Pauta Auxiliar 6

P1 a) $\sin(2x) = \dfrac{2\tan(x)}{1+\tan(x)^2}$

$$\frac{2\tan(x)}{1+\tan(x)^2} = \frac{2\frac{\sin(x)}{\cos(x)}}{1+\frac{\sin(x)^2}{\cos(x)^2}} = \frac{2\frac{\sin(x)}{\cos(x)}}{\frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)}} = \frac{2\frac{\sin(x)}{\cos(x)}}{\frac{1}{\cos^2(x)}}$$

$$= 2\frac{\sin(x)}{\cos(x)} \cdot \cos^2(x) = \sin(2x)$$

b) $\cos(2x) = \dfrac{1-\tan(x)^2}{1+\tan(x)^2}$

$$\frac{1-\tan(x)^2}{1+\tan(x)^2} = \frac{1-\frac{\sin(x)^2}{\cos(x)^2}}{1+\frac{\sin(x)^2}{\cos^2(x)}} = \frac{\frac{\cos^2(x)-\sin^2(x)}{\cos^2(x)}}{\frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)}} = \cos^2(x)-\sin^2(x)$$

$$= \cos(2x)$$

c) $\operatorname{Sin}(3x) = 3\operatorname{Sin}(x) - 4\operatorname{Sin}^3(x)$

$$\operatorname{Sin}(3x) = \operatorname{Sin}(x)\cdot\operatorname{Cos}(2x) + \operatorname{Sin}(2x)\cdot\operatorname{Cos}(x)$$

$$= \operatorname{Sin}(x)\left[\operatorname{Cos}^2(x) - \operatorname{Sin}^2(x)\right] + 2\operatorname{Sin}(x)\operatorname{Cos}^2(x)$$

$$= \operatorname{Sin}(x)\operatorname{Cos}^2(x) - \operatorname{Sin}^3(x) + 2\operatorname{Sin}(x)\operatorname{Cos}^2(x)$$

$$= 3\operatorname{Sin}(x)\operatorname{Cos}^2(x) - \operatorname{Sin}^3(x)$$

$$= 3\operatorname{Sin}(x)\left[1 - \operatorname{Sin}^2(x)\right] - \operatorname{Sin}^3(x)$$

$$= 3\operatorname{Sin}(x) - 4\operatorname{Sin}^3(x)$$

d) $\operatorname{Cos}(3x) = 4\operatorname{Cos}^3(x) - 3\operatorname{Cos}(x)$

$$\operatorname{Cos}(3x) = \operatorname{Cos}(2x)\cdot\operatorname{Cos}(x) - \operatorname{Sin}(2x)\cdot\operatorname{Sin}(x)$$

$$= \left[\operatorname{Cos}^2(x) - \operatorname{Sin}^2(x)\right]\operatorname{Cos}(x) - 2\operatorname{Sin}^2(x)\cdot\operatorname{Cos}(x)$$

$$= \operatorname{Cos}^3(x) - 3\operatorname{Sin}^2(x)\operatorname{Cos}(x)$$

$$= \operatorname{Cos}^3(x) - 3\operatorname{Cos}(x) + 3\operatorname{Cos}^3(x)$$

$$= 4\operatorname{Cos}^3(x) - 3\operatorname{Cos}(x)$$

P2 $\tan(4u) = \dfrac{4\tan(u) - 4\tan^3(u)}{1 - 6\tan^2(u) + \tan^4(u)}$

$$\tan(4u) = \frac{\sin(4u)}{\cos(4u)} = \frac{2\sin(2u)\cdot\cos(2u)}{\cos(2u)^2 - \sin(2u)^2}$$

$$= \frac{4\sin(u)\cos(u)\left[\cos(u)^2 - \sin(u)^2\right]}{\left[\cos^2(u) - \sin^2(u)\right]^2 - \left[2\sin(u)\cos(u)\right]^2}$$

$$= \frac{4\sin(u)\cos^3(u) - 4\sin^3(u)\cos(u)}{\cos^4(u) - 6\cos^2(u)\sin^2(u) + \sin^4(u)}$$

$\dfrac{\frac{1}{\cos^4(u)}}{\frac{1}{\cos^4(u)}}$

$$\frac{4\tan(u) - 4\tan^3(u)}{1 - 6\tan^2(u) + \tan^4(u)}$$

P3) Hint

probar $\tan(x+y) = \dfrac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$

$$\frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin(x)\cos(y)+\sin(y)\cos(x)}{\cos(x)\cos(y)-\sin(x)\sin(y)} \Big/ \frac{\frac{1}{\cos(x)\cos(y)}}{\frac{1}{\cos(x)\cos(y)}}$$

$$= \frac{\tan(x)+\tan(y)}{1-\tan(x)\cdot\tan(y)}$$

Con lo que se concluye

$\tan(\alpha) = \frac{a}{x}$, $\tan(\alpha+\beta) = \frac{a+b}{x}$, $\tan(2\alpha+\beta) = \frac{a+b+c}{x}$

$$\frac{a+b+c}{x} = \tan(2\alpha+\beta) \overset{\text{Hint}}{=} \frac{\tan(\alpha)+\tan(\alpha+\beta)}{1-\tan(\alpha)\cdot\tan(\alpha+\beta)} = \frac{\frac{a}{x}+\frac{a+b}{x}}{1-\frac{a}{x}\cdot\frac{a+b}{x}}$$

$$\Rightarrow \frac{(a+b+c)}{x} = \frac{\frac{2a+b}{x}}{\frac{x^2-a(a+b)}{x^2}} \Rightarrow (a+b+c) = \frac{(2a+b)x^2}{x^2-a(a+b)}$$

$$\Rightarrow x^2(a+b+c) - a(a+b)(a+b+c) = (2a+b)x^2$$

$$\Rightarrow x^2(c-a) = a(a+b)(a+b+c)$$

$$\Rightarrow \boxed{x = \sqrt{\frac{a(a+b)(a+b+c)}{|c-a|}}}$$

P4

a) $\sin^4(\alpha) + 4\cos^2(\alpha) = \left(1+\cos^2(\alpha)\right)^2$

$$\sin^4(\alpha) + 4\cos^2(\alpha) = \left(1-\cos^2(\alpha)\right)^2 + 4\cos^2(\alpha)$$

$$= 1 - 2\cos^2(\alpha) + \cos^4(\alpha) + 4\cos^2(\alpha)$$

$$= 1 + 2\cos^2(\alpha) + \cos^4(\alpha)$$

$$= \left(1+\cos^2(\alpha)\right)^2$$

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b) $\sqrt{\sin^4(\alpha) + 4\cos^2(\alpha)} = \sqrt{\left(1+\cos^2(\alpha)\right)^2} = 1+\cos^2(\alpha)$ / OBS: es positivo

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$$\left(\sqrt{\sin^4(\alpha)+4\cos^2(\alpha)} - \cos(2\alpha)\right)^2 = \left(1+\cos^2(\alpha) - \cos(2\alpha)\right)^2$$

$$= \left(1+\sin^2(\alpha)\right)^2$$

$$= 1 + 2\sin^2(\alpha) + \sin^4(\alpha)$$

$$= 1 + 2\sin^2(\alpha) + \left(1-\cos^2(\alpha)\right)^2$$

$$= 1 + 2\sin^2(\alpha) + 1 - 2\cos^2(\alpha) + \cos^4(\alpha)$$

$$= 4\sin^2(\alpha) + \cos^4(\alpha)$$