Pauta Auxiliar 13

$\boxed{P1}$ $\lim_{x\to\infty} \dfrac{x+\frac{1}{x^2}}{2x^2+\sqrt{x}+\frac{1}{x}} \overset{\frac{1/x^2}{1/x^2}}{=} \lim_{x\to\infty} \dfrac{\frac{1}{x}+\frac{1}{x^4}}{2+\frac{1}{x^{\frac{3}{2}}}+\frac{1}{x^3}} \overset{\text{Usando álgebra de límites}}{=} \dfrac{0+0}{2+0+0}=0$

b) $$\lim_{x\to\infty} \frac{x\ln\left(\cos\left(\frac{1}{2x}\right)\right)}{\sin\left(\frac{1}{x}\right)} = \lim_{x\to\infty} x\cdot \frac{\ln\left(\cos\left(\frac{1}{2x}\right)\right)}{\cos\left(\frac{1}{2x}\right)-1}\cdot\frac{\cos\left(\frac{1}{2x}\right)-1}{\sin\left(\frac{1}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{\ln\left(\cos\left(\frac{1}{2x}\right)\right)}{\cos\left(\frac{1}{2x}\right)-1}\cdot\frac{\cos\left(\frac{1}{2x}\right)-1}{\left(\frac{1}{2x}\right)^2}\cdot\frac{\left(\frac{1}{2x}\right)\cdot\frac{1}{2x}\cdot x}{\sin\left(\frac{1}{x}\right)}$$

$$= \lim_{x\to\infty} \underbrace{\frac{\ln\left(\cos\left(\frac{1}{2x}\right)\right)}{\cos\left(\frac{1}{2x}\right)-1}}_{1}\cdot\underbrace{\frac{\cos\left(\frac{1}{2x}\right)-1}{\left(\frac{1}{2x}\right)^2}}_{-\frac{1}{2}}\cdot\underbrace{\frac{\frac{1}{x}}{\sin\left(\frac{1}{x}\right)}}_{1}\cdot\frac{1}{4}$$

$$= \boxed{-\frac{1}{8}}$$

$$\lim_{x\to\infty} x\ln\left(\frac{x+a}{x-a}\right) = \lim_{x\to\infty} \frac{\ln\left(\frac{x+a}{x-a}\right)}{\left(\frac{x+a}{x-a}\right)-1} \cdot \frac{\left(\frac{x+a}{x-a}\right)-1}{1}\, x$$

$$= \lim_{x\to\infty} \underbrace{\frac{\ln\left(\frac{x+a}{x-a}\right)}{\frac{2a}{x-a}}}_{\to 1} \cdot \underbrace{\frac{2a}{x-a}\, x}_{\to 2a}$$

$$= \boxed{2a}$$

P4 a)

$f(x) = x^2 \sin\left(\frac{1}{x}\right)$ $\quad$ Dom $= \mathbb{R} - \{0\}$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \underbrace{x^2}_{\text{Nula}} \cdot \underbrace{\sin\left(\frac{1}{x}\right)}_{\text{Acotada}} = 0 \Longrightarrow \text{No hay Verticales}$$

$\sin(y) \in [-1, 1]$

$$\lim_{x\to\infty} x^2 \sin\left(\frac{1}{x}\right) = \lim_{x\to\infty} x \cdot \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \infty$$

Similar con $-\infty$ $\Longrightarrow$ No hay horizontales

$$\lim_{x\to\infty} x \cdot \sin\left(\frac{1}{x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = 1 = m_{\infty}$$

$$\lim_{x\to-\infty} x \sin\left(\frac{1}{x}\right) = 1 = m_{-\infty}$$

$$\lim_{x\to\infty} x^2 \sin\left(\frac{1}{x}\right) - x$$

(Solo se me ocurrio con l'hopital)

$$= \lim_{u\to 0^+} \frac{1}{u^2} \sin(u) - \frac{1}{u}$$

$u = \frac{1}{x}$

$$= \lim_{u\to 0^+} \frac{\sin(u) - u}{u^2} \overset{L'H}{=} \lim_{u\to 0^+} \frac{\cos(u) - 1}{2u} \overset{\text{conocido}}{=} 0$$

El hacia $\lim -\infty$ es Analogo

$\Rightarrow$ Tiene la Asintota oblicua a ambos infinitos de la forma $\boxed{y = x}$

b) $\lim \frac{x^2\left(e^{\frac{2}{x}}-1\right)}{x+1} = f(x)$

Dom : $\mathbb{R} - \{0,-1\}$

$\lim_{x\to 0^-} f(x) = 0 \frac{\left(e^{-\infty}-1\right)}{1} = 0$ (con $e^{-\infty} \to 0$)

$\lim_{x\to 0^+} f(x) = \frac{0\left(e^{\infty}\right)}{1} = \infty$ exponencial gana

$\Longrightarrow$ $x=0$ Asíntota Vertical

$\lim_{x\to -1^+} \frac{x^2\left(e^{\frac{2}{x}}-1\right)}{x+1} = \frac{1\ \left(e^{2}-1\right)}{0^+} = \infty \rightarrow x=1$ Asíntota Vertical

Horizontal

$\lim_{x\to\infty} \frac{x^2\left(e^{\frac{2}{x}}-1\right)}{x+1} = \lim_{x\to\infty} \frac{x^2}{x+1} \cdot \left(e^{\frac{2}{x}}-1\right)$

$= \lim_{x\to\infty} \frac{x}{x+1} \cdot \frac{\left(e^{\frac{2}{x}}-1\right)}{\frac{2}{x}} \cdot \frac{2}{1} = \boxed{2}$

($\frac{x}{x+1} \to 1$, $\frac{e^{2/x}-1}{2/x} \to 1$, $\to 2$)

$y=2$ Horizontal al ∞

Para $-\infty$ es Análogo $\Longrightarrow$ $\boxed{y=2 \text{ para ambos}}$

Por lo Tanto no hay Oblicuas

c) $(20-e^{-x})(x+5)$

$Dom = \mathbb{R} \Rightarrow$ No hay vertical

$$\lim_{x\to\infty} (20-e^{-x})(x+5) = \infty$$

Similar

$x \to -\infty \Rightarrow$ No hay horizontales

$$\lim_{x\to-\infty} (20-e^{-x})(x+5) = (-\infty)(\infty) = -\infty$$

$$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} (20-e^{-x})\left(\frac{\cancel{x}}{\cancel{x}}+\frac{5}{x}\right)$$

$$= \lim_{x\to\infty} (20-0)(1+0) = 20 = m_\infty$$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} (20-\infty)(1+0) = -\infty \text{ No hay}$$

$$\lim_{x\to\infty} f(x) - m_\infty x = \lim_{x\to\infty} \cancel{20x}+100-e^{-x}x-e^{-x}5-\cancel{20x}$$

$$= \boxed{100}$$

$\Rightarrow$ hacia el ∞ hay asíntota $\boxed{y = 20x+100}$