Pauta Auxiliar 14

$\boxed{P1}$ a) $\displaystyle \text{Cosh}(x) = \frac{e^{x}+e^{-x}}{2}$

$$\boxed{\text{Sinh}(x) = \frac{e^{x}-e^{-x}}{2}}$$

$$\text{Cosh}'(x) = \lim_{h\to 0} \frac{\text{Cosh}(x+h)-\text{Cosh}(x)}{2h} = \lim_{h\to 0} \frac{e^{x+h}+e^{-x-h}-e^{x}-e^{-x}}{2h}$$

$$= \lim_{h\to 0} \frac{\dfrac{e^{x+h}-e^{x}}{h} + \dfrac{e^{-x-h}-e^{-x}}{h}}{2}$$

$$= \lim_{h\to 0} \frac{\underbrace{\dfrac{e^{x+h}-e^{x}}{h}}_{\to e^{x}} - \underbrace{\dfrac{e^{-x-h}-e^{-x}}{-h}}_{\to e^{-x}}}{2} = \frac{e^{x}-e^{-x}}{2}$$

$$= \text{Sinh}(x)$$

b) $(x\sin(x))' = \lim_{h\to 0} \dfrac{(x+h)\sin(x+h) - x\sin(x)}{h}$

$= \lim_{h\to 0} \dfrac{x(\sin(x+h) - \sin(x))}{h} + \sin(x+h)$

$= \lim_{h\to 0} \dfrac{x(\sin(x)\cdot\cos(h) + \sin(h)\cdot\cos(x) - \sin(x))}{h} + \sin(x+h)$

$= \lim_{h\to 0} x\sin(x)\left[\dfrac{\cos(h)-1}{h}\right] + x\cos(x)\cdot\dfrac{\sin(h)}{h} + \sin(x+h)$

$\dfrac{\cos(h)-1}{h} \to 0 \qquad \dfrac{\sin(h)}{h} \to 1 \qquad \sin(x+h) \to \sin(x)$

$= \cancel{x\sin(x)}\cdot 0 + \boxed{x\cos(x) + \sin(x)}$

c) $(\tan(x))' = \lim_{h\to 0} \dfrac{\dfrac{\sin(x+h)}{\cos(x+h)} - \dfrac{\sin(x)}{\cos(x)}}{h} = \lim_{h\to 0} \dfrac{\dfrac{\sin(x)\cdot\cos(h) + \sin(h)\cdot\cos(x)}{\cos(x+h)} - \dfrac{\sin(x)}{\cos(x)}}{h}$

$= \lim_{h\to 0} \sin(x) \dfrac{\left[\dfrac{\cos(h)}{\cos(x+h)} - \dfrac{1}{\cos(x)}\right]}{h} + \dfrac{\sin(h)}{h}\cdot\dfrac{\cos(x)}{\cos(x+h)}$

$$= \lim_{h\to 0} \frac{\sin(x)}{h}\left[\frac{\cos(h)\cos(x) - \cos(x+h)}{\cos(x+h)\cdot\cos(x)}\right] + \frac{\sin(h)}{h}\cdot\frac{\cos(x)}{\cos(x+h)}$$

$$= \lim_{h\to 0} \sin(x)\left[\frac{\sin(x+h)}{\underbrace{\cos(x+h)\cdot\cos(x)}_{\to \cos(x)}}\cdot\underbrace{\frac{\sin(h)}{h}}_{\to 1}\right] + \underbrace{\frac{\sin(h)}{h}}_{\to 1}\cdot\underbrace{\frac{\cos(x)}{\cos(x+h)}}_{\to 1}$$

$$= \frac{\sin(x)^2}{\cos^2(x)} + 1 = \frac{1}{\cos^2(x)} = \boxed{\sec^2(x)}$$

$\boxed{P2}$

a) $$(\tanh(x))' = \left(\frac{\sinh(x)}{\cosh(x)}\right)' \overset{\text{División}}{=} \frac{(\sinh(x))'\cosh(x) - (\cosh(x))'\sinh(x)}{(\cosh(x))^2}$$

$$= \frac{\cosh(x)^2 - \sinh(x)^2}{\cosh(x)^2} = \frac{1}{\cosh(x)^2} = \boxed{\operatorname{sech}(x)^2}$$

b) $$\left(x^3(x^2-1)^2\right)' = (x^3)'(x^2-1)^2 + x^3\left[(x^2-1)^2\right]'$$

$$= 3x^2(x^2-1)^2 + x^3\cdot 2(x^2-1)(x^2-1)'$$

$$= 3x^2(x^2-1)^2 + x^3\cdot 2(x^2-1)2x$$

c) $$\left(\frac{2x^4}{6^2-x^2}\right)' = \frac{(2x^4)'(6^2-x^2) - 2x^4(6^2-x^2)'}{(6^2-x^2)^2}$$

$$= \boxed{\frac{8x^3(6^2-x^2) - 2x^4(-2x)}{(6^2-x^2)^2}}$$

P3

a) $$(\cos(x))^{(n)} = \begin{cases} (\cos(x))^{(4n)} = \cos(x) \\ (\cos(x))^{(4n+1)} = -\sin(x) \\ (\cos(x))^{(4n+2)} = -\cos(x) \\ (\cos(x))^{(4n+3)} = \sin(x) \end{cases}$$

