P1 · objeto masa m atado a resorte de cte k en contacto con el piso coef. μ_c y μ_e

· $x(0) = x_0 > 0$ $v(0) = 0$ $\quad 2\mu_c < \mu_e$

c/r al equilibrio (l_0)

a) · qué debe cumplir x_0 para que haya mov. debido al resorte? (suponer N conocido)

$$F_{\parallel} \leq \mu_e N \longrightarrow \text{no hay mov.}$$

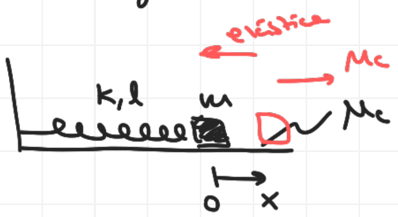
fzas tangenciales

para que haya mov, $\quad F > \mu_e N$

inicialmente $\vec{F} = -kx_0\hat{x} \longrightarrow$ condición para movimiento

$$\boxed{kx_0 > \mu_e N}$$

b) calc. x y v:



hay mov. $\longrightarrow \ddot{x} \neq 0$

$$m\ddot{x} = -kx + \mu_c N$$

$$m\ddot{y} = 0 = N - mg$$

$$\ddot{x} = -\frac{k}{m}x + \mu_c g \qquad \left(x_0 > \frac{\mu_e mg}{k}\right)$$

$$\ddot{x} + \omega^2 x = \mu_c g$$

$$x = x_h + x_p$$

$$x_h = A\cos\omega t + B\sin\omega t$$

$$x_p = C \longrightarrow \omega^2 C = \mu_c g \rightarrow C = \frac{\mu_c g}{\omega^2}$$

$$\boxed{x(t) = A\cos\omega t + B\sin\omega t + \frac{\mu_c g}{\omega}}$$

$$x(0) = x_0 = A + \frac{\mu_c g}{\omega^2} \longrightarrow \boxed{A = x_0 - \frac{\mu_c g}{\omega^2}}$$

$$\dot{x}(t) = -A\omega\sin\omega t + B\omega\cos\omega t$$

$\dot{x}(0)=0=B\omega \longrightarrow B=0$

$$\boxed{\begin{gathered} x(t)=\left(x_0-\frac{\mu_c g}{\omega^2}\right)\cos\omega t+\frac{\mu_c g}{\omega} \\ x_0 > \frac{\mu_c g}{\omega^2} \\ \omega^2=k/m \\ v(t)=-\left(x_0-\frac{\mu_c g}{\omega^2}\right)\omega\sin\omega t \end{gathered}}$$

por primera vez?

c) · cuánto tarda en cambiar de dirección? cuándo ocurre v_{max}?
· en qué posición ocurre?

$v(t^*)=0 \longrightarrow \sin\omega t=0$

$\omega t=n\pi \qquad n=1 \qquad (1^a \text{ vez})$

$$\boxed{t_1=\frac{\pi}{\omega}}$$

$\dot{v}=-\left(x_0-\frac{\mu_c g}{\omega^2}\right)\omega^2\cos\omega t=0$

$\cos\omega t=0$

$\omega t=n\pi+\frac{\pi}{2} \qquad n=0 \qquad (1^a \text{ vez})$

$$\boxed{t_{max}=\frac{\pi}{2\omega}}$$

$* \quad x_0 > \frac{\mu_c g}{\omega^2}$

$x(t_1)=\left(x_0-\frac{\mu_c g}{\omega^2}\right)\cos\pi+\frac{\mu_c g}{\omega^2}$

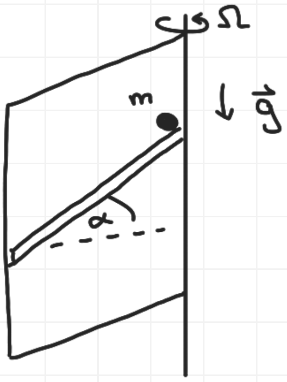
$=\frac{\mu_c g}{\omega^2}-x_0+\frac{\mu_c g}{\omega^2}=-x_0+\frac{2\mu_c g}{\omega^2}<-x_0+\frac{\mu_c g}{\omega^2}<0$

$$x(t_{máx}) = \left(x - \frac{\mu_c g}{\omega^2}\right)\cancel{\cos\frac{\pi}{2}} + \frac{\mu_c g}{\omega^2}$$

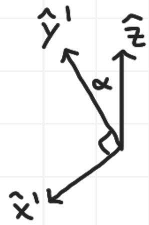
$$= \frac{\mu_c g}{\omega^2}$$

$$\left|\overbrace{-x_0 + \frac{2\mu_c g}{\omega^2}}^{<0}\right| = x_0 - \frac{2\mu_c g}{\omega^2} < |x_0|$$

