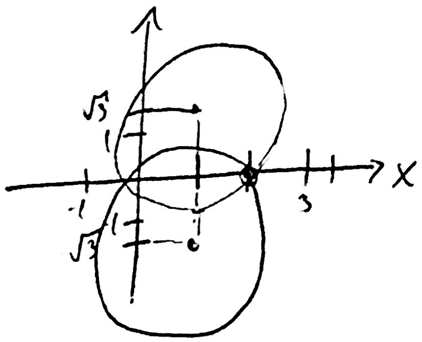
P1 a) $(x-1)^2+(y-2)^2=4$

b) $(x-1)^2+(y-y_c)^2=4$, como pasa por $(2,0)$

$\Rightarrow (2-1)^2+(y_c)^2=4 \Rightarrow y_c^2=3 \Rightarrow y_c=\pm\sqrt{3}$

Hay 2 soluciones $\left.\begin{array}{l}(x-1)+(y-\sqrt{3})=4 \\ (x-1)+(y+\sqrt{3})=4\end{array}\right\}$ Recomiendo Graficar y ver los 2 casos



Hay 2

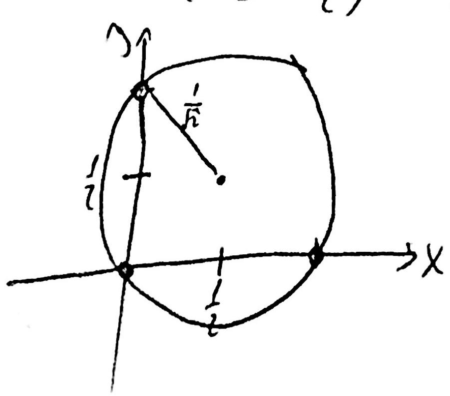
c) $(x-x_c)^2+(y-y_c)^2=R^2$, Reemplazando

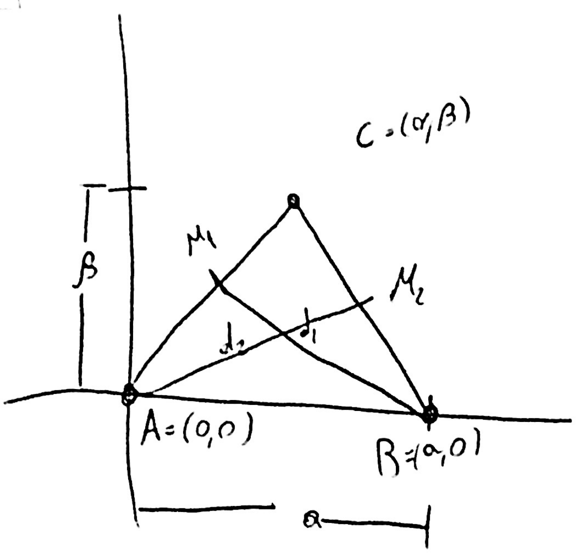
$$x_c^2+y_c^2=R^2 \quad (1)$$
$$(1-x_c)^2+y_c^2=R^2 \quad (2)$$
$$x_c^2+(1-y_c)^2=R^2 \quad (3)$$

Restando (1) y (2) $\Rightarrow 1-2x_c=0 \Rightarrow x_c=\frac{1}{2}$

" (2) y (3) $\Rightarrow 1-2y_c=0 \Rightarrow y_c=\frac{1}{2}$ $\Big\} \Rightarrow R=\frac{1}{\sqrt{2}}$

$\Rightarrow (x-\frac{1}{2})^2+(y-\frac{1}{2})^2=\frac{1}{2}$ Única





$$\Delta = \frac{\text{Base} \cdot \text{Altura}}{2} = \frac{a \cdot \beta}{2}$$

$$M_1 = \left(\frac{\alpha}{2}, \frac{\beta}{2}\right), \quad M_2 = \left(\frac{a+\alpha}{2}, \frac{\beta}{2}\right)$$

$$d_1^2 = \left(a - \frac{\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2, \quad d_2^2 = \left(\frac{a+\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2$$

Imponiendo $d_1^2 + d_2^2 = 5\Delta \iff \boxed{\left(a - \frac{\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2 + \left(\frac{a+\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2 = \frac{5}{2} a\beta}$

