = x_{jp} \qquad \forall j \neq T_l | (i,j) \in \mathcal{A}_l, l \in \mathcal{L}, \quad p \in \mathcal{P}$
  - $\sum_{p\in\mathcal{P}} a_{ijp} \leq 1 \qquad (i,j) \in \mathcal{A}_l, \quad l \in \mathcal{L}$

Details in: *Yarmuch, J. L., Brazil, M., Rubinstein, H., & Thomas, D. A. (2021). A model for open - pit pushback design with operational constraints. Optimization and Engineering*

Solution method

We use a series of preprocessing routines (computation of the ultimate pit limit, early start pushback for each block, rectangular template boundaries, and MIP warm start) and a sliding window heuristic to solve instances of tens of thousands of blocks to near optimality.

Algorithm 1:

begin

STEP 0: Compute an augmented ultimate pit limit (UPL^A).

Rule out all variables that are outside the augmented UPL^A.

STEP 1: Compute the minimum pushback for each block $b \in UPL^A$ using the Early Start algorithm. Delete all variables for which pushback p is less than $ES(b)$ (delete $x_{b,p} \quad \forall (b,p) | p < ES(b)$).

STEP 2: Formulate $SPPM^*$ by relaxing the mining width constraints of $SPPM$).

Solve $SPPM^*$ using $SWH(Iwin, gap)$.

$\mathcal{X}^*_{sol} \leftarrow SPPM^*$

STEP 3: For all $w \in \mathcal{W}$, compute $p_{min}(w)$ and $p_{max}(w)$ from $\mathcal{X}^*_{sol}$.

STEP 4: Formulate a $SPPM$ and delete the z (rectangular templates) variables such that p is larger than $p_{max(w)}$ or is smaller than $p_{min}(w)$ (delete $z_{wp} \quad \forall (w,p) | p > p_{max}(w) || p < p_{min}(w)$). We call this model $SPPM^B$.

STEP 5: Solve $SPPM^B$ using $SWH(Iwin, gap)$.

$\mathcal{X}^B_{sol} \leftarrow SPPM^B$

STEP 6: Formulate a $SPPM$ and load $\mathcal{X}^B_{sol}$ as a MIP warm start. Solve $SPPM$.

$\mathcal{X}_{sol} \leftarrow SPPM$

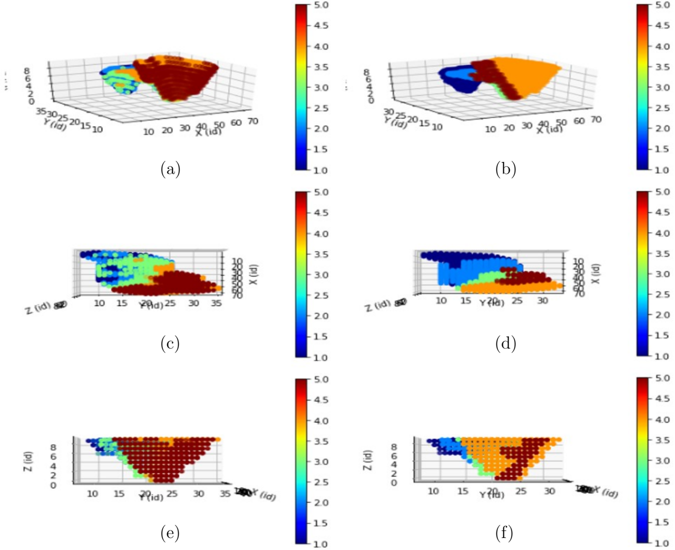
Results

- Instance I_3 is a modification of the KD instance (Minelib).
- Number of blocks: 4,682 blocks.
- Optimisation parameters:
  - Mining width template of 3 by 2 blocks.
  - Discount factor of 20%
  - Max. of 5 pushbacks.
  - Min. and max. tonnage per pushback: 10,000,000 and 15,000,000 tonnes, respectively.

Experiment	Obj. Function Value [US$]	Running Time [sec]	linear relax. gap [%]
$I_3(1,1)$	215,452,113	2,414	6.6
$I_3(1,2)$	218,204,764	4,971	5.4
I_3 *Opt**	214,356,412	86,400	7.1
I_3 *LP*	230,735,261	11,900	-

Table 1: Computational experiments for problem instance I_3. The value presented in I_3 *Opt** is the best solution found by the solver within the time limit of 24 hours.

