Aux 8

P1

$\Theta = \mathbb{R}^2 \longrightarrow \Theta_0 = \{(\mu,\nu) \in \mathbb{R}^2 \mid \mu^2 + \nu^2 = 1\}$

$\Theta_1 = \{(\mu,\nu) \in \mathbb{R}^2 \mid (\mu,\nu) \notin \Theta_0\}$

$\mathcal{X} = \mathbb{R}^{2n}$

$P = \mathcal{N}\left(\begin{pmatrix} \mu \\ \nu \end{pmatrix}, I_{d_{2n}}\right)$

- $\theta = (\mu, \nu)$; $z = (x, y)$

$$\Rightarrow L(\theta, z) = \frac{1}{(2\pi)^n} e^{-1/2\left[\sum (x_i-\mu)^2 + \sum (y_i-\nu)^2\right]}$$

1) Irrestricto: $\hat{\mu} = \overline{X}$; $\hat{\nu} = \overline{y}$

$$\hookrightarrow L(\hat{\theta}, z) = 1/(2\pi)^n \, e^{-1/2\left[\sum (x_i-\overline{x})^2 + \sum (y_i-\overline{y})^2\right]}$$

2) Tenemos restricción $\mu^2 + \nu^2 = 1$

$$L(\theta, z) = \frac{1}{(2\pi)^n} e^{-\frac{1}{2}\left[\underbrace{\sum (x_i-\mu)^2 + \sum (y_i - \nu)^2}_{\circledast}\right]}$$

$$\circledast = \sum \left(x_i^2 - 2x_i\mu + \mu^2\right) + \sum \left(y_i^2 - 2y_i\nu + \nu^2\right)$$

$$= \sum \left(x_i^2 + y_i^2\right) + n\underbrace{\left(\mu^2 + \nu^2\right)}_{=1} - 2\mu n\bar{x} - 2\nu n \bar{y}$$

$$= \sum \left(x_i^2 + y_i^2\right) + n - 2\mu n\bar{x} - 2\sqrt{1-\mu^2}\, n\bar{y}$$

Ahora, queremos calcular $\sup_{\theta \in \Theta_0} L(\theta, z)$

$$\sup_{\theta \in \Theta_0} \frac{1}{(2\pi)^n} e^{-\frac{1}{2}\left[\sum (x_i^2 + y_i^2) + n - \left(2n\mu\bar{x} + 2n\sqrt{1-\mu^2}\,\bar{y}\right)\right]}$$

↳ Equivalente a:

$$\sup_{\mu \in [-1,1]} \mu\bar{x} + \sqrt{1-\mu^2}\,\bar{y} \xrightarrow{\partial/\partial\mu = 0} \bar{x} + \cancel{\tfrac{1}{2}}(1-\mu^2)^{-1/2} \cdot -\cancel{2}\mu\,\bar{y} = 0$$

$$\Rightarrow \bar{x} - \frac{\mu}{\sqrt{1-\mu^2}}\,\bar{y} = 0$$

$$\Rightarrow \bar{x}\sqrt{1-\mu^2} = \mu\bar{y} \quad / \; (\;)^2$$

$$\Rightarrow \bar{x}^2(1-\mu^2) = \mu^2\bar{y}^2$$

$$\Rightarrow \bar{x}^2 - \mu^2\bar{x}^2 = \mu^2\bar{y}^2$$

$$\Rightarrow \mu = \frac{\bar{x}}{\sqrt{\bar{x}^2+\bar{y}^2}} \qquad \therefore \hat{\mu}_0 = \frac{\bar{x}}{\sqrt{\bar{x}^2+\bar{y}^2}}$$

$$\Rightarrow \hat{\nu}_0 = \frac{\bar{y}}{\sqrt{\bar{x}^2+\bar{y}^2}}$$

$$\therefore \; L(\hat{\theta}_0, z) = \frac{1}{(2\pi)^n} e^{-\frac{1}{2}\left[\sum (x_i^2 + y_i^2) - n\left(\frac{2\,\bar{x}^2}{\sqrt{\bar{x}^2+\bar{y}^2}} + \frac{2\,\bar{y}^2}{\sqrt{\bar{x}^2+\bar{y}^2}} - 1\right)\right]}$$

