P1 $\dfrac{||x|-|x-2||}{x^2-1} \leq 2$

Notar que si $x \in (-1,1) \Rightarrow x^2-1<0 \rightarrow \dfrac{||x|-|x-2||}{x^2-1} < 0 \leq 2$

$\Rightarrow (-1,1)$ es parte de la solución.

Casos importantes $x=0$, $x=2$

ya está listo

<u>Caso 1</u> $x<-1 \Rightarrow |x|=-x$

$|x-2| = -(x-2)$

$$\Rightarrow \frac{|-x+x-2|}{x^2-1} \leq 2 \Leftrightarrow \frac{2}{x^2-1} \leq 2 \Leftrightarrow \frac{1}{x^2-1} \leq 1$$

Usando que $x^2-1>0 \overset{\cdot(x^2-1)}{\Longrightarrow} 1 \leq x^2-1 \Leftrightarrow 2 \leq x^2$

$\Leftrightarrow x \in (-\infty,-\sqrt{2}] \cup [\sqrt{2},\infty)$

Como $x<-1 \Rightarrow Sol_{\text{caso } 1}\ (-\infty,-\sqrt{2}]$

<u>Caso 2</u>: $x \in (1,2) \Rightarrow |x|=x$

$|x-2| = -(x-2)$

$x+1>0$
$x-1>0$

$$\Rightarrow \frac{|x+x-2|}{x^2-1} \leq 2 \Leftrightarrow \frac{2|x-1|}{x^2-1} \leq 2 \overset{x-1>0}{\Longleftrightarrow} \frac{x-1}{x^2-1} \leq 1 \Leftrightarrow \cancel{1} \leq x+\cancel{1}$$

$\Leftrightarrow 0 \leq x$

$\Rightarrow \text{Sol}_{\text{Caso } 2} = (1,2)$

<u>Caso 3 $\quad x \geq 2$</u> $\Rightarrow$ $|x| = x$

$|x-2| = x-2$

$$\frac{|x-(x-2)|}{x^2-1} \leq 2 \iff \frac{2}{x^2-1} \leq 2 \iff \frac{1}{x^2-1} \leq 1 \overset{x^2-1>0}{\iff} 1 \leq x^2-1$$

$$\iff 2 \leq x^2 \iff x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$$

$$\Rightarrow \text{Sol}_{\text{Caso } ③} = [2, \infty)$$

$$\Rightarrow \text{Solución} = (-\infty, -\sqrt{2}] \cup (-1, 1) \cup (1, \infty)$$

$$\frac{1}{a^{-1}+b^{-1}} < \frac{a+b}{2}$$

Desarrollar a la rapida

$$\frac{ab}{a+b} < \frac{a+b}{2} \Rightarrow 2ab < (a+b)^2 \Rightarrow 0 < a^2+b^2$$

Hacer bien demostración volviendo

$$\left.\begin{array}{l} a \in \mathbb{R}_+^* \xRightarrow{\text{clausura } \cdot} a^2 \in \mathbb{R}_+^* \\ b \in \mathbb{R}_+^* \qquad\qquad b^2 \in \mathbb{R}_+^* \end{array}\right\} \text{clausura } + \quad a^2+b^2 \in \mathbb{R}_+^*$$

$$\xRightarrow{\text{0 neutro aditivo}} a^2+b^2+0 \in \mathbb{R}_+^* \xRightarrow{\text{inverso aditivo}} a^2+b^2+2ab-(2ab) \in \mathbb{R}_+^*$$

$$\xRightarrow{\text{Notación}} (a+b)^2-(2ab) \in \mathbb{R}_+^*$$

~~Como~~ $\cancel{a \in \mathbb{R}_+^* \xRightarrow{\text{inverso}} a^{-1} \in \mathbb{R}_+^*,\ b \in \mathbb{R}_+^* \Rightarrow b^{-1} \in \mathbb{R}_+^*}$ ~~clausura~~ $\cancel{\cdot \quad a^{-1}b^{-1} \in \mathbb{R}_+^*}$

~~Por clausura~~ $\cancel{\cdot \Rightarrow (a+b)^2 a^{-1}b^{-1} - 2 \in \mathbb{R}_+^*}$ $\cancel{a \cdot a^{-1}=1}$ $\cancel{b \cdot b^{-1}=1}$

