$$f(x) = c^t x = \sum_{i=1}^{N} c_i x_i \quad , \qquad \frac{\partial f}{\partial x_1}(x) = \frac{\partial}{\partial x_1}(c_1 x_1 + c_2 x_2 + \dots + c_N x_N)$$

$$= c_1$$

$$\vdots$$

$$\frac{\partial f}{\partial x_i}(x) = \frac{\partial}{\partial x_i}(c_1 x_1 + c_2 x_2 + \dots + c_i x_i + \dots + c_N x_N)$$

$$= c_i$$

$$\frac{\partial f}{\partial x_N}(x) = \frac{\partial}{\partial x_N}(c_1 x_1 + \dots + c_N x_N)$$

$$= c_N$$

$$\Rightarrow \boxed{\nabla f(x) = \begin{pmatrix} c_1 \\ \vdots \\ c_i \\ \vdots \\ c_N \end{pmatrix} = c}$$

$$g(x) = x^t A x = \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} x_i x_j$$

$$\frac{\partial g}{\partial x_1}(x) = \sum_{i=1}^{N} a_{ij} \frac{\partial}{\partial x_1} \begin{pmatrix} a_{11} x_1 x_1 + a_{12} x_1 x_2 + \dots + a_{1N} x_1 x_N + \\ a_{21} x_2 x_1 + a_{22} x_2 x_2 + \dots + a_{2N} x_2 x_N + \\ \vdots \\ a_{N1} x_N x_1 + a_{N2} x_N x_2 + \dots + a_{NN} x_N x_N \end{pmatrix}$$

$$= 2 a_{11} x_1 + a_{12} x_2 + \dots + a_{1N} x_N + a_{21} x_2 + \dots + a_{N1} x_N$$

$$= \sum_{i=1}^{N} a_{i1} x_i + \sum_{j=1}^{N} a_{1j} x_j = \sum_{K=1}^{N} (a_{K1} + a_{1K}) x_K$$

En general $\frac{\partial}{\partial x_i} g(x) = \sum_{K=1}^{N} (a_{Ki} + a_{iK}) x_K$

$$\Rightarrow \nabla g(x) = \begin{pmatrix} \sum_{K=1}^{N} (a_{K1} + a_{1K}) x_K \\ \vdots \\ \sum_{K=1}^{N} (a_{Ki} + a_{iK}) x_K \\ \vdots \\ \sum_{K=1}^{N} (a_{KN} + a_{NK}) x_K \end{pmatrix} = \begin{pmatrix} \sum_{K=1}^{N} (a_{K1} + a_{1K}) \\ \vdots \\ \sum_{K=1}^{N} (a_{Ki} + a_{iK}) \\ \vdots \\ \sum_{K=1}^{N} (a_{KN} + a_{NK}) \end{pmatrix} \circ \begin{pmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_N \end{pmatrix}$$

$$Ax = \begin{pmatrix} \sum_{K=1}^{N} a_{1K} x_K \\ \vdots \\ \sum_{K=1}^{N} a_{iK} x_K \\ \vdots \\ \sum_{K=1}^{N} a_{NK} x_K \end{pmatrix} \qquad A^t x = \begin{pmatrix} \sum_{K=1}^{N} a_{K1} x_K \\ \vdots \\ \sum_{K=1}^{N} a_{KN} x_K \end{pmatrix}$$

$$\Rightarrow \boxed{(A + A^t) x = Ax + A^t x = \nabla g(x)}$$