Resumen!! funciones inversas trigonométricas

arcsin : $[-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$ $\quad x \to \arcsin(x) = y \iff x = \sin y$

arccos : $[-1,1] \to [0, \pi]$ $\quad x \to \arccos(x) = y \iff x = \cos(y)$

arctan : $(-\infty, \infty) \to (-\frac{\pi}{2}, \frac{\pi}{2})$ $\quad x \to \arctan(x) = y \iff x = \tan(y)$

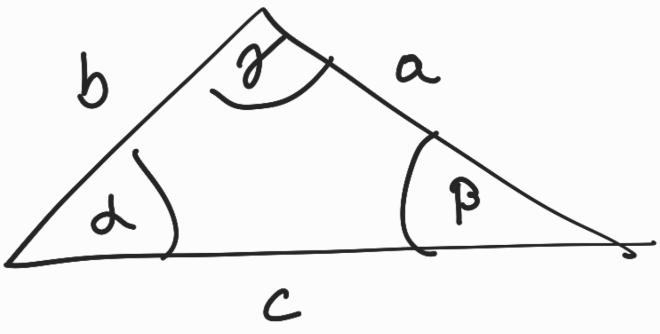
Ecuaciones $\boxed{\text{para } |a| \leq 1}$ $\boxed{b \in \mathbb{R}}$

$$\cos x = a \Rightarrow x = 2k\pi \pm \arcsin(a)$$

$$\sin x = a \Rightarrow x = k\pi + (-1)^k \arcsin(a)$$

$$\tan x = b \Rightarrow x = k\pi + \arctan(b)$$

Teoremas seno y coseno



Para Resolver ec. Siempre dejarlas de esta forma! Puesto que conocemos su solución!

Teo sin:

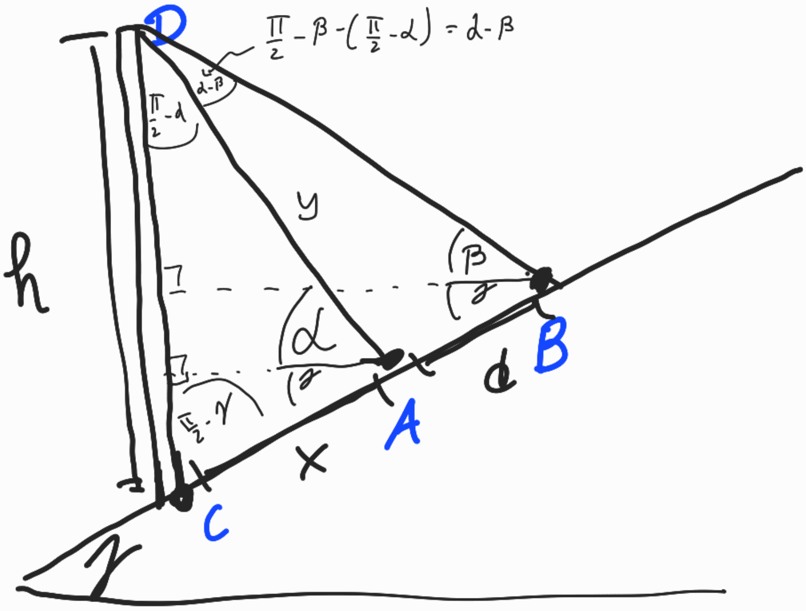
$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Teo cos:

$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos\alpha \\ b^2 &= a^2 + c^2 - 2ac\cos\beta \\ c^2 &= b^2 + a^2 - 2ba\cos\gamma \end{aligned}}$$

P2

Solución 1:



La idea es encontrar y para tener una relación entre $\boxed{h}$ y $\boxed{d}$

Usando el teorema del seno en el Δ ADC

$$\frac{\sin\left(\frac{\pi}{2}-\gamma\right)}{y} = \frac{\sin(\alpha+\gamma)}{h} \Rightarrow \boxed{y = \frac{h\cos(\gamma)}{\sin(\alpha+\gamma)}}$$