Otra forma $$\boxed{(\cos(x))^{(n)} = \sin\left(x + (n-1)\frac{\pi}{2}\right)}$$

(No es obvio, prefiero deducir la primera)

b) $(e^{5x}\ln(x))^{(n)}$ = Usar leibnitz

$$(e^{5x})^{(k)} = e^{5x}\cdot 5^k, \quad k \geq 0$$

$$(\ln(x))^{(k)} = \left(\frac{1}{x}\right)^{(k-1)}$$

$$\left(\frac{1}{x}\right)^{(l)} = \left(x^{-1}\right)^{(l)} = x^{-l-1} \cdot l!\,(-1)^{l} \quad , \; l \geq 0$$

$$\Rightarrow \left(\ln(x)\right)^{(k)} = \left(\frac{1}{x}\right)^{(k-1)} = x^{-k} \cdot (k-1)!\,(-1)^{k-1} \quad , \; k \geq 1$$

(La formula no sirve para $k=0$)

$$\left(e^{sx} \cdot \ln(x)\right)^{(N)} = \sum_{k=0}^{N} \binom{N}{k} \cdot \left(e^{sx}\right)^{(k)} \cdot \left(\ln(x)\right)^{(N-k)}$$

$$= \binom{N}{N} \left(e^{sx}\right)^{(N)} \cdot \ln(x)^{(0)} + \sum_{k=1}^{N-1} \binom{N}{k} \left(e^{sx}\right)^{(k)} \left(\ln(x)\right)^{(N-k)}$$

Ahora vale la formula

$$= \boxed{e^{sx} \cdot s^{N} \cdot \ln(x) + \sum_{k=1}^{N-1} \binom{N}{k} e^{sx} \cdot s^{k} \cdot x^{-N+k} (N-k-1)!\,(-1)^{N-k-1}}$$

Dejar Así

c) $\left(\sin(x)\cdot x^3\right)^{(n)}$

$$\left(x^3\right)^{(k)} = \begin{cases} x^3 & k=0 \\ 3x^2 & k=1 \\ 6x & k=2 \\ 6 & k=3 \\ 0 & k\geq 4 \end{cases}$$

$$\left(\sin(x)\right)^{(k)} = \sin\left(x+\frac{k\pi}{2}\right)$$

Por Leibnitz

$$= \sum_{k=0}^{n} \binom{n}{k} \left(x^3\right)^{(k)} \cdot \left(\sin(x)\right)^{(n-k)}$$

$$\left(\sin(x)\cdot x^3\right)^{(n)} = 1\cdot x^3\cdot \sin\left(x+n\frac{\pi}{2}\right) + n\cdot 3x^2\cdot \sin\left(x+\frac{(n-1)\pi}{2}\right)$$

$$+ \frac{n\cdot(n-1)}{2}\cdot 6x\cdot \sin\left(\frac{(n-2)\pi}{2}\right) +$$

$$\frac{n\cdot(n-1)(n-2)}{\cancel{3}\cdot 2}\cdot 6\cdot \sin\left(\frac{(n-3)}{2}\pi\right)$$

Dejar con el remplazo

P4 ⓐ $f \circ f^{-1}(x) = x$

$$\Rightarrow \left(f \circ f^{-1}(x)\right)' = 1$$

$$\Rightarrow f'(f^{-1}(x)) \cdot f^{-1\,\prime}(x) = 1$$

$$\Rightarrow \boxed{\left(f^{-1}(x)\right)' = \frac{1}{f'(f^{-1}(x))}} \quad \text{Usar}$$

$\mathrm{Arcos}(x) = f^{-1}(x)$ para $f(x) = \mathrm{Cos}(x)$

$f'(x) = -\mathrm{Sin}(x)$

$$\Rightarrow \left(\mathrm{Arcos}(x)\right)' = \frac{1}{-\mathrm{Sin}(\mathrm{Arcos}(x))} = \cancel{\frac{1}{\ldots}} = \frac{1}{-\sqrt{1-\mathrm{Cos}^2(\mathrm{Arcos}(x))}}$$

$$= \boxed{\frac{1}{-\sqrt{1-x^2}}}$$

b) $(\tan(x))' = \sec^2(x)$

recordar

$1+\tan^2(x) = \sec^2(x)$

$(\operatorname{ArcTan}(x))' = \sec^2(\operatorname{ArcTan}(x)) = (1+\tan^2(\operatorname{ArcTan}(x)))^{-1}$

$= (1+x^2)^{-1}$

$= \boxed{\dfrac{1}{1+x^2}}$

c) $\log_2(x) = f^{1}(x)$ para $2^x = f(x)$

$(2^x)' = (e^{x\ln(2)})' = e^{x\ln(2)} \cdot (\cancel{x\ln(2)})'$

$= 2^x \cdot \dfrac{2}{x}$

$(\log_2(x))' = \dfrac{1}{2^{\log_2(x)} \cdot 2} \cdot \log_2(x) = \dfrac{\log_2(x)}{2x}$

i) $\log_2(x) = f^{-1}(x)$ para $f(x) = 2^x$

$$(2^x)' = \left(e^{\ln(2^x)}\right)' = \left(e^{x\cdot\ln(2)}\right)' = e^{x\ln(2)}\cdot(x\ln(2))'$$

$$= 2^x\cdot\ln(2)$$

$$(\log_2(x))' = \frac{1}{2^{\log_2(x)}\cdot\ln(2)} = \boxed{\frac{1}{x\ln(2)}}$$