P2

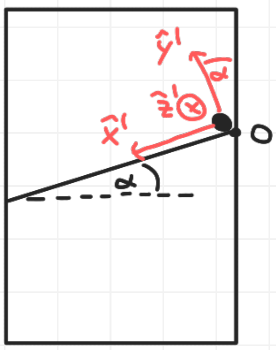


$$\vec{\Omega} = \Omega \hat{z}$$



$$\hat{z} = \hat{z}\cdot\hat{x}'\,\hat{x}' + \hat{z}\cdot\hat{y}'\,\hat{y}'$$
$$= \cos(\pi/2 + \alpha)\,\hat{x}' + \cos\alpha\,\hat{y}'$$
$$= -\sin\alpha\,\hat{x}' + \cos\alpha\,\hat{y}'$$

a)



$$\vec{r}' = x\hat{x}' \longrightarrow \vec{v}' = \dot{x}\hat{x}' \longrightarrow \vec{a}' = \ddot{x}\hat{x}'$$

$$\vec{r}_{O'} = \vec{0} \longrightarrow \vec{a}_{O'} = \vec{0}$$

<u>fuerzas reales</u>:

peso: $m\vec{g} = -mg\hat{z} = mg\sin\alpha\,\hat{x}' - mg\cos\alpha\,\hat{y}'$

normal vara: $\vec{N}_v = N_v\hat{y}'$

normal puerta: $\vec{N}_p = -N_p\hat{z}'$

<u>fuerzas inerciales</u>:

Coriolis: $\vec{F}_{cor} = -2m\vec{\Omega}\times\vec{v}' = -2m\Omega\dot{x}\,\hat{z}\times\hat{x}'$

$$= -2m\Omega\dot{x}\left(\cancel{-\sin\alpha\,\hat{x}'} + \cos\alpha\,\hat{y}'\right)\times\hat{x}'$$

$$= 2m\Omega\dot{x}\cos\alpha\,\hat{z}'$$

centrífuga: $\vec{F}_{cent} = -m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') = -m\Omega^2 x\,\hat{z}\times(\hat{z}\times\hat{x}')$

$$= -m\Omega^2 x\,\hat{z}\times(-\cos\alpha\,\hat{z}')$$

$$= m\Omega^2 x\cos\alpha\left(-\sin\alpha\,\hat{x}' + \cos\alpha\,\hat{y}'\right)\times\hat{z}'$$

$$= m\Omega^2 x\cos\alpha\left(\sin\alpha\,\hat{y}' + \cos\alpha\,\hat{x}'\right)$$

tangencial: $\vec{F}_t = -m\dot{\vec{\Omega}}\times\vec{r}\,' = \vec{0}$

b) suma de fuerzas

$$m\vec{a} = \vec{F}$$

$$m(\vec{a}\,' + \cancel{\vec{a}_{o'}}^{\,0} + 2\vec{\Omega}\times\vec{v}\,' + \vec{\Omega}\times(\vec{\Omega}\times\vec{r}\,') + \cancel{\dot{\vec{\Omega}}\times\vec{r}\,'}^{\,0}) = m\vec{g} + \vec{N}_v + \vec{N}_p$$

$\hat{x}'$ | $\quad m\ddot{x} - m\Omega^2 x\cos^2\alpha = mg\sin\alpha \qquad (1)$

$\hat{y}'$ | $\quad -m\Omega^2 x\cos\alpha\sin\alpha = N_v - mg\cos\alpha \qquad (2)$

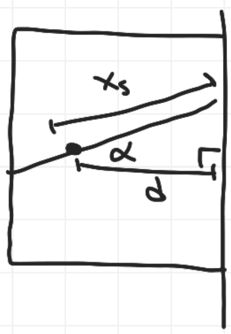
$\hat{z}'$ | $\quad -2m\Omega\dot{x}\cos\alpha = -N_p \qquad (3)$

c) · separación de la vara: $N_v = 0$

$$(2) \longrightarrow \Omega^2 x_s \cancel{\cos\alpha}\sin\alpha = g\cancel{\cos\alpha}$$

$$x_s = \frac{g}{\Omega^2\sin\alpha}$$

dist. al eje de rotación:



$$\cos\alpha = \frac{d}{x_s} \rightarrow \boxed{d = x_s\cos\alpha}$$

d) · fza puerta ejerce: N_p

$$(3) \rightarrow \quad N_p = 2m\Omega\dot{x}\cos\alpha \qquad (\dot{x}?)$$

$(1) \longrightarrow \quad \ddot{x} - \Omega^2 x \cos^2\alpha = g\sin\alpha$

$$\ddot{x} = x\Omega^2\cos^2\alpha + g\sin\alpha \qquad \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{dx}\,\frac{dx}{dt}$$

$$\dot{x}\,\frac{d\dot{x}}{dx} = x\,\Omega^2\cos^2\alpha + g\sin\alpha$$

$$\dot{x}\,d\dot{x} = \Omega^2 x\cos^2\alpha\,dx + g\sin\alpha\,dx$$

$$\frac{1}{2}(\dot{x}^2 - \underbrace{\dot{x}_0^2}_{0}) = \Omega^2\cos^2\alpha\cdot\frac{1}{2}(x^2 - \cancel{x_0^2}^{\,0}) + g\sin\alpha\,(x - \cancel{x_0}^{\,0})$$

(repouso)

$$\dot{x}^2 = \Omega^2\cos^2\alpha\,x^2 + 2g\sin\alpha\,x$$

$$\dot{x}_s^2 = \Omega^2\cos^2\alpha\left(\frac{g}{\Omega^2\sin\alpha}\right)^2 + 2g\cancel{\sin\alpha}\left(\frac{g}{\Omega^2\cancel{\sin\alpha}}\right)$$

$$= \cot^2\alpha\,\frac{g^2}{\Omega^2} + 2\,\frac{g^2}{\Omega^2}$$

$$N_p = 2m\Omega\cos\alpha\,\frac{g}{\Omega}\sqrt{\cot^2\alpha + 2}$$

$$\boxed{N_p = 2mg\cos\alpha\sqrt{\cot^2\alpha + 2}}$$