y lo mismo en el Δ ABD

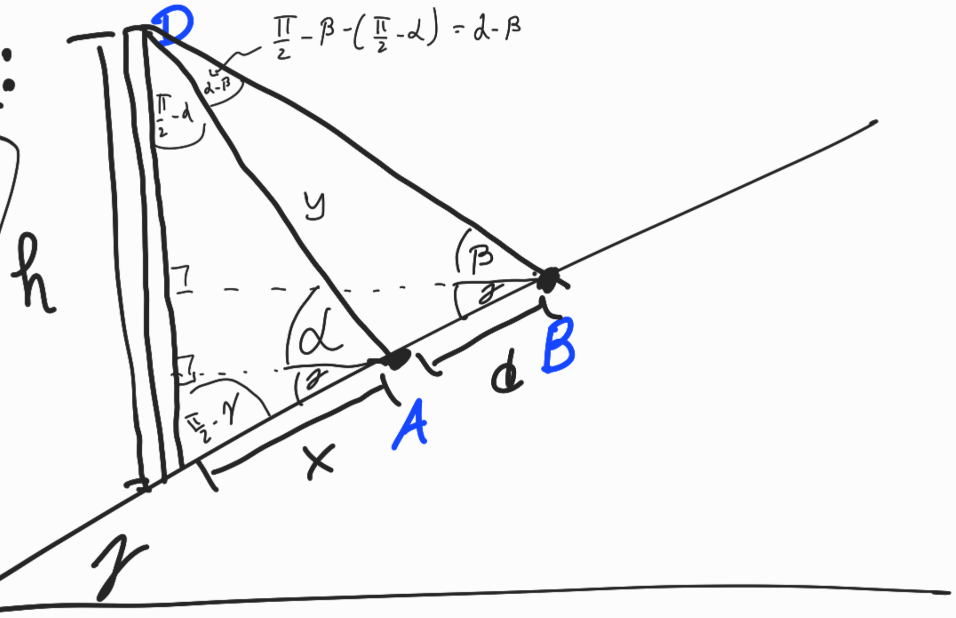
$$\frac{\sin(\alpha-\beta)}{d} = \frac{\sin(\beta+\gamma)}{y} \Rightarrow y = \frac{d\sin(\beta+\gamma)}{\sin(\alpha-\beta)}$$

igualando:
$$\frac{d\sin(\beta+\gamma)}{\sin(\alpha-\beta)} = \frac{h\cos\gamma}{\sin(\alpha+\gamma)}$$

$$\boxed{h = d\sin(\beta+\gamma)\sin(\alpha+\gamma)\sec\gamma\csc(\alpha-\beta)}$$

P2 | Sol 2:

Busquemos $\boxed{x}$ para tener algun tipo de relacion!



Por teo del seno

$$\boxed{\frac{\sin(\alpha+\gamma)}{h}=\frac{\cos(\alpha)}{x}} \Rightarrow \boxed{x=\frac{h\cos\alpha}{\sin(\alpha+\gamma)}}$$

$$\frac{\sin(\beta+\gamma)}{h}=\frac{\cos(\beta)}{d+x}$$

$$d+x=\frac{h\cos\beta}{\sin(\beta+\gamma)} \Rightarrow x=\frac{h\cos\beta}{\sin(\beta+\gamma)}-d$$

$$h\left(\frac{\cos\alpha}{\sin(\alpha+\gamma)} - \frac{\cos\beta}{\sin\beta+\gamma}\right) = -d$$

$$h\left(\sin(\beta+\gamma)\cos\alpha - \sin(\alpha+\gamma)\cos\beta\right)$$

$$= -d\sin(\alpha+\gamma)\sin(\beta+\gamma)$$

$$\boxed{\begin{aligned} \sin(\beta+\gamma) &= \sin\beta\cos\gamma + \cos\beta\sin\gamma \\ \sin(\alpha+\gamma) &= \sin\alpha\cos\gamma + \cos\alpha\sin\gamma \end{aligned}}$$

$$\cos\alpha\sin(\beta+\gamma) = \cos\alpha\sin\beta\cos\gamma + \cos\alpha\cos\beta\sin\gamma$$

$$\cos\beta\sin(\alpha+\gamma) = \cos\beta\sin\alpha\cos\gamma + \cos\beta\cos\alpha\sin\gamma \quad (-)$$

la resta! $= \cos\alpha\sin\beta\cos\gamma - \cos\beta\sin\alpha\cos\gamma$

$$= -\cos\gamma\left(\sin\alpha\cos\beta - \sin\beta\cos\alpha\right)$$

$$= -\cos\gamma\,\sin(\alpha-\beta)$$

$$\Rightarrow h(-\cos\gamma)\sin(\alpha-\beta) = -d\sin(\alpha+\gamma)\sin(\beta+\gamma)$$

$$\Rightarrow \boxed{h = d\sin(\alpha+\gamma)\sin(\beta+\gamma)\sec(\gamma)\operatorname{cosec}(\alpha-\beta)}$$

P1 $\boxed{\sin 2x - \cos\frac{x}{2} = 0}$

Identidade ÚTIL!

