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Taylor en Torno a $x_0 = 0$ (Mc Laurin)

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k = f(0) + f'(0) \cdot x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3} x^3 \ldots$$

$$+ \frac{f^n(0)}{n!} \cdot x^n$$

$f(x) = e^x$ $f(0) = 1$ $P_n^f(x) = \sum_{k=0}^{n} \frac{f_0^{(k)}}{\cancel{k}!} x^{\cancel{k}} =$

$f'(x) = e^x$ $f'(0) = 1$

$$\sum_{k=0}^{n} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!}$$

$f^{(n)}(x) = e^x$ $f^{(n)}(0) = 1$

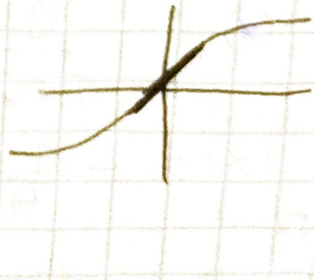
Para $f(x) = \operatorname{sen} x$, $f(0) = 0$

$f'(x) = \cos(x)$; $f'(0) = 1$ $P_n(x) = 0 + x + \frac{0x^2}{2!} - \frac{x^3}{3!}$

$f''(x) = -\operatorname{sen} x$; $f''(0) = 1$

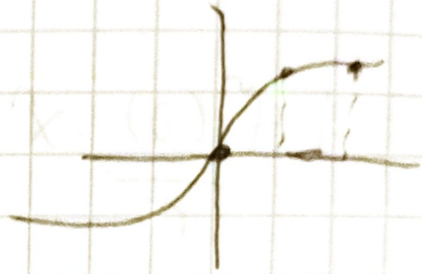
$f'''(x) = -\cos x$, $f'''(0) = -1$

$f''''(x) = \operatorname{sen} x$



$= \sum_{n=0}^{n} (-1)^{n} \frac{x^{2n+1}}{(2n+1)!}$ $\qquad \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k$

$P_n(x) = \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!}$



$f(x)=\sqrt{x}$ en torno a $x_0=4$

$\to f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2$

$+ \frac{f'''(x_0)}{3!}(x-x_0)^3 + \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$

$4 \cdot 4$

$(2^2)^3 \to 2^6$

$f(x)=\sqrt{x} \to f(4)=\sqrt{4}=2$ $\qquad \sqrt[2]{4^3} \to 2^2 2^2 2^2$

$\frac{1}{2}x^{-\frac{1}{2}}$ $f'(x) = \frac{1}{2\sqrt{x}} \to f'(4) = \frac{1}{4}$

$f''(x) = -\frac{1}{4}x^{-3/2} = -\frac{1}{4x^{3/2}} = f''(4) = -\frac{1}{32}$

$\sqrt{x}$

$P(x) = 2 + \frac{1}{4}(x-4) - \frac{\frac{1}{32}}{2!}(x-4)^2$

$f(x) = x \ln(1+x)$ en torno a $x_0 = 0$ de orden 3

$f(x) = x \ln(1+x) \rightarrow f(0) = 0$

$f'(x) = \ln(1+x) + \dfrac{x}{1+x} \rightarrow f'(0) = 0$

$f''(x) = \dfrac{1}{1+x} + \dfrac{1}{(1+x)^2}, \quad f''(0) = 2$

$f'''(x) = \dfrac{-1}{(1+x)^2} - \dfrac{2(1+x)}{(1+x)^4} = -\dfrac{1}{(1+x)^2} + \dfrac{2}{(1+x)^3}, \quad f'''(0) = -3$

$P_3(x) = \cancel{\dfrac{x^3}{3!}}$?? $\quad x^2 - \dfrac{1}{3}x^3$

$f(x) = \text{sen } x$ en torno $x_0 = \dfrac{\pi}{2}$ y orden 4

$$P_4^{\text{sen}(x)}(x) = f\left(\tfrac{\pi}{2}\right) + \frac{f'\left(\tfrac{\pi}{2}\right)}{1}\left(x-\tfrac{\pi}{2}\right) + \frac{f''(\pi/2)}{2!}(x-\pi/2)^2$$

$$+ \frac{f'''\left(\tfrac{\pi}{2}\right)}{3!}\left(x-\tfrac{\pi}{2}\right)^3 + \frac{f^{(4)}\left(\tfrac{\pi}{2}\right)}{4!}\left(x-\tfrac{\pi}{2}\right)^4$$

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Regla de Leibnitz

$$(f \pm g)^{(n)} = f^{(n)} \pm g^{(n)}$$

$$\left(f(x)\, g(x)\right)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f(x)^{(k)}\, g(x)^{(n-k)}$$

$$(xe^x)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} x^{(k)} (e^x)^{(n-k)} =$$

$$= \binom{n}{0} x^{(0)} \cdot (e^x)^{(n)} + \binom{n}{1} x' (e^x)^{n-1}$$

$$+ \binom{n}{2} x'' (e^x)^{(n-2)} \to 0$$

$$(xe^x)^{(n)} = xe^x + ne^x$$

$$(xe^x)^{(n)} = e^x(x+n)$$

$$(xe^x)^{(n)}(0) = n$$

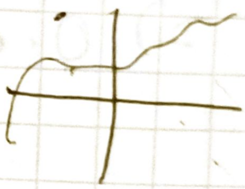
$(x^2 \operatorname{sen}(x))^{(n)} = \sum \binom{n}{k} (x^2)^{(k)} \operatorname{sen}(x)^{(n-k)}$

