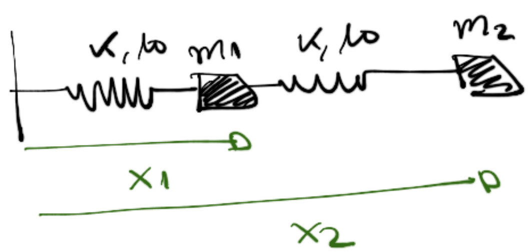
Pauta Aux Extra C3

P1)

Tenemos:



Anotemos las ecuaciones de mov

$$m_1 \ddot{x}_1 = -k(x_1 - l_0) + k((x_2 - x_1) - l_0)$$

$$m_2 \ddot{x}_2 = -k(x_2 - x_1 - l_0)$$

Cambiando de variables a $\quad y_1 = x_1 - l_0$

$$y_2 = x_2 - 2l_0$$

queda:

$$m_1 \ddot{y}_1 = -2k y_1 + k y_2$$

$$m_2 \ddot{y}_2 = -k y_2 + k y_1$$

$$\ddot{y}_1 = -\frac{k}{m_1}(2y_1 - y_2) \qquad \frac{k}{m_1} = \omega_1^2$$

$$\ddot{y}_2 = \frac{-k}{m_2}(y_2 - y_1) \qquad \frac{k}{m_2} = \omega_2^2$$

Matricialmente:

$$\begin{pmatrix} \ddot{y}_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -2\omega_1^2 & \omega_1^2 \\ \omega_2^2 & -\omega_2^2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

Como buscamos oscilaciones: $y_1 = A\cos(\omega t + \phi)$

$$\rightarrow \ddot{y}_1 = -\omega^2 y_1$$

lo mismo con y_2 $\longrightarrow$ $\ddot{y}_2 = -\omega^2 y_2$

$$-\omega^2 \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -2\omega_1^2 & \omega_1^2 \\ \omega_2^2 & -\omega_2^2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 2\omega_1^2 - \omega^2 & -\omega_1^3 \\ -\omega_2^2 & \omega_2^2 - \omega^2 \end{pmatrix}} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = 0$$

buscamos soluciones de ω $\rightarrow$ $\det(\quad) = 0$

(valores propios)

$$(2\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2) - \omega_1^2\omega_2^2 = 0$$

$$2\omega_1^2\omega_2^2 - 2\omega_1^2\omega^2 - \omega_2^2\omega^2 + \omega^4 - \omega_1^2\omega_2^2 = 0$$

$$\omega^4 - \omega^2 \cdot 2\omega_1^2 - \omega^2\omega_2^2 + \omega_1^2\omega_2^2 = 0$$

$$\omega^4 - \omega^2(2\omega_1^2 + \omega_2^2) + \omega_1^2\omega_2^2 = 0$$

$$\omega^2 = \frac{(2\omega_1^2+\omega_2^2) \pm \sqrt{(2\omega_1^2+\omega_2^2)^2 - 4\omega_1^2\omega_2^2}}{2}$$

$$= \frac{(2\omega_1^2+\omega_2^2) \pm \sqrt{4\omega_1^4 + \cancel{4\omega_1^2\omega_2^2} + \omega_2^2 - \cancel{4\omega_1^2\omega_2^2}}}{2}$$

$$\boxed{\omega^2 = \frac{2\omega_1^2 + \omega_2^2 \pm \sqrt{4\omega_1^4 + \omega_2^4}}{2}}$$

$\rightarrow$ frecuencias de oscilación !!

Para sacar los modos: vectores propios:

con $\omega^2 = \dfrac{2\omega_1^2 + \omega_2^2 + \sqrt{4\omega_1^4 + \omega_2^4}}{2}$

$$\rightarrow \begin{pmatrix} 2\omega_1^2 - \dfrac{2\omega_1^2+\omega_2^2+\sqrt{4\omega_1^4+\omega_2^4}}{2} & -\omega_1^2 \\ \omega_2^2 & \omega_2^2 - \dfrac{2\omega_1^2+\omega_2^2+\sqrt{4\omega_1^4+\omega_2^4}}{2} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0$$

usando la primera fila. A_1 A_2

$$\left(2\omega_1^2 - \frac{2\omega_1^2+\omega_2^2+\sqrt{4\omega_1^4+\omega_2^4}}{2}\right) A_1 - \omega_1^2 A_2 = 0$$