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$$

$$\cos\underbrace{\left(\frac{\pi-4x}{2}\right)}_{\alpha} - \cos\underbrace{\left(\frac{x}{2}\right)}_{\beta} = 0$$

$$\alpha+\beta = \frac{\pi-3x}{2}$$

$$\alpha-\beta = \frac{\pi-5x}{2}$$

$$\boxed{-2\sin\left(\frac{\pi-3x}{4}\right)\sin\left(\frac{\pi-5x}{4}\right) = 0}$$

$$\sin\left(\frac{\pi-3x}{4}\right) = 0 \lor \sin\left(\frac{\pi-5x}{4}\right) = 0$$

Por formula:

$$\frac{\pi - 3x}{4} = k\pi \qquad \frac{\pi - 5x}{4} = k\pi$$

$$-3x = (4k-1)\pi \qquad -5x = (4k-1)\pi$$

$$\boxed{x = (1-4k)\frac{\pi}{3}} \qquad \boxed{x = (1-4k)\frac{\pi}{5}}$$

Sol 1 Sol 2

$\boxed{\text{con } k \in \mathbb{Z}}$

2) $3\tan^2 + 5 = \frac{7}{\cos x}$

$$3\frac{\sin^2 x}{\cos^2 x} + 5 = \frac{7}{\cos x} \quad / \cdot \cos^2 x$$

$$3\sin^2 x + 5\cos^2 x - 7\cos x = 0$$

$$3(\underbrace{\cancel{\sin^2 x + \cos^2 x}}_{\to 1}) + 2\cos^2 x - 7\cos x = 0$$

$$2\cos^2 x - 7\cos x + 3 = 0$$

Una cuadrática en $\cos(x)$!

$$\cos x = \frac{7 \pm \sqrt{49 - 8 \cdot 3}}{4}$$

$$= \frac{7 \pm \sqrt{25}}{4} = \frac{7 \pm 5}{4}$$

$$\underbrace{\cos x = 3}_{} \quad \vee \quad \cos x = \frac{1}{2}$$

No tiene sol!

$$\boxed{\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}}$$

$$\cos x = \frac{1}{2} \Rightarrow \boxed{x = 2k\pi \pm \frac{\pi}{3}}$$

3) $\cos x + \sin x = 1 \quad / \cdot \frac{1}{\sqrt{2}}$

Truco:

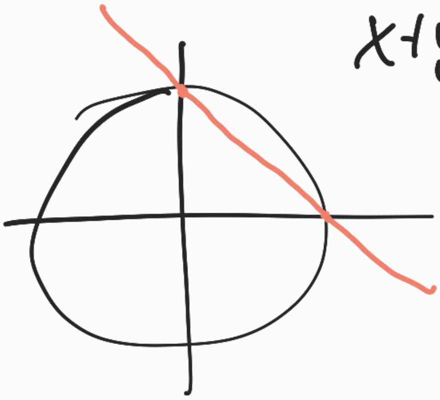
$$\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x = \sin \frac{\pi}{4} \cos x + \cos \frac{\pi}{4} \sin x$$

$$= \boxed{\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}}$$

Luego como $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} \Rightarrow \boxed{x + \frac{\pi}{4} = k\pi + (-1)^k \frac{\pi}{4}}$

$$\Rightarrow \boxed{x = k\pi + (-1)^k \frac{\pi}{4} - \frac{\pi}{4}} \qquad \boxed{\begin{array}{ll} k \text{ par} & x = 2n\pi \\ k \text{ impar} & x = (2n+1)\pi - \frac{\pi}{2} \end{array}}$$

$x + y = 1 \cap C(0,1)$



otra forma:

$t = \tan\frac{x}{2}$

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}$$

$$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} - \frac{(1+t^2)}{1+t^2} = 0$$

$$\frac{2t + \cancel{1} - t^2 - \cancel{1} - t^2}{1+t^2} = \frac{2t(1-t)}{1+t^2} = 0$$

$$\Rightarrow 2t = 0 \quad \vee \quad 1 - t = 0$$

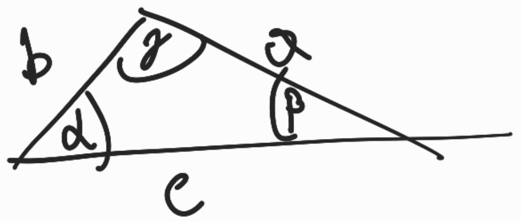
$$\tan\frac{x}{2} = 0 \quad \vee \quad \tan\frac{x}{2} = 1$$

$\frac{x}{2} = k\pi \quad \vee \quad \frac{x}{2} = k\pi + \frac{\pi}{4}$

$\boxed{x = 2k\pi} \qquad \boxed{x = 2k\pi + \frac{\pi}{2}}$

las mismas soluciones!