$(\operatorname{sen} x)^{(k)} = \operatorname{sen}\left[x + \frac{k\pi}{2}\right]$ $\qquad \frac{n(n-1)}{\cancel{2}} \cancel{2}$

Ej $x^2 + 3xy^2 - \cos(y) = 0$ en $P(0, \frac{\pi}{2})$

$x^2 + 3x f^3_{(x)} - \cos(f(x))$. $\qquad y' = f'_{(x)}$

$((f(x))^3)' = 3f(x)^2 \cdot f'(x)$ $\qquad \infty$



$()'$

$$\boxed{2x + 3(y^3 + 3xy^2y') + \operatorname{sen}(y)\, y' = 0}$$

$y' = -\frac{2x + 3y^3}{9xy^2 + \operatorname{sen} y}$ en $P(0, \pi/2)$

$x^2 + y^2 = R^2$

$2x + 2yy' = 0$

$\boxed{y' = \frac{-x}{y}}$

$y'_{(0,\frac{\pi}{2})} = \frac{-\frac{3\pi^3}{8}}{1} = -\frac{3}{8}\pi^3$

Recta tgte

$(y - \frac{\pi}{2}) = -\frac{3}{8}\pi^{-3}(x - 0)$

$y = \sqrt{2^2 - x^2}$

$y' = \frac{-2x}{2\sqrt{R - x}} = \frac{-x}{y}$

Javier Santidrian

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$$\lim_{x\to B} \frac{f(x)}{g(x)} \xrightarrow{0 \cdot \infty} 0\cdot\infty \; ; \; \lim_{x\to B} \frac{f'(x)}{g'(x)} = L \Rightarrow \lim_{x\to B} \frac{f(x)}{g(x)} = L$$

con $B = \{-\infty, \infty, x_0, x_0^+, x_0^-\}$

Formas de L'Hopital

1) $\frac{0}{0}, \frac{\infty}{\infty}$

2) $0 \cdot \infty = \frac{0}{1/0} = \frac{\infty}{\infty}$

2) $0 \cdot \infty = \frac{0}{1/\infty} = \frac{0}{0}$

3) $1^{\infty} \xrightarrow{\ln} \infty \ln(1) = \infty \cdot 0$

Si $1^{\infty} \to L$

Si $\ln f(x) \to L$

$\Rightarrow f(x) \to e^{L}$

$$\lim_{x\to\infty} \frac{x^4}{x^4} = \lim \frac{4x^3}{1} \to \infty$$

$\frac{x}{x^4}$ $\quad e^0 = 1$

$e^0 \; 1 \cdot 1 \quad \ln(1) = 0$

$$\lim_{x\to 0} \frac{e^{\operatorname{sen} x} - 1}{x} \overset{L'H}{=} \lim_{x\to 0} \frac{e^{\operatorname{sen} x} \cdot \cos x}{1} = 1$$

$$\lim_{x\to 0} \frac{\operatorname{sen} x - x}{x^2} \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x - 1}{2x} = 0$$

$\to \frac{-\operatorname{sen} x}{2} \to \frac{0}{2} = 0$

$$\lim_{x\to 0^+} \frac{\text{sen}^2(\sqrt{x})}{\ln\cos(\sqrt{x})} \overset{L'H}{=} \lim_{x\to 0^+} \frac{2\,\cancel{\text{sen}(\sqrt{x})}\cdot\cos(\sqrt{x})\cdot\cancel{\frac{1}{2\sqrt{x}}}}{\frac{1}{\cos(\sqrt{x})}\cdot -\cancel{\text{sen}(\sqrt{x})}\cdot\cancel{\frac{1}{2\sqrt{x}}}}$$

$$= \frac{2}{-1} = -2$$

$$\lim_{h\to 0} \frac{\frac{\text{sen}\,h}{h}-1}{h} = \lim_{h\to 0} \frac{\text{sen}\,h - h}{h^2}$$

$$= \lim_{h\to 0} \frac{\cosh - 1}{2h} = 0$$

$\ln g \to L$
$g \to e^L$

$$\lim_{x\to\infty} \left(e^{2x}+e^{x}+x\right)^{1/x}$$

$(\infty)^0 \xrightarrow{\ln} 0 \ln\infty$
$= 0\cdot\infty =$

e^2

$$\lim_{x\to\infty} \exp\left(\frac{\ln(e^{2x}+e^{x}+x)}{x}\right) \overset{L'H}{\to} \frac{2e^{2x}+e^{x}+1}{e^{2x}+e^{x}+x}$$

$$\to \frac{4e^{2x}+e^{x}}{2e^{2x}+e^{x}+1} \to \frac{8e^{2x}+e^{x}}{4e^{2x}+e^{x}} \to \frac{\cancel{e^{2x}}(8+\cancel{e^{-x}})}{\cancel{e^{2x}}(4+\cancel{e^{-x}})} \to \frac{8}{4} = 2$$