$$\frac{2\omega_1^2-\omega_2^2-\sqrt{4\omega_1^4+\omega_2^4}}{2} A_1 = \omega_1^2 A_2$$

$$\rightarrow \boxed{A_2 = \frac{2\omega_1^2-\omega_2^2-\sqrt{4\omega_1^4+\omega_2^4}}{2\omega_1^2} A_1}$$

$$\text{Vector propio: } \begin{pmatrix} 1 \\ \dfrac{2\omega_1^2-\omega_2^2-\sqrt{4\omega_1^4+\omega_2^4}}{2\omega_1^2} \end{pmatrix}$$

haciendo lo mismo con

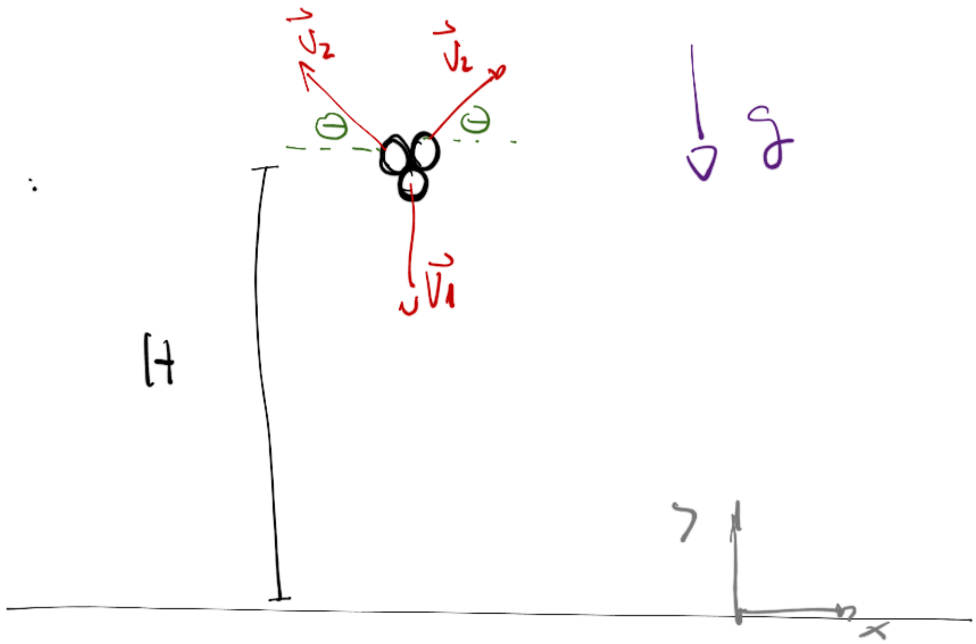
$$\omega^2 = \frac{2\omega_1^2 + \omega_2^2 - \sqrt{4\omega_1^4 + \omega_2^4}}{2}$$

Quedará:

$$\boxed{\left(\frac{1}{\frac{2\omega_1^2 - \omega_2^2 + \sqrt{4\omega_1^4 + \omega_2^4}}{2\omega_1^2}}\right)}$$

P2)

Tenemos:



Sabemos que un fragmento llega en t_1 al suelo y los otros dos en t_2.

Sabemos que en el eje y hay mov rec. unifor. acelerado.

$$\longrightarrow \boxed{\begin{aligned} y_1(t) &= H - v_1 t - g\frac{t^2}{2} \\ y_2(t) &= H + v_2 \operatorname{sen}\theta - g\frac{t^2}{2} \end{aligned}}$$

y sabemos que en t_1 $\quad y_1(t_1) = 0$

$t_2 \quad y_2(t_2) = 0$

$$\boxed{\begin{aligned} 0 &= H - v_1 t_1 - g\frac{t_1^2}{2} \\ 0 &= H + v_2 \operatorname{sen}\theta\, t_2 - g\frac{t_2^2}{2} \end{aligned}}$$

Tenemos 2 ecuaciones pero 3 incógnitas H, v_1, v_2

→ Antes de explotar $\vec{V}_{CM}$ (velocidad CM $= 0$)

→ $\vec{P}_{CM} = 0$ { se conserva

→ $0 = m\vec{v}_1 + m\vec{v}_2^{\,1} + m\vec{v}_2^{\,2}$

$= -v_1\hat{y} + (v_2 \cancel{\cos\theta}\,\hat{x} + v_2 \operatorname{sen}\theta\,\hat{y}) + (v_2 \cancel{(-\cos\theta)}\hat{x}$
$+ v_2 \operatorname{sen}\theta\,\hat{y})$