Results



(a) (b)

(c) (d)

(e) (f)

Results

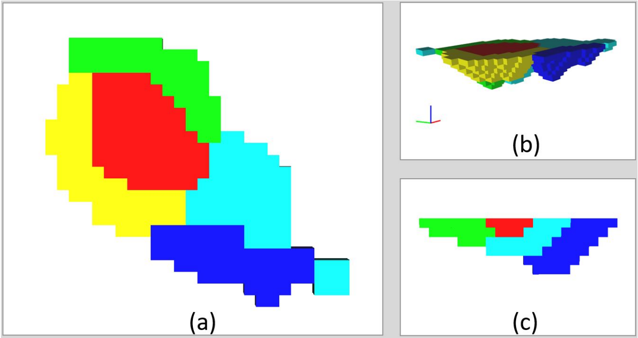


Figure 20: Visualisation of the instance I_1. Plots (a), (b) and (c) are XY section view, isometric and YZ views for the instance I_1, respectively. Colour scale represents different pushbacks (pushback 1: red, pushback 2: yellow, pushback 3: green, pushback 4: light blue, pushback 5: blue).

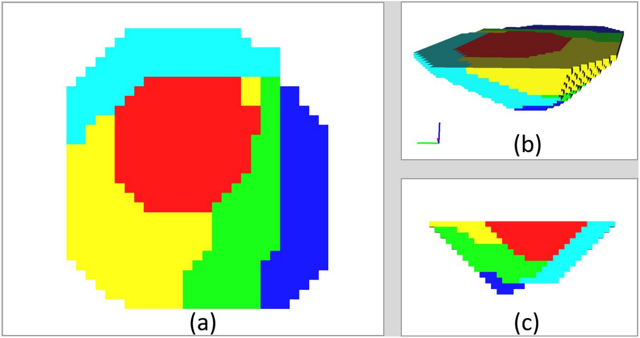
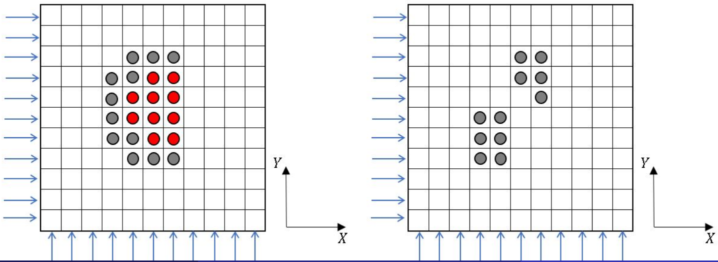


Figure 21: Visualisation of the instance I_2. Plots (a), (b) and (c) are XY section view, isometric and XZ views for the instance I_1, respectively. Colour scale represents different pushbacks (pushback 1: red, pushback 2: yellow, pushback 3: green, pushback 4: light blue, pushback 5: blue).

Summary

Semi-practical pushbacks

- Pros
  - The model is able to generate semi-practical pushbacks (no λ).
  - The use of Algorithm 1 allows solving instances up to 50,000 blocks.
- Cons
  - The model optimises an approximate discounted cash-flow.
  - The mining width is constrained to the shape of the templates.
  - The model does not account for the production schedule.
  - Limitations of the connectivity approach:



What if we re-think the problem? (patented by UoM)

A physical model of the soap bubbles applied to the pushback problem.

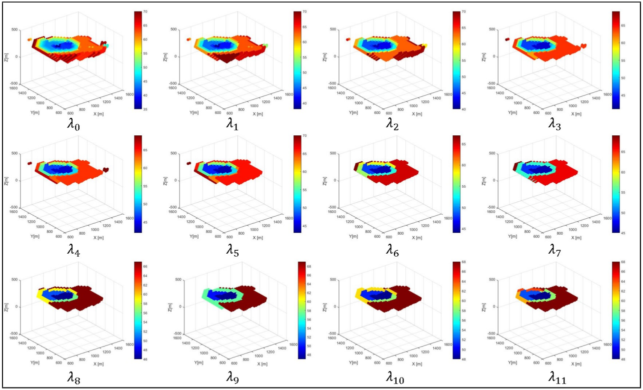


Figure 22: For every value of λ, the relaxed sub-problem can be solved in **polynomial** time (In this case: < 1 minute for an instance with 4000 blocks).

https://research.unimelb.edu.au/work-with-us/case-studies/improving-mining-functionality-using-an-algorithm-based-on-soap-bubbles .

The same technique can be used for the open pit mine production scheduling problem (OPMPSP)

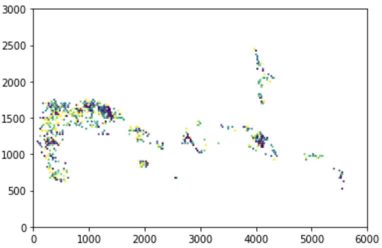


Figure 23: Output of the current model for the OPMPSP.

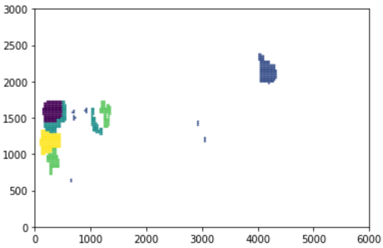


Figure 24: Proposed method for an operational OPMPSP.

AMIRA project 1210: **Optimal pushback design** ⌣̈