Trabajar la ecuación y obtener relación entre (α,β)

$$\iff a^2 - a\alpha + \frac{\alpha^2}{4} + \frac{\beta^2}{4} + \frac{a^2}{4} + \frac{a\alpha}{2} + \frac{\alpha^2}{4} + \frac{\beta^2}{4} = \frac{5}{2} a\beta$$

$$\iff \frac{\alpha^2}{2} + \frac{a\alpha}{2} + \frac{\beta^2}{2} - \frac{5}{2} a\beta = -\frac{5a^2}{4} \qquad /\cdot 2$$

$$\iff \alpha^2 + 2\cdot\alpha\cdot\left(\frac{a}{2}\right) + \beta^2 - 2\cdot\beta\cdot\left(\frac{5a}{2}\right) = -\frac{5a^2}{2}$$

$$\iff \alpha^2 + 2\alpha\left(\frac{a}{2}\right) + \frac{a^2}{4} + \beta^2 - 2\cdot\beta\left(\frac{5}{2}a\right) + \frac{25}{4}a^2 = \frac{28}{4}a^2 - \frac{5a^2}{2}$$

$$\iff \left(\alpha + \frac{a}{2}\right)^2 + \left(\beta - \frac{5}{2}a\right)^2 = (2a)^2$$

$\iff (\alpha,\beta)$ es una circunferencia de centro $\left(-\frac{a}{2}, \frac{5}{2}a\right)$ y

Radio $2a$

P3 a) $\frac{x^2}{25} + \frac{y^2}{9} = 1 \iff \frac{(x-0)^2}{5^2} + \frac{(y-0)^2}{3^2} = 1 \iff$ elipse $a=5$, $b=3$ y centro $(0,0)$

$\Rightarrow$ Horizontal

$$a \cdot e = \sqrt{a^2 - b^2} = 4 \Rightarrow \boxed{\text{Focos} = (\pm 4, 0)}$$

Como la parabola pasa por $(\pm 4, 0)$

$$\Rightarrow 4p(0 - y_0) = (4 - x_0)^2 \iff -4py_0 = 16 - 8x_0 + x_0^2 \quad (1)$$

$$4p(0 - y_0) = (-4 - x_0)^2 \iff -4py_0 = 16 + 8x_0 + x_0^2 \quad (2)$$

Restando (1) y (2) $\Rightarrow 0 = 16x_0 \Rightarrow \boxed{x_0 = 0}$

Remplazando $\overset{(1)}{\Rightarrow} -4py_0 = 16 \iff \boxed{-py_0 = 4}$

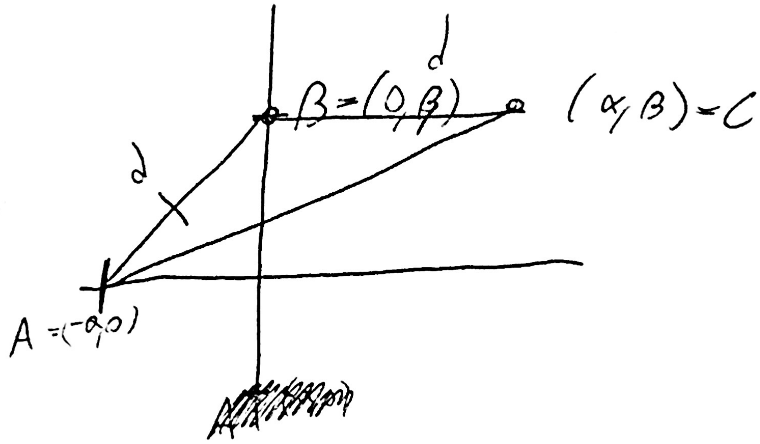
b) Directriz $y = -5 \Rightarrow y_0 - p = -5$, como $p \neq 0$

$$\Rightarrow py_0 - p^2 = -5p \Rightarrow 0 = p^2 - 5p + 4 \Rightarrow p = 4 \text{ o } p = 1$$

Usando que $p \geq 2 \Rightarrow \boxed{p = 4} \to \boxed{y_0 = -1}$

P4 | Asumir

$a > 0$



Buscar relación (α, β)

$$\left.\begin{aligned} d^2 &= a^2 + \beta^2 \\ d^2 &= \alpha^2 \end{aligned}\right\} \Rightarrow \alpha^2 = a^2 + \beta^2$$

$$\Rightarrow \alpha^2 - \beta^2 = a^2 \Rightarrow \frac{\alpha^2}{a^2} - \frac{\beta^2}{a^2} = 1$$

Hiperbola centrada en $(0,0)$ con $a = b$