Como $a \in \mathbb{R}_+^*$, $b \in \mathbb{R}_+^* \xRightarrow[\text{clausura } +]{} a+b \in \mathbb{R}_+^* \Rightarrow (a+b)^{-1} \in \mathbb{R}_+^*$

Por clausura $\cdot$ $\quad (a+b) - 2ab(a+b)^{-1} \in \mathbb{R}_+^*$

Clausura $\cdot \Rightarrow \quad \frac{(a+b)}{2} - ab(a+b)^{-1} \in \mathbb{R}_+^*$

$\Rightarrow \quad \frac{a+b}{2} > ab(a+b)^{-1}$

PDQ $\quad ab(a+b)^{-1} = (a^{-1}+b^{-1})^{-1}$

Vamos a probar que $(a^{-1}+b^{-1})\,ab(a+b)^{-1} = 1$

$$(a^{-1}+b^{-1})\,ab(a+b)^{-1} \overset{\text{Ax Dist}}{=} a^{-1}\cdot ab(a+b)^{-1} + b^{-1}ab(a+b)^{-1}$$

$$\overset{\text{Ax Conmut y Asoc}}{=} (a^{-1}a)\,b(a+b)^{-1} + a(b^{-1}b)(a+b)^{-1}$$

$$= b(a+b)^{-1} + a(a+b)^{-1} \quad / \text{ Usando } a\cdot a^{-1}=1,\ bb^{-1}=1 \text{ y neutro mult}$$

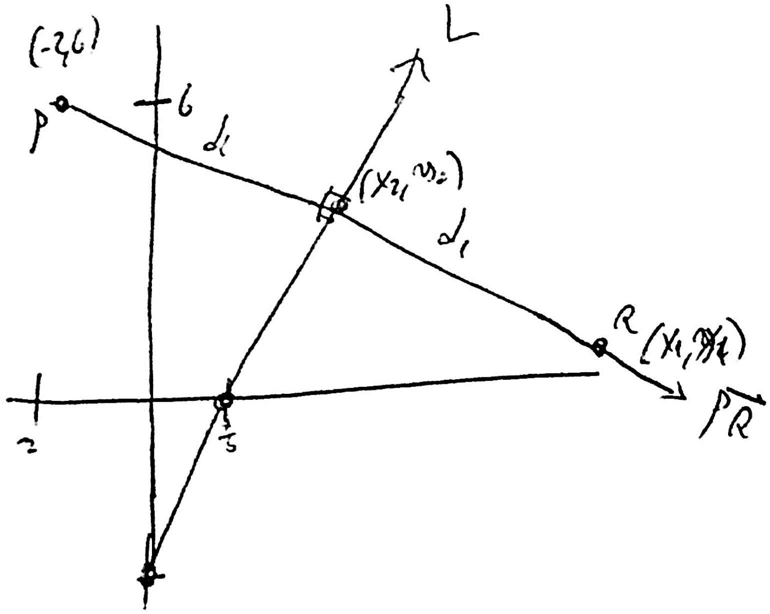
$$= (a+b)(a+b)^{-1} \quad / \text{ Ax Dist, Ax Conm}$$

$$= 1 \quad / \text{ inverso mult}$$

Por unicidad inverso $\Rightarrow \quad (a^{-1}+b^{-1})^{-1} = ab(a+b)^{-1}$

$$\Rightarrow \boxed{\frac{a+b}{2} > (a^{-1}+b^{-1})^{-1}}$$

P3



$$m_L = \frac{5}{2} \Rightarrow m_{\overrightarrow{PR}} = -\frac{2}{5}$$

$$y_1 = -\frac{2}{5}(x_1+2)+6 \quad (1)$$

$$\left.\begin{aligned} y_2 &= -\frac{2}{5}(x_2+2)+6 \quad (2) \\ y_2 &= \frac{5}{2}(x_2) - \frac{7}{2} \quad (3) \end{aligned}\right\}$$

