P1

a) $\cos(x) = \frac{\sqrt{3}}{2} \iff \boxed{x = 2k\pi \pm \frac{\pi}{3}}$, $k \in \mathbb{Z}$

b) $\sin(3x) = \frac{1}{2} \iff 3x = \frac{\pi}{6}(-1)^k + k\pi$, $k \in \mathbb{Z}$

$$\boxed{\iff x = \frac{\pi}{18}(-1)^k + k\frac{\pi}{3}}$$

c) $(\cos(x))^3 + (\sin(x))^3 + 1 - \frac{1}{2}\sin(2x) = 0$

$\iff (\cos(x) + \sin(x))(\cos^2(x) - \cos(x)\sin(x) + \sin^2(x)) + 1 - \sin(x)\cos(x) = 0$

$\Rightarrow (\cos(x) + \sin(x))(1 - \cos(x)\sin(x)) + (1 - \cos(x)\sin(x)) = 0$

$\iff \underbrace{(\cos(x) + \sin(x) + 1)}_{①}\underbrace{(1 - \cos(x)\sin(x))}_{①} = 0$

Sol ① $\cos(x) + \sin(x) = -1$

$\iff \frac{\sqrt{2}}{2}\cos(x) + \frac{\sqrt{2}}{2}\sin(x) = -\frac{\sqrt{2}}{2}$

$\iff \sin(\frac{\pi}{4})\cos(x) + \sin(x)\cos(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$\iff \sin(x + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2} \iff x + \frac{\pi}{4} = \frac{3\pi}{4}(-1)^k + k\pi$, $k \in \mathbb{Z}$

$\iff x = \frac{3\pi}{4}(-1)^k + k\pi - \frac{\pi}{4}$, $k \in \mathbb{Z}$

Sol II

$1 = \cos(x)\sin(x) \iff 2\sin(x)\cos(x) = 2$

$\iff \sin(2x) = 2$

$\iff$ Imposible, pues $|\sin(x)| \leq 1$

$\Rightarrow$ $\boxed{Sol = \frac{3\pi}{4}(-1)^k + k\pi - \frac{\pi}{4}, \quad k \in \mathbb{Z}}$

P2 a) $\sin(x) + \sqrt{3}\cos(x) = 1$

$\iff \frac{1}{2}\sin(x) + \cos(x)\cdot\frac{\sqrt{3}}{2} = \frac{1}{2}$ / $\cos(\frac{\pi}{3}) = \frac{1}{2}$, $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

$\iff \sin(x + \frac{\pi}{3}) = \frac{1}{2} \iff x + \frac{\pi}{3} = \frac{\pi}{6}(-1)^k + k\pi, \quad k \in \mathbb{Z}$

$\iff x = \frac{\pi}{6}(-1)^k + k\pi - \frac{\pi}{3}, \quad k \in \mathbb{Z}$

b) $2\cos^2(x) + 2\cos^2(x)\cos(2x) - 20\cos^2(\frac{x}{2} - \frac{\pi}{4})\sin^2(\frac{x}{2} - \frac{\pi}{4}) + 1 = 0$

$\iff 2\cos^2(x)(1 + \cos(2x)) - 5[\sin(x - \frac{\pi}{2})]^2 + 1 = 0$

$\iff 4\cos^2(x)(\sin^2(x)) - 5\cos^2(x) + 1 = 0$

$\iff 4\cos^2(x) - 4\cos^4(x) - 5\cos^2(x) + 1 = 0$

$\iff 4\cos^4(x) + \cos^2(x) - 1 = 0$

$\cos(x) = \frac{-1 \pm \sqrt{1+16}}{8} = \frac{-1-\sqrt{17}}{8} \vee \frac{-1+\sqrt{17}}{8}$ Ambos entre $[-1,1]$

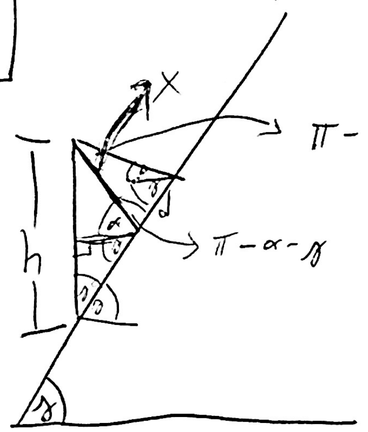
$\Rightarrow$ $\underline{Sol_{I}}$ $\quad X = 2K\pi \pm \text{ArcCos}\left(\frac{-1-\sqrt{17}}{8}\right)$

$\underline{Sol_{II}}$ $\quad X = 2K\pi \pm \text{ArcCos}\left(\frac{-1+\sqrt{17}}{8}\right)$

$\Big\}, K \in \mathbb{Z}$

$$Sol = Sol_{I} \cup Sol_{II}$$

P3



$\pi - (\pi-\alpha-\gamma) - (\gamma+\beta) = (\alpha-\beta)$

$\pi - \alpha - \gamma$

Usando Teo Seno

1) $$\frac{X}{\text{Sin}(\gamma+\beta)} = \frac{d}{\text{Sin}(\alpha-\beta)}$$

$$\Rightarrow X = \frac{\text{Sin}(\gamma+\beta)}{\text{Sin}(\alpha-\beta)} \cdot \quad X = \text{Sin}(\gamma+\beta)\,\text{Cosec}(\alpha-\beta)\,d$$

2) $$\frac{h}{\text{Sin}(\gamma+\alpha)} = \frac{X}{\text{Sin}(\frac{\pi}{2}-\beta)} = \frac{X}{\text{Cos}(\gamma)} \Rightarrow h = X \cdot \text{Sin}(\gamma+\alpha) \cdot \text{Sec}(\beta)$$

$$\Rightarrow \boxed{h = d \cdot \text{Sin}(\alpha+\beta)\,\text{Sin}(\gamma+\beta) \cdot \text{Sec}(\gamma) \cdot \text{Cosec}(\alpha+\beta)}$$

Creado con Tiny Scanner

P4

1) $\sin^2(y) = 1 \iff \sin(y) = 1$ o $\sin(y) = -1$

$\iff y = \frac{\pi}{2}(-1)^k + k\pi$ o $y = -\frac{\pi}{2}(-1)^k + k\pi$

Con $k \in \mathbb{Z}$

Opcionalmente se puede decir que $y = \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$

2) Como $\pi \cos(2x) = y \implies y \in [-\pi, \pi]$

O sea solo sirven $-\frac{\pi}{2}$ y $\frac{\pi}{2}$

$\implies \underbrace{\cos(2x) = -\frac{1}{2}}_{①}$ o $\underbrace{\cos(2x) = \frac{1}{2}}_{②}$

3) ① $2x = \pm \frac{2\pi}{3} + 2k\pi \iff \boxed{x = k\pi \pm \frac{\pi}{3}}$

② $2x = \pm \frac{\pi}{3} + 2k\pi \iff \boxed{x = k\pi \pm \frac{\pi}{6}}$

Solución