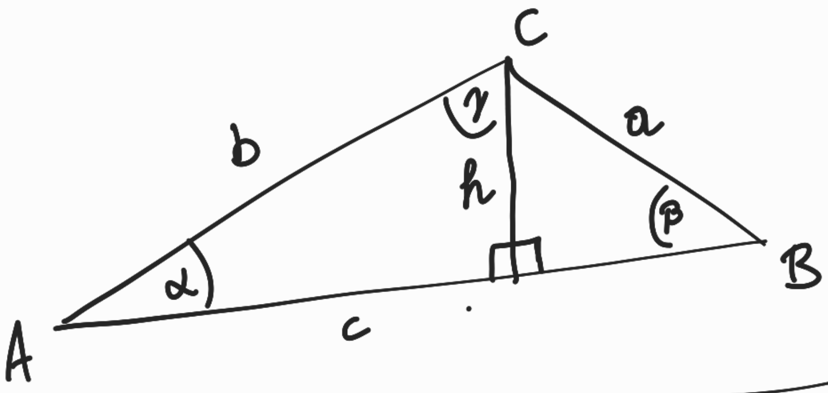
P1



Area $= A$

Notemos que $\boxed{h = b\sin\alpha = a\sin\beta}$

y Notar que $A = \dfrac{h\cdot c}{2}$ $\left(\dfrac{\text{Base}\cdot\text{altura}}{2}\right)$

luego $\Rightarrow$ $\boxed{2A = h\cdot c}$

y Notar que $c = b\cos\alpha + a\cos\beta$

$$\Rightarrow 2A = b\,h\cos\alpha + a\,h\cos\beta$$

$$= b^2\sin\alpha\cos\alpha + a^2\sin\beta\cos\beta$$

y recordar que $\boxed{\sin 2\alpha = 2\sin\alpha\cos\alpha}$

$$\Rightarrow 2A = b^2\frac{\sin 2\alpha}{2} + a^2\frac{\sin 2\beta}{2}$$

$$\boxed{4A = b^2\sin 2\alpha + a^2\sin 2\beta}$$

P2

2) $\boxed{\tan^2 x + 1 = \sec^2 x}$

Veamos: $\dfrac{\sin^2 x}{\cos^2 x} + 1 = \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} = \sec^2 x$ $\square$

3) $\sin 2x = \dfrac{2\tan x}{1+\tan^2 x}$

Veamos:

$\to \dfrac{2\tan x}{1+\tan^2 x} = \dfrac{2\tan x}{\sec^2 x} = 2\dfrac{\sin x}{\cancel{\cos x}} \cdot \cos^{\cancel{2}} x$

$= 2\sin x \cos x = \sin 2x$ $\square$

4) $1+\sin 2x = (\sin x + \cos x)^2$

$\boxed{1 = \cos^2 x + \sin^2 x}$

$\to 1 + \sin 2x = \cos^2 x + \sin 2x + \sin^2 x$

$= \cos^2 x + 2\sin x \cos x + \sin^2 x$

$= (\cos x + \sin x)^2$ $\square$

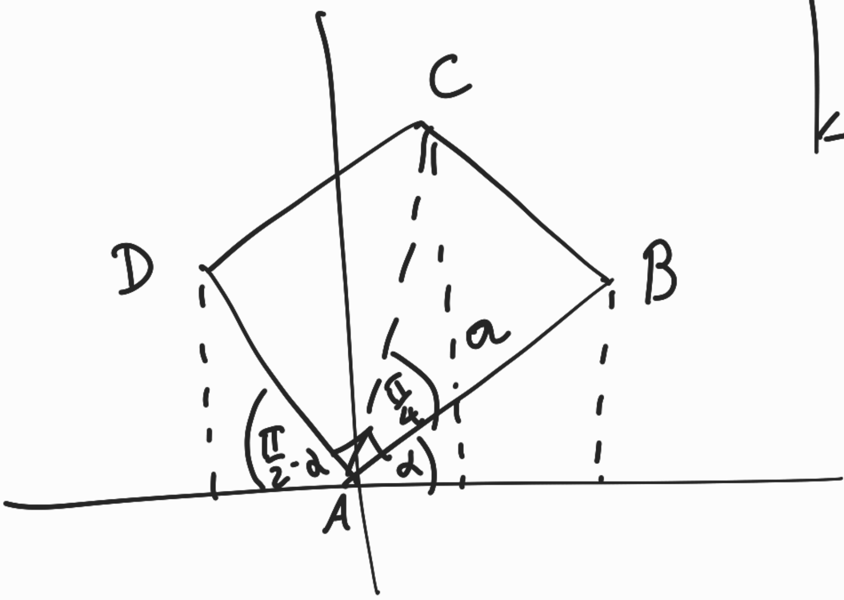
1) $\tan\alpha - \tan\beta = \dfrac{\sin(\alpha-\beta)}{\cos\alpha \cos\beta}$

