Pauta Auxiliar 10

P1

$$\lim \frac{\frac{4}{n} + \frac{5}{\sqrt{n}}\cdot \sin(n^n) + \frac{1-3n}{n+5}}{\frac{5^n}{n!} + \frac{(-1)^n}{n^2} + \frac{1}{1-\frac{2n!}{n^n}}}$$

$\frac{4}{n} \to 0$ (Nula $\frac{1}{n}$, por Acotada 4)

$\frac{5}{\sqrt{n}}\cdot \sin(n^n) \to 0$ (Nula $\frac{1}{\sqrt{n}}$, por Acotada $5\sin(n^n) \in [-1,1]$)

$\frac{1-3n}{n+5} \longrightarrow -3$ (Igual grado)

$\frac{5^n}{n!} \longrightarrow 0$ $\left(\frac{a^n}{n!} \to 0\right)$

$\frac{(-1)^n}{n^2} \to 0$ (Nula $\frac{1}{n^2}$ Acotada $(-1)^n$)

$2\frac{n!}{n^n} \to 0 \implies \frac{1}{1-\frac{2n!}{n^n}} \xrightarrow{\text{Algebra}} 1$

Usando Algebra de limites

$$\longrightarrow \frac{0+0+(-3)}{0+0+1} \longrightarrow \boxed{-3}$$

$\boxed{P2}$ a) $u_1 = 1 < 6$ ✓

Suponiendo $u_n < 6$

$$u_{n+1} = \frac{1}{3}u_n + 4 < \frac{1}{3}\cdot 6 + 4 = 6$$ ✓

b) Sea $n \in \mathbb{N}$

$$u_{n+1} - u_n = \frac{1}{3}u_n + 4 - u_n = 4 - \frac{2}{3}u_n > 4 - \frac{2}{3}\cdot 6 = 0$$

$$\Longrightarrow \boxed{u_n < u_{n+1}}$$

estrictamente creciente

c) El limite cumple que $l = \frac{1}{3}l + 4 \Longrightarrow \boxed{l = 6}$

Creado con Tiny Scanner

P3

a)

$$3 = \sqrt[n]{3^n} < \sqrt[n]{3^n + 2^n} < \sqrt[n]{3^n + 3^n} = \sqrt[n]{2} \cdot \sqrt[n]{3^n} =$$

$$= 3 \cdot \sqrt[n]{2}$$

$\longrightarrow$ Por Sandwich

$$\boxed{\sqrt[n]{3^n + 2^n} \longrightarrow 3}$$

b) $\sqrt[n]{n + \sin(n!)}$

$$\sqrt[n]{n-1} < \sqrt[n]{n + \sin(n!)} < \sqrt[n]{n+1}$$

$$\vee \qquad\qquad\qquad\qquad\qquad \vee$$

$$\cancel{\sqrt[n]{1}} \to 1 < \qquad \cancel{\cdot} \qquad < \sqrt[n]{2n} = \underbrace{\sqrt[n]{2}}_{\to 1} \cdot \underbrace{\sqrt[n]{n}}_{\to 1}$$

$\xrightarrow{\text{Sandwich}}$ $\boxed{\sqrt[n]{n + \sin(n!)} \longrightarrow \underline{1}}$

c)

$$\frac{-3n^2 - 2}{2n^2 + n + 1} \longrightarrow \boxed{-\frac{3}{2}}$$

Cociente igual grado.

d) $$\frac{6^n}{8^n+3^n}$$

$$0 \leq \frac{6^n}{8^n+3^n} \leq \left(\frac{6}{8}\right)^n \longrightarrow 0$$

$$\Longrightarrow \frac{6^n}{8^n+3^n} \longrightarrow 0$$

e) $$\frac{1}{n} \sum_{k=0}^{n} \left(\frac{k}{n}\right)^3 = \frac{1}{n^4} \cdot \sum_{k=0}^{n} k^3$$

$$= \frac{1}{n^4} \cdot \frac{\cancel{n^2}(n-1)^2}{\cancel{n^4}} = \frac{n^2 - 2n+1}{n^2} \longrightarrow 1$$

f) $$\frac{4n+(-1)^n}{n-\cos(n!)} < \frac{4n+1}{n-1} \longrightarrow 4$$

$$\frac{4n+(-1)^n}{n-\cos(n!)} > \frac{4n-1}{n+1} \longrightarrow 4$$

Sandwich

$$\frac{4n+(-1)^n}{n+\cos(n!)} \to 4$$

g) $\sqrt[n]{4^n + n2^n + 4\cdot 3^n}$

$$4 \leq \sqrt[n]{4^n} \leq \sqrt[n]{4^n + n2^n + 4\cdot 3^n} < \sqrt[n]{3\cdot 4^n} = \sqrt[n]{3}\cdot 4$$

$$\Rightarrow \sqrt[n]{4^n + n\cdot 2^n + 4\cdot 3^n}$$

h) $$\sum_{k=1}^{n} \frac{n}{n^2+k} < \sum_{k=1}^{n} \frac{1}{n} = \underline{1} \qquad \frac{n}{n^2} = \frac{n}{n^2+0}$$

$$\sum_{k=1}^{n} \frac{n}{n^2+k} > \sum_{k=1}^{n} \frac{n}{n^2+n} = \frac{n^2}{n^2+n} \longrightarrow 1$$

$\Rightarrow$ Por Sandwich Converge a $\underline{1}$