(2) y (3) $\Rightarrow -\frac{2}{5}(x+2)+6 = \frac{5}{2}x_2 - \frac{7}{2} \Rightarrow -4x_2 - 8 + 60 = 25x_2 - 35$

$$\Rightarrow \boxed{x_2 = \frac{87}{29} = 3} \Rightarrow \boxed{y_2 = 4}$$

$$d_1 = \sqrt{(3+2)^2 + (4-6)^2} \Rightarrow d_1^2 = 29$$

$$\cancel{d_1^2 = (3-x_2)^2 + \left(4 - \frac{5x_2}{2} + \frac{7}{2}\right)^2} \quad \text{Sorry ☺}$$

$$\cancel{d_1^2 = (3-x_2)^2\left[1+\left(\frac{5}{2}\right)^2\right] = (3-x_2)^2\,\frac{29}{4}}$$

$$\cancel{\Rightarrow 4 = (3-x_2)^2 \Rightarrow x_2 = 5 \text{ o } x_2 = 1}$$

$$29 = d_1^2 = (3-x_1)^2 + \left(4 + \tfrac{2}{5}(x_1+2) - 6\right)^2$$

$$= (3-x_1)^2 + \left(\tfrac{2}{5}x_1 - \tfrac{6}{5}\right)^2$$

$$= (3-x_1)^2\left[1 + \left(\tfrac{2}{5}\right)^2\right]$$

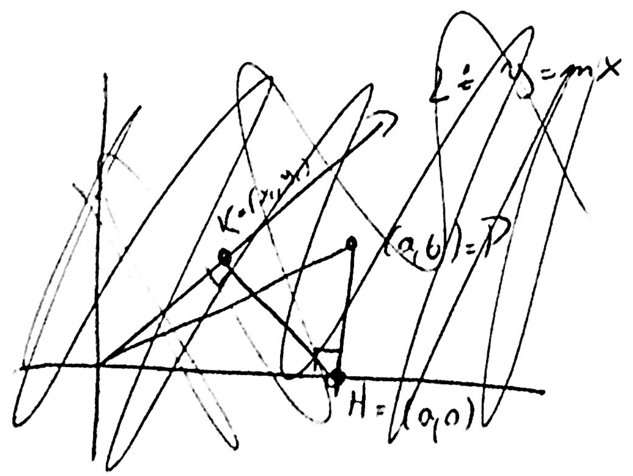
$$= (3-x_1)^2 \left[\tfrac{29}{25}\right]$$