• Finalmente:

$$\lambda(z) = \frac{L(\hat{\theta}, z)}{L(\hat{\theta}_0, z)} = e^{-\frac{1}{2}\left[\cancel{\sum(x_i^2+y_i^2)} + n(\bar{x}^2+\bar{y}^2) - 2n\bar{x}^2 - 2n\bar{y}^2 \cancel{- \sum(x_i^2+y_i^2)} + n\left(2\sqrt{\bar{x}^2+\bar{y}^2} - 1\right)\right]}$$

$$= e^{-\frac{1}{2}\left[-n\bar{x}^2 - n\bar{y}^2 + n\left(2\sqrt{\bar{x}^2+\bar{y}^2} - 1\right)\right]}$$

$$= e^{\frac{n}{2}\left[\bar{x}^2 - 2\sqrt{\bar{x}^2+\bar{y}^2} + \bar{y}^2 + 1\right]}$$

$$= e^{\frac{n}{2}\left[\left(\sqrt{\bar{x}^2+\bar{y}^2} - 1\right)^2\right]}$$

Si : $\lambda(z) > K \iff \left(\sqrt{\bar{x}^2+\bar{y}^2}-1\right)^2 > K'$

$$\hat{R} = \{ z \in \mathbb{R}^{2n} \mid \lambda(z) > K \} = \{ z \in \mathbb{R}^{2n} \mid \left(\sqrt{\bar{x}^2+\bar{y}^2}-1\right)^2 > K' \}$$

P2) $\Theta = \{\theta > \text{Feromona}\} = \mathbb{R}_+$

$\mathcal{X} = \mathbb{R}_+^n \qquad P \sim \text{Unif}(0, \theta)$

Con todo eso, las hipótesis son:

$H_0: \theta = 1 \quad v/s \quad H_1: \theta \neq 1$

- Irrestricto: $\hat{\theta} = \max X_i$

$L(\hat{\theta}|x) = \frac{1}{\hat{\theta}^n} \mathbb{1}_{\max x_i \leq \hat{\theta}} = \frac{1}{(\max X_i)^n}$ //

- Tenemos restricción $\theta = 1$

$$\Rightarrow L(\hat{\theta}_0, x) = L(1, x) = \mathbb{1}_{\max x_i \leq 1}$$

- Con esto, calculamos:

$$\lambda(x) = \frac{L(\hat{\theta}, x)}{L(\hat{\theta}_0, x)} = \frac{1}{(\max x_i)^n} \cdot \frac{1}{\mathbb{1}_{\max x_i \leq 1}} \quad \begin{cases} +\infty & \max x_i > 1 \\ 1/(\max x_i)^n & \max x_i \leq 1 \end{cases}$$

$$\Rightarrow \hat{R} = \{ x \in \mathbb{R}^n \mid \lambda(x) > k \}$$

- Caso 1: Rechazo siempre en el caso $\max x_i > 1$

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• Caso 2: $\max X_i \leq 1$

$$\Rightarrow \hat{R} = \{ x \in \mathbb{R}^n \mid 1/(\max X_i)^n > K \} = \{ x \in \mathbb{R}^n \mid (\max X_i)^n < K' \}$$

¿Quién es K'?

$$\underbrace{\sup_{\theta \in \Theta_0} \mathbb{P}(X \in \hat{R})}_{= \alpha} = \mathbb{P}_{\theta=1}\left((\max X_i)^n < K' \right) = \mathbb{P}_{\theta=1}\left(\max X_i < (K')^{1/n} \right)$$

$$= \prod_{i=1}^{n} \underbrace{\mathbb{P}_{\theta=1}\left(X_i < (K')^{1/n} \right)}_{F_{X_i}((K')^{1/n})} = \prod_{i=1}^{n} (K')^{1/n} = K' \doteq \alpha$$

$$\therefore \hat{R} = \{ x \in \mathbb{R}^n \mid \max X_i < \alpha^{1/n} \} \quad \boxed{}$$