P3) $\boxed{b\cos\gamma - c\cos\beta = \frac{1}{a}(b^2 - c^2)}$



$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta \quad \cdot \frac{1}{a}$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma \quad / \cdot \frac{1}{a}$$

la idea es construir la igualdad con el teorema del coseno!!

$$\frac{c^2}{a} = \frac{a^2}{a} + \frac{b^2}{a} - 2b\cos\gamma$$

$$\boxed{\frac{-c^2 + a^2 + b^2}{2a} = b\cos\gamma}$$

$$\boxed{\frac{a^2 + c^2 - b^2}{2a} = c\cos\beta}$$

$$b\cos\gamma - c\cos\beta = \frac{\cancel{a^2} + b^2 - c^2 + b^2 - \cancel{a^2} - c^2}{2a}$$

$$= \frac{\cancel{2}(b^2 - c^2)}{\cancel{2}a} = \frac{1}{a}(b^2 - c^2) \quad \square$$

P4) $2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ | PROPUESTO

$= 2\left(\cos\frac{\alpha}{2}\cos\frac{\beta}{2} - \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right)$

$\cdot\left(\cos\frac{\alpha}{2}\cos\frac{\beta}{2} + \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right)$

$= 2\left(\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2} - \sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}\right)$

$= 2\left(\left(\frac{\cos\alpha+1}{2}\right)\left(\frac{\cos\beta+1}{2}\right) - \left(\frac{1-\cos\alpha}{2}\right)\left(\frac{1-\cos\beta}{2}\right)\right)$

$\cos 2x = \cos^2 x - \sin^2 x$
$= 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$

$= \frac{1}{2}\left(\cancel{\cos\alpha\cos\beta} + \cos\alpha + \cos\beta + \cancel{1} - \left(\cancel{1} - \cos\alpha - \cos\beta + \cancel{\cos\alpha\cos\beta}\right)\right)$

$= \frac{1}{\cancel{2}}\left(\cancel{2}\left(\cos\alpha + \cos\beta\right)\right) = \boxed{\cos\alpha + \cos\beta}$ $\square$

2) $1 + \underline{\cos x} + \cos 2x + \underline{\cos 3x} = 0$

$\cos x + \cos 3x = 2\cos 2x \cos x$

$2\cos 2x\cos x + \underline{1 + \cos 2x}$

$1 = \cos 0 \Rightarrow \boxed{\cos 2x + \cos 0} = 2\cos\left(\frac{2x}{2}\right)\cos\left(\frac{2x}{2}\right)$

$= 2\cos^2(x)$

$$= 2\cos 2x \cos x + 2\cos^2 x$$

$$= 2\cos x\left(\cos 2x + \cos x\right) = 0$$

$$\cos x = 0 \quad \vee \quad \cos 2x + \cos x = 0$$

$$\boxed{x = 2\pi k \pm \frac{\pi}{2}} \qquad 2\cos\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right) = 0$$

$$\cos\frac{3x}{2} = 0 \quad \vee \quad \cos\frac{x}{2} = 0$$

$$\frac{3x}{2} = 2k\pi \pm \frac{\pi}{2} \quad \vee \quad \frac{x}{2} = 2k\pi \pm \frac{\pi}{2}$$

$$\boxed{x = \frac{4k\pi}{3} \pm \frac{\pi}{3}} \quad \vee \quad \boxed{x = 4k\pi \pm \pi}$$

P5] Proposto!

$$\cos(x+y) = 0 \Rightarrow \cos^2(x+y) = 0 \Rightarrow \boxed{\sin^2(x+y) = 1}$$

$$\sin(x+2y) = \sin(2(x+y) - x)$$

$$= \sin(2(x+y))\cos(x) - \sin(x)\cos(2(x+y))$$

$$= 2\sin(x+y)\cancel{\cos(x+y)}^{\to 0}\cos x - \sin x\left(\cancel{\cos^2(x+y)}^{\to 0} - \sin^2(x+y)\right)$$

$$= \sin x \, \cancel{\sin^2(x+y)}^{\to 1}$$

$$= \sin x \quad \square$$

ii) $$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$- \quad \cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$\frac{1}{\sqrt{2}} /$ $$0 = \frac{2}{\sqrt{2}}\cos\left(\frac{\alpha-\beta}{2}\right)\left(\frac{1}{\sqrt{2}}\sin\left(\frac{\alpha+\beta}{2}\right) - \frac{1}{\sqrt{2}}\cos\left(\frac{\alpha+\beta}{2}\right)\right)$$

$$\cos\frac{\alpha-\beta}{2} = 0 \Rightarrow \boxed{\frac{\alpha-\beta}{2} = \frac{\pi}{2}}$$ No es un Triangulo!!

$\vee$

$$-\cos\left(\frac{\alpha+\beta}{2} + \frac{\pi}{4}\right) = 0$$

Ojo, No me interesan las otras soluciones pq quiero que sea un Δ

$\arccos(\cdot) \Rightarrow$

$$\frac{\alpha+\beta}{2} + \frac{\pi}{4} = \frac{\pi}{2}$$

$$\boxed{\alpha+\beta = \frac{\pi}{2}}$$ y como $\boxed{\alpha+\beta+\gamma = \pi}$

y $\therefore$ $\boxed{\gamma = \frac{\pi}{2}}$ luego es Δ rectangulo!