$\Rightarrow (3-x_1)^2 = 25 \Rightarrow \underbrace{x_1 = -2}$ o $x_1 = 8$

es el punto $(-2,6)$ $\qquad$ $\downarrow$ $(8,2)$

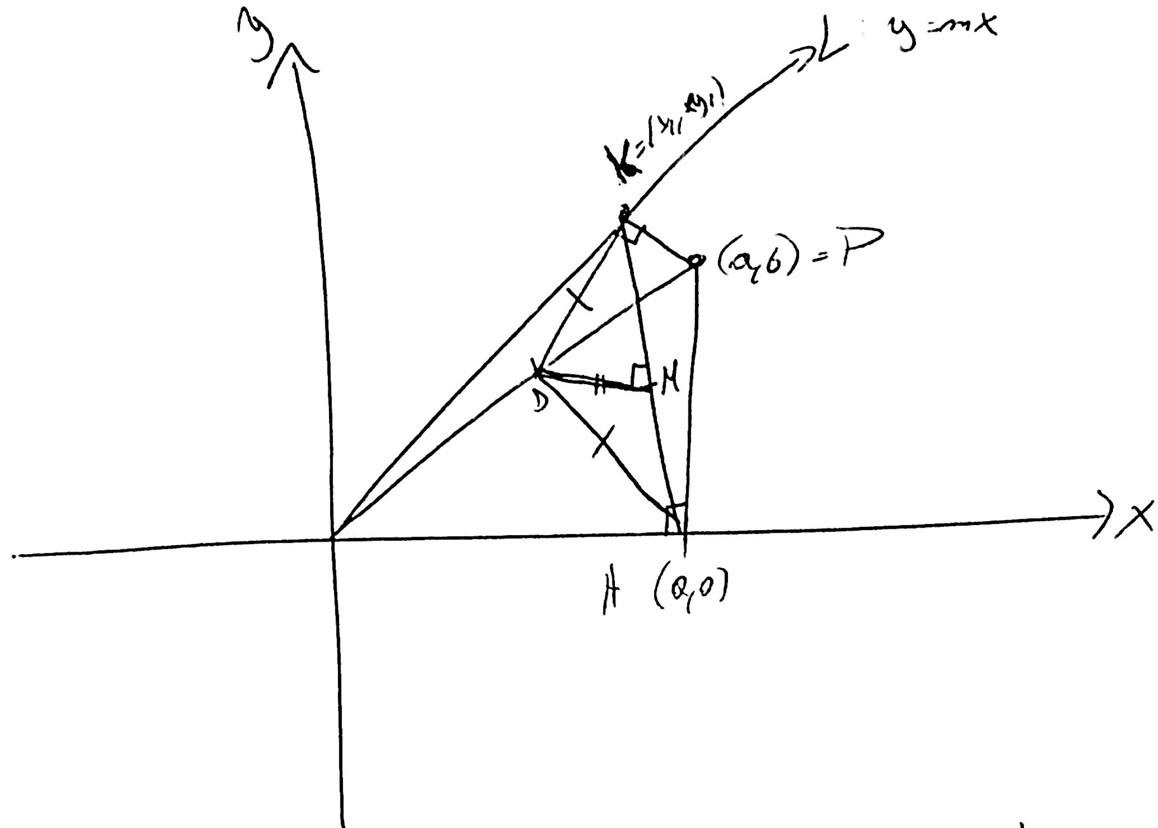
$$\Rightarrow \boxed{Q = (8,2)}$$

d)



P4

Idea



Perpendicular to L

$P=(a,b)$, $H=(a,0)$, $K=(x_1, mx_1)$ $\quad L_{PK}$: then $m_{L_{PK}} = \frac{-1}{m}$

L_{PK} $y-b = \frac{-1}{m}(x-a) \Rightarrow mx_1 - b = -\frac{1}{m}(x_1-a) \Rightarrow m^2x_1 - mb = -x_1 + a$

$\Rightarrow x_1(m^2+1) = mb+a \Rightarrow \boxed{x_1 = \frac{mb+a}{m^2+1}} \Rightarrow \boxed{y_1 = \frac{m}{m^2+1}(mb+a)}$

$D = \left(\frac{a}{2}, \frac{b}{2}\right)$, $M = \left(\frac{a + \frac{mb+a}{m^2+1}}{2}, \frac{m}{2}\,\frac{(mb+a)}{m^2+1} \right)$

$$m_{HK} = \frac{\frac{m(a+mb)}{\cancel{m^2+1}}}{\frac{mb+a-(m^2+1)a}{m^2+1}} = \frac{\cancel{m}(a+mb)}{\cancel{m}(b-ma)} = \frac{a+mb}{b-ma}$$

$$m_{DM} = \frac{\frac{1}{2}\left[\frac{m(mb+a)}{m^2+1} - b\right]}{\frac{1}{2}\left[\frac{mb+a}{m^2+1}\right]} = \frac{ma-b}{mb+a}$$

$m_{HK} \cdot m_{DM} = -1$

$\Rightarrow HK$ perpendicular DM ✓

$$DK = \sqrt{\left(\frac{a}{2} - \left(\frac{mb+a}{m^2+1}\right)\right)^2 + \left(\frac{b}{2} - \left(\frac{m}{m^2+1}\right)(mb+a)\right)^2}$$

$$= \sqrt{\left(\frac{a}{2}\right)^2 - a\left(\frac{mb+a}{m^2+1}\right) + \left(\frac{mb+a}{m^2+1}\right)^2 + \left(\frac{b}{2}\right)^2 - mb\left(\frac{mb+a}{m^2+1}\right) + m^2\left(\frac{mb+a}{m^2+1}\right)^2}$$

$$= \sqrt{\frac{a^2+b^2}{4} - \frac{(mb+a)^2}{(m^2+1)} + \cancel{(m^2+1)}\left(\frac{mb+a}{\cancel{m^2+1}}\right)^2}$$

$$= \sqrt{\frac{a^2+b^2}{4} - \cancel{\frac{(mb+a)^2}{m^2+1}} + \cancel{\frac{(mb+a)^2}{m^2+1}}} = \sqrt{\frac{a^2+b^2}{4}}$$

$$DH = \sqrt{\left(\frac{a}{2}-a\right)^2 + \left(\frac{b}{2}-0\right)^2} = \sqrt{\frac{a^2+b^2}{4}} = DK \implies DH = DK \checkmark$$