$0 = -v_1 + 2v_2 \operatorname{sen}\theta$

$$\boxed{v_1 = 2v_2 \operatorname{sen}\theta}$$

Usando esto tenemos:

$$0 = H - 2v_2 \operatorname{sen}\theta\, t_1 - \frac{g t_1^2}{2} \quad (1)$$

$$0 = H - V_2 \,\text{sen}\theta\, t_2 - g\frac{t_2^2}{2} \qquad (2)$$

$$\to \quad (1)\cdot t_2 - 2(2)\cdot t_1$$

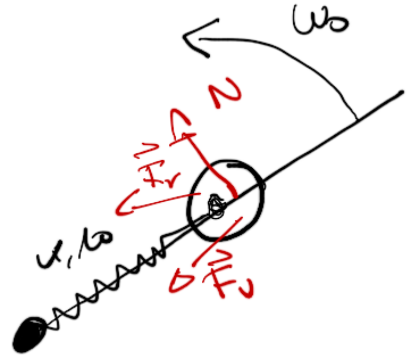
$$0 = Ht_2 - \cancel{2V_2 \,\text{sen}\theta\, t_1 t_2} - g\frac{t_1^2 t_2}{2} - 2Ht_1$$

$$+ \cancel{2V_2 \,\text{sen}\theta\, t_1 t_2} + g\frac{t_1 t_2^2}{2}$$

$$0 = H(t_2 - 2t_1) - g/_2 \,(t_1^2 t_2 - t_1 t_2^2)$$

$$\boxed{H = \frac{g}{2}\frac{t_1 t_2 (t_2 - t_1)}{(t_2 - 2t_1)}}$$

P3)



Ecuaciones de mov en cilíndrica:

$$m((\ddot\rho - \rho\dot\theta^2)\hat\rho + (2\dot\rho\dot\theta + \rho\ddot\theta)\hat\theta) = N\hat\theta - k(\rho - l_0)\hat\rho - c\dot\rho\hat\rho$$

→ 2 ecuaciones

$$m(\ddot\rho - \rho\cancelto{\omega_0^2}{\dot\theta^2}) = -k(\rho - l_0) - c\dot\rho$$

$$m(2\dot\rho\cancelto{\omega_0^2}{\dot\theta} + \cancelto{0}{\rho\ddot\theta}) = N$$

debemos resolver:

$$\ddot\rho - \omega_0^2\rho = -\frac{k}{m}(\rho - l_0) - \frac{c\dot\rho}{m}$$

$$\underbrace{\ddot{\rho} + \frac{c}{m}\dot{\rho} + \left(\frac{k}{m} - \omega_0^2\right)\rho = \frac{k l_0}{m}}$$

sol homogenea + sol particular

$\hookrightarrow \rho = \frac{k l_0}{m} \cdot \frac{1}{\left(\frac{k}{m} - \omega_0^2\right)}$

$$\boxed{\ddot{\rho} + \frac{c}{m}\dot{\rho} + \left(\frac{k}{m} - \omega_0^2\right)\rho = 0}$$

definiendo $2\gamma = \frac{c}{m}$

$\omega^2 = \frac{k}{m} - \omega_0^2$

$$\underbrace{\ddot{\rho} + 2\gamma\dot{\rho} + \omega^2\rho = 0}$$

tiene solución: $\rho(t) = e^{-\gamma t} \cdot A e^{i\left(\sqrt{\omega^2 - \gamma^2}\, t + \delta\right)}$

si

1) $\omega^2 > \gamma^2$ (es decir $\frac{k}{m} - \omega_0^2 > 4c^2$)

$$\boxed{p(t) = A \cdot e^{-\gamma t} \cdot \cos\left(\sqrt{\omega^2-\gamma^2}\, t + \phi\right)}$$

2) $\omega^2 = \gamma^2$ (es decir $\frac{k}{m} - \omega_0^2 = 4c^2$)

$$\boxed{p(t) = A \cdot e^{-\gamma t}}$$

3) $\omega^2 < \gamma^2 \longrightarrow p(t) = A e^{-\gamma t} \cdot e^{i\left(\sqrt{\omega^2-\gamma^2}\, t + \delta\right)}$

$\boxed{B = A \cdot e^{i\delta}}$

$$= B e^{-\gamma t}\, e^{-\sqrt{\gamma^2-\omega^2}\, t}$$