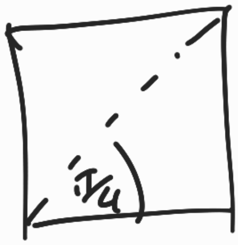
$$\frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta} = \frac{\sin\alpha\cos\beta - \sin\beta\cos\alpha}{\cos\alpha\cos\beta}$$

$$= \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} \quad \square$$

<u>Propuesto ver el crecimiento!</u>

<u>P3)</u>





$B(a\cos\alpha, a\sin\alpha)$

$D\left(-a\cos\left(\frac{\pi}{2}-\alpha\right), a\sin\left(\frac{\pi}{2}-\alpha\right)\right)$

$= D(-a\sin\alpha, a\cos\alpha)$

Notar que como C es la diagonal dentro del cuadrado tiene un angulo de $\frac{\pi}{4}$ y largo $\sqrt{2}a$

$$\Rightarrow \quad C = \left(\sqrt{2}\,a\cos\left(\alpha+\frac{\pi}{4}\right),\ \sqrt{2}\,a\sin\left(\alpha+\frac{\pi}{4}\right)\right)$$

$$\cos\left(\alpha+\frac{\pi}{4}\right) = \cos\alpha\cos\frac{\pi}{4} - \sin\alpha\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$$

$$\sin\left(\alpha+\frac{\pi}{4}\right) = \sin\alpha\cos\frac{\pi}{4} + \cos\alpha\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha)$$

$$C = \left(a(\cos\alpha - \sin\alpha),\ a(\sin\alpha + \cos\alpha)\right)$$

$$m_{AC} = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha}$$

$$L_{AC}:\ y = m_{OC}x$$

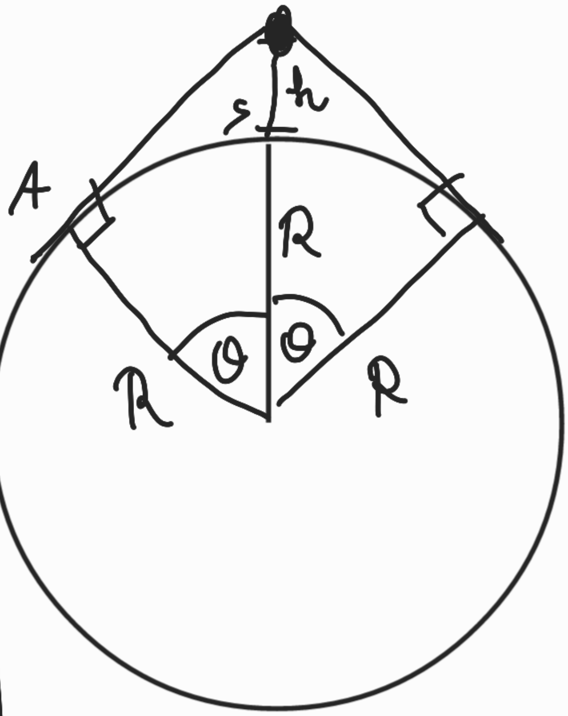
$$y(\cos\alpha - \sin\alpha) = (\sin\alpha + \cos\alpha)x$$

$$\boxed{(\sin\alpha + \cos\alpha)x + y(\sin\alpha - \cos\alpha) = 0}$$

$\square$

P5

Notar que
el maximo es
cuando sea
tangente la recta
de Visión !



Notar que $\boxed{\cos\theta = \dfrac{R}{R+h}}$

$$\theta = \arccos\left(\frac{R}{R+h}\right)$$

y el largo del arco es :

$$\boxed{\overset{\frown}{AS} = R\theta = R\arccos\left(\frac{R}{R+h}\right)}$$

P2)

los de $\tan\left(\frac{\alpha}{2}\right)$

5) $\sin(\alpha) = \sin\left(2\,\frac{\alpha}{2}\right)$

$$= 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

$$= \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}} \cdot \frac{\frac{1}{\cos^2\frac{\alpha}{2}}}{\frac{1}{\cos^2\frac{\alpha}{2}}}$$

$$= \frac{2\,\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}}{\frac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}+1}$$

$$= \frac{2\tan\frac{\alpha}{2}}{\tan^2\frac{\alpha}{2}+1} \quad \square$$

б) $\cos\alpha = \cos\left(2\frac{\alpha}{2}\right) = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}$

$$= \frac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}} \cdot \frac{\frac{1}{\cos^2\frac{\alpha}{2}}}{\frac{1}{\cos^2\frac{\alpha}{2}}}$$

$$= \frac{1 - \tan^2\frac{\alpha}{2}}{1 + \tan^2\frac{\alpha}{